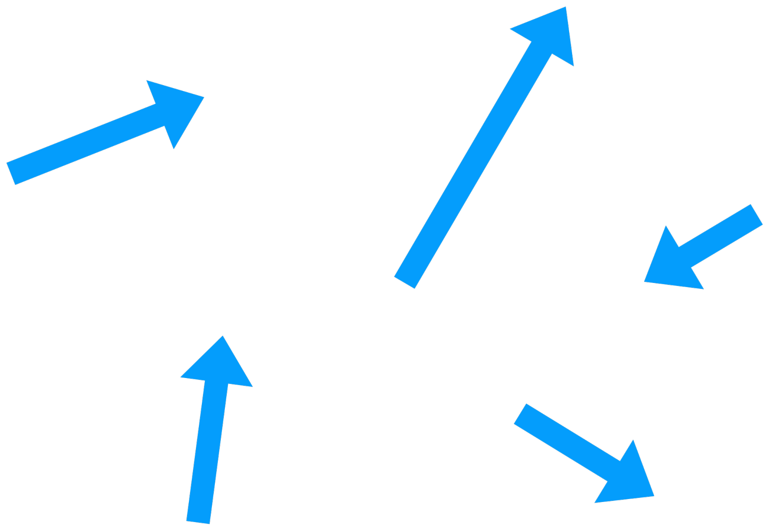
# Vectors

Objects with **magnitude (length)** and **direction $\boldsymbol{\theta}$**



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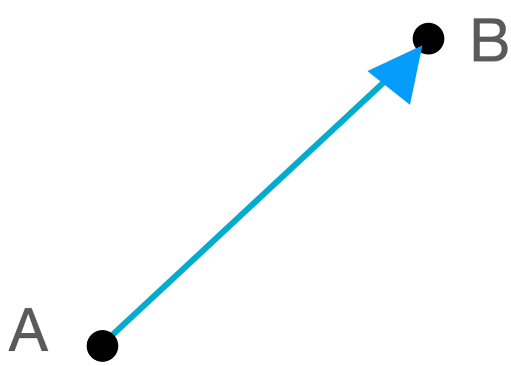
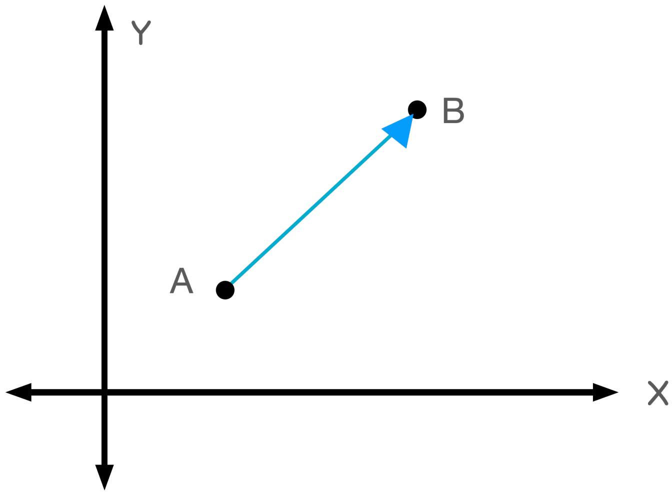
**Geometric Description of a Vector $\vec{v}$**

$$\vec{\boldsymbol{v}} = \overrightarrow{\boldsymbol{AB}}$$



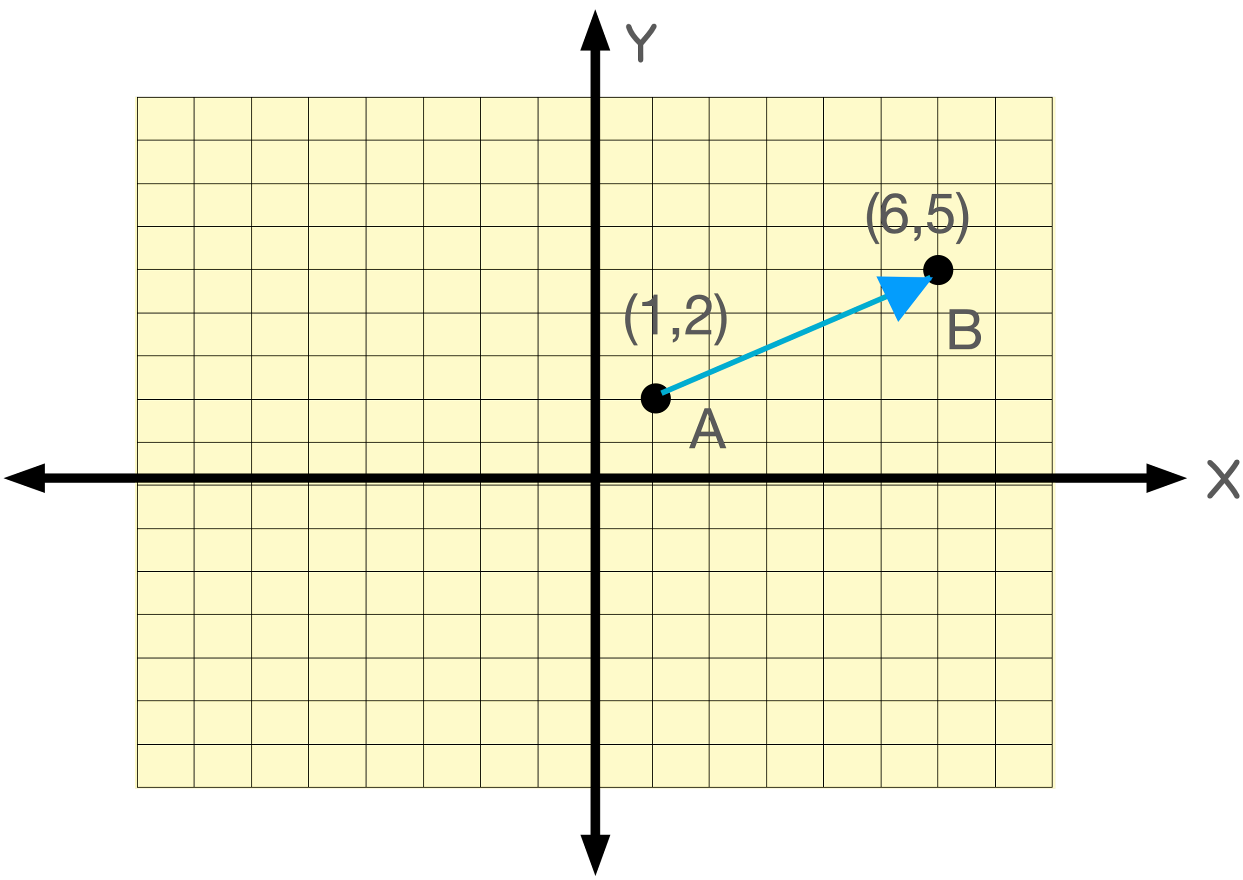

We call A an initial point and B a terminal point with the magnitude represented as $|\vec{v}|$ and direction represented by an angle $\theta$.

**Vectors in the Cartesian Coordinate Plane or Rectangular Coordinate System**

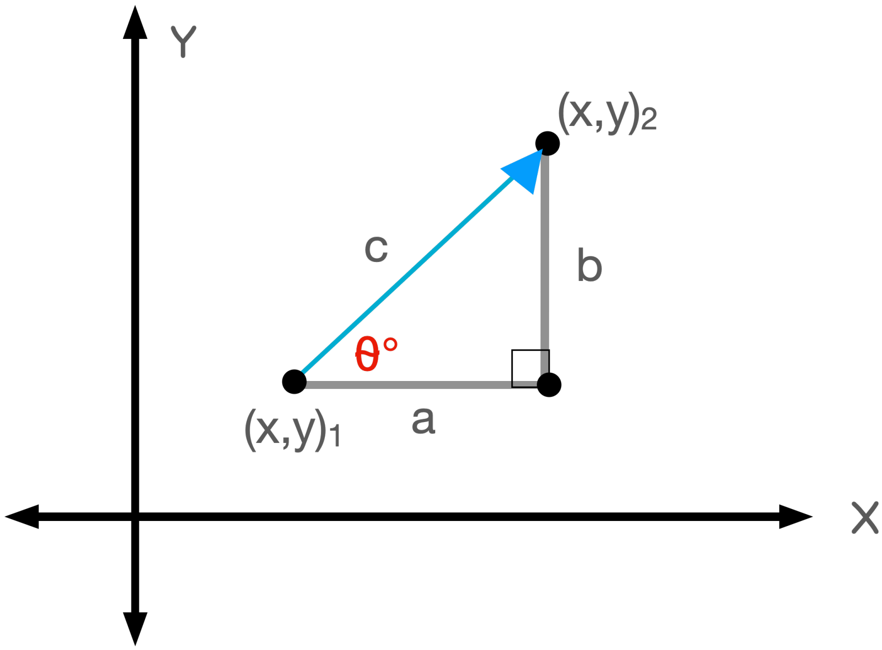



We can plot points A and B to draw a vector in cartesian coordinates.

**Example**- Let $A = (1,2)$ and $B = (6,5)$ to determine vector $\vec{\boldsymbol{v}} = \overrightarrow{\boldsymbol{AB}}$



We can determine the **magnitude (length)** of our vector and it's **direction $\boldsymbol{\theta}$** by considering two quantities. The horizontal component of a vector and the vertical component of a vector.




**Horizontal Component**

$$a = x_2 - x_1$$

**Vertical Component**

$$b = y_2 - y_1$$

**Pythagorean Theorem**

$$c^2 = a^2 + b^2$$

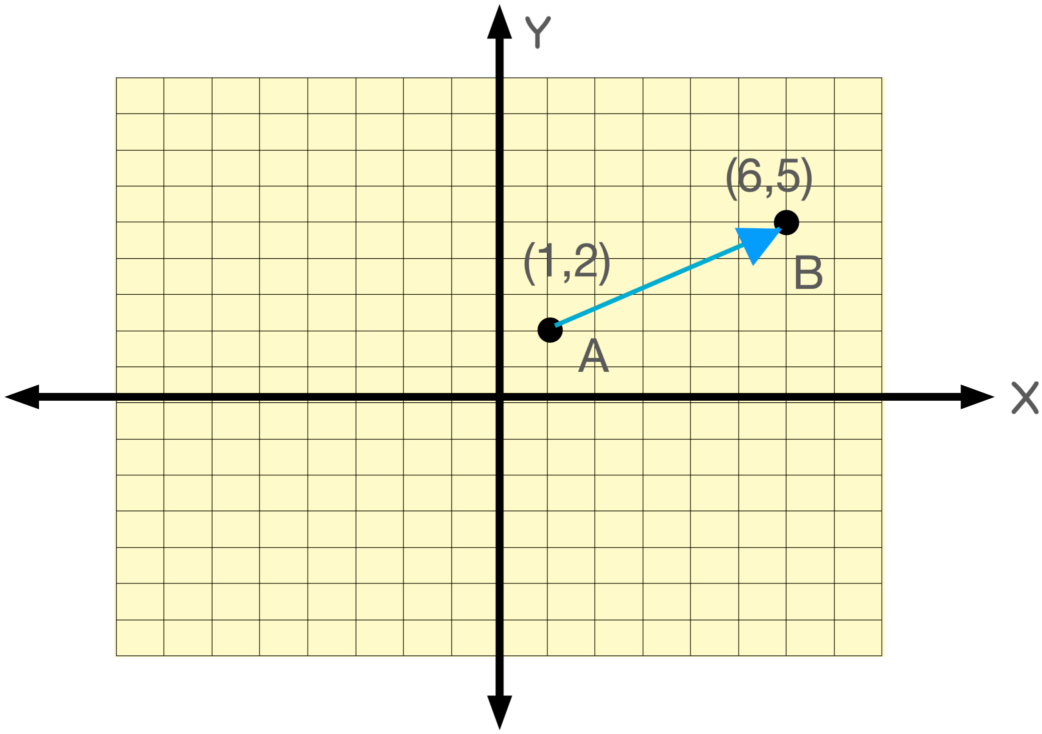
**Right Angle Geometry**

$$tan(\theta) = \frac{b}{a}$$

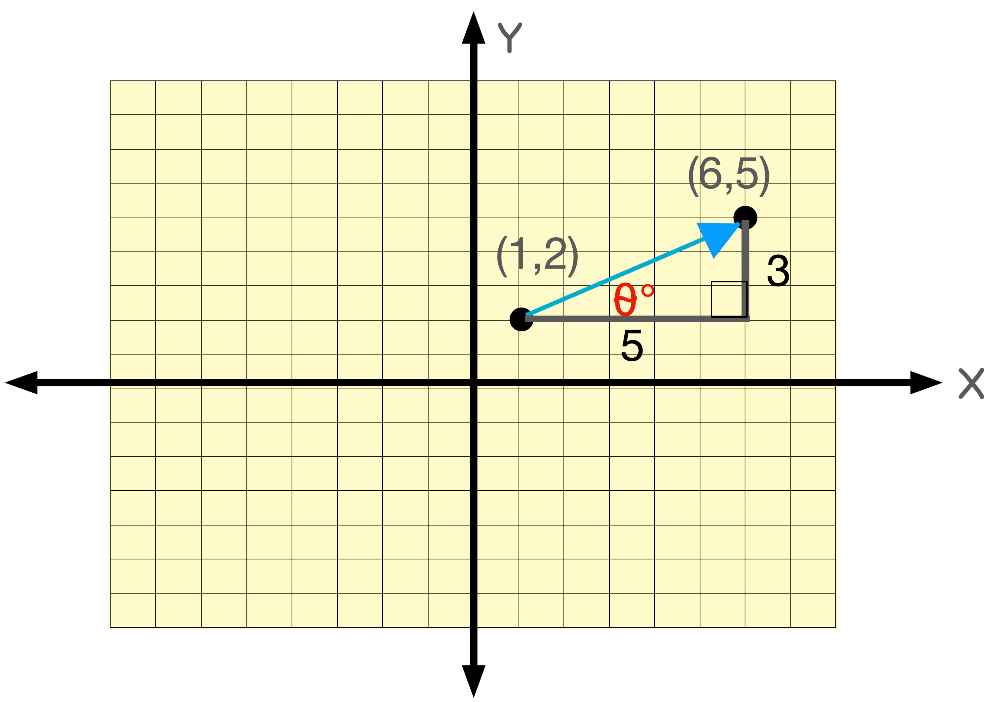
Component From of Vector $\vec{\boldsymbol{v}} = \overrightarrow{\boldsymbol{AB}}$

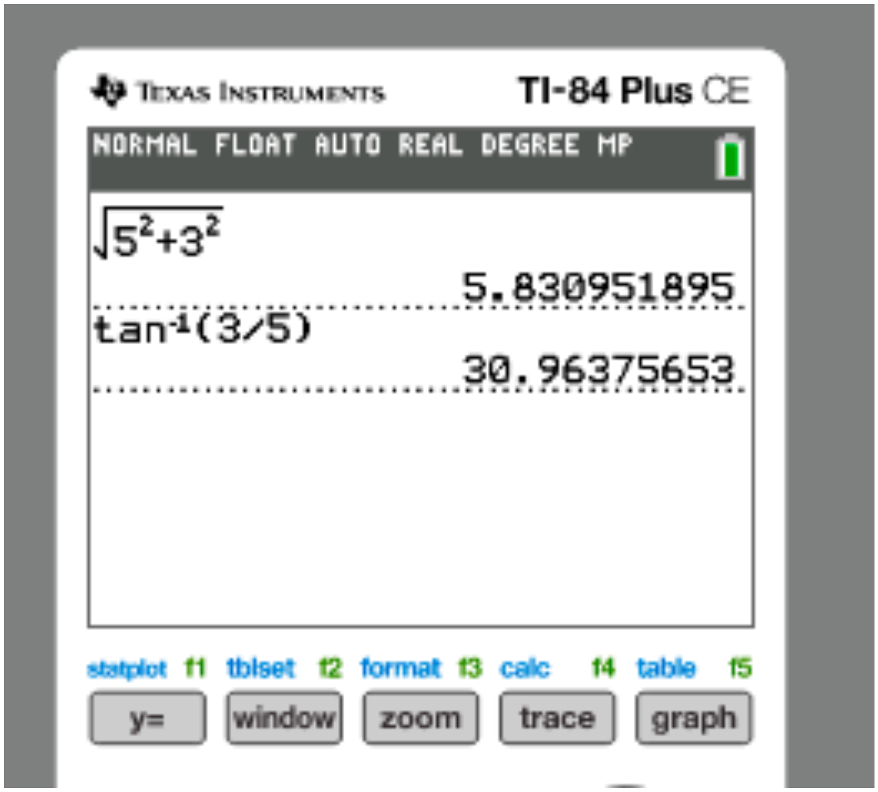
$$\vec{\boldsymbol{v}} =< x_2 - x_1, y_2 - y_1 >=< a, b >$$

**Example**- Let $A = (1,2)$ and $B = (6,5)$ to determine vector $\vec{\boldsymbol{v}} = \overrightarrow{\boldsymbol{AB}}$




$$a = 6 - 1 = 5$$
$$b = 5 - 2 = 3$$





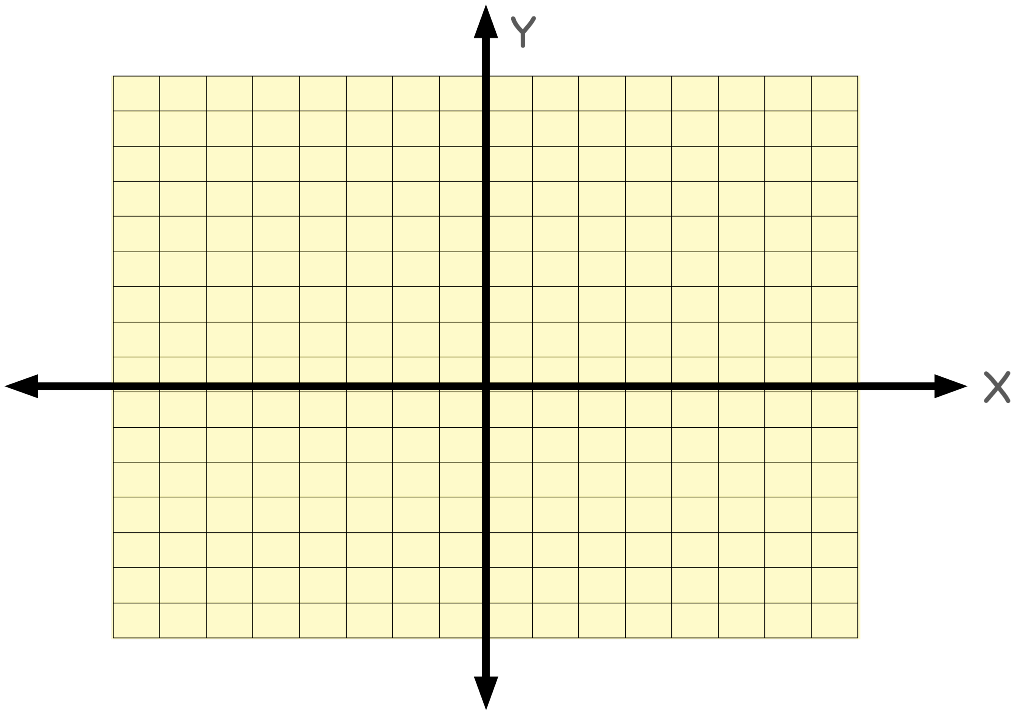

$c = \sqrt{5^2 + 3^2} \approx 5.8$; **Magnitude** $|\vec{v}| \approx \mathbf{5.8}$

$\theta = \tan^{-1}\left(\frac{3}{5}\right) \approx 31^{\circ}$; **Direction** $\boldsymbol{\theta} \approx \mathbf{31}^{\circ}$

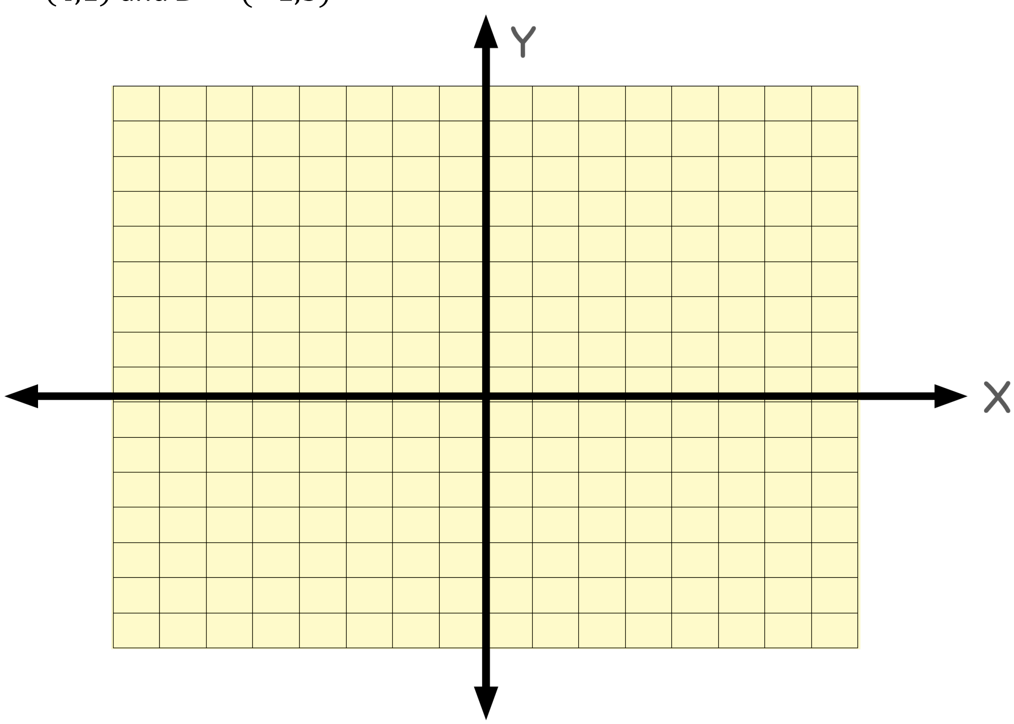
$$\vec{\boldsymbol{v}} = < 5,3 >$$

Sketch the following Vectors and determine the magnitude, direction, and component form.

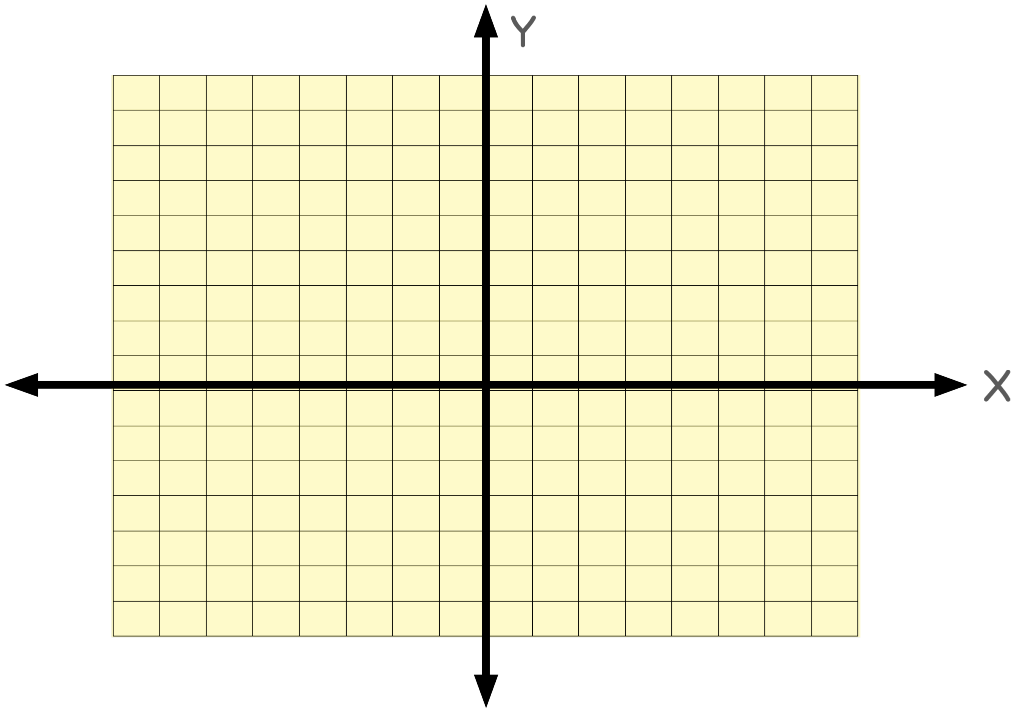
1. $A = (0, -2)$ and $B = (4,2)$



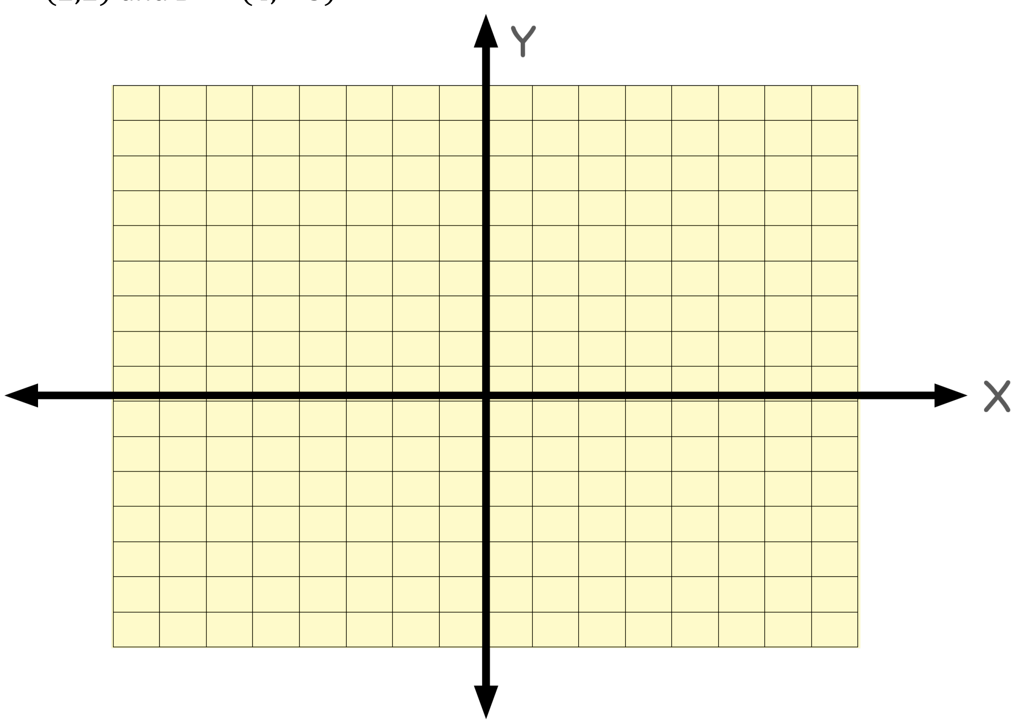

2. $A = (4,1)$ and $B = (-1,5)$



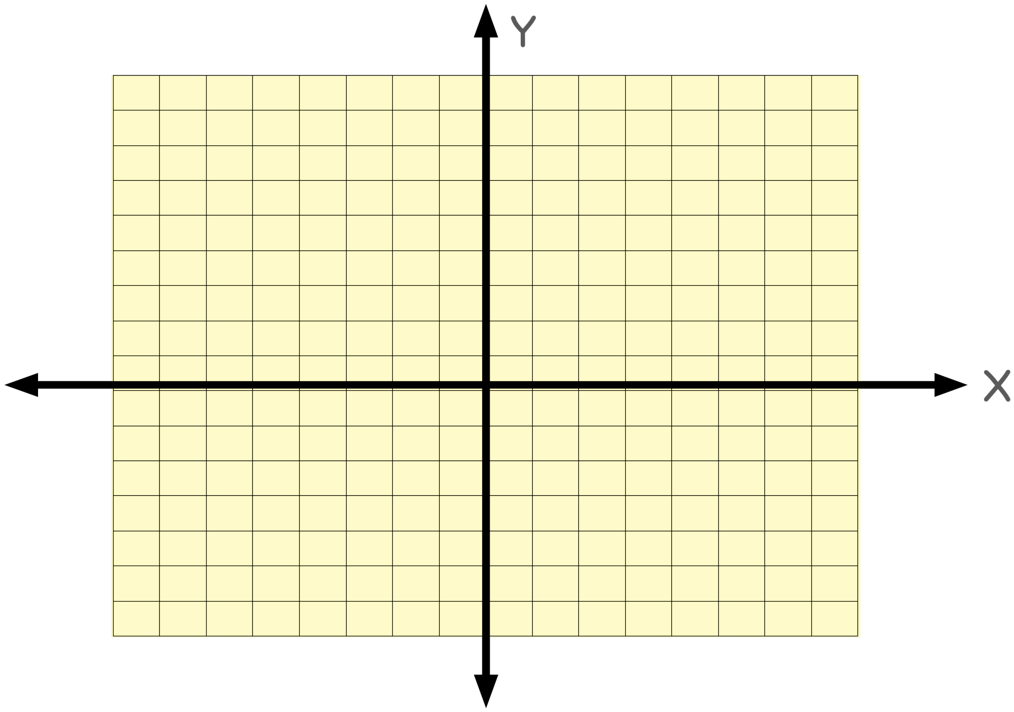
3. $A = (-3,5)$ and $B = (0,6)$



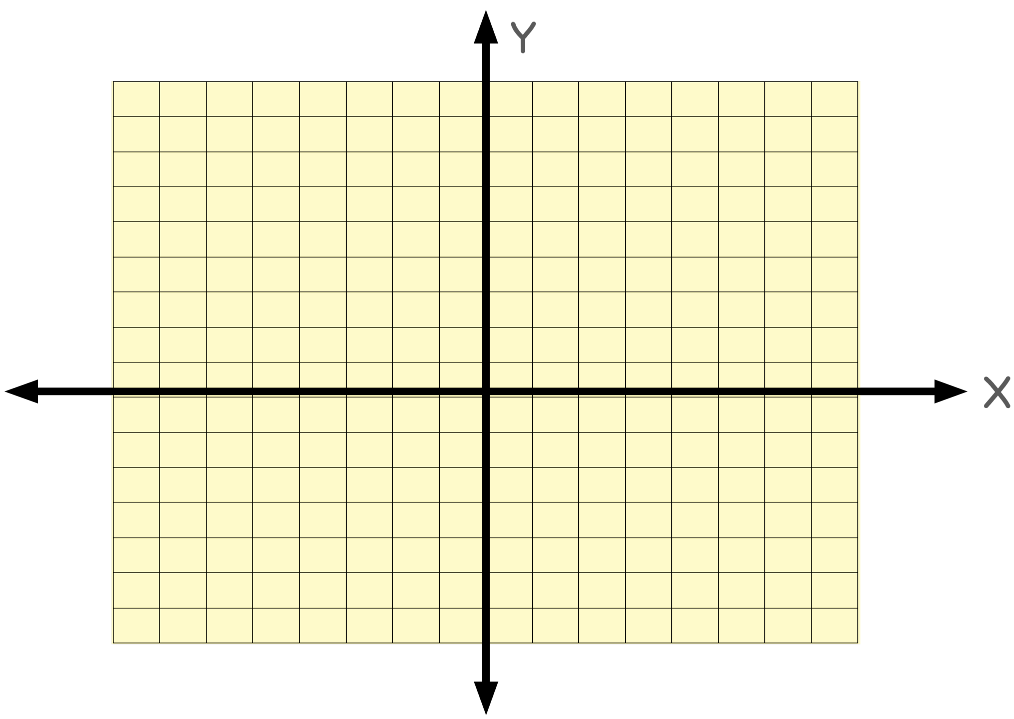

4. $A = (2,2)$ and $B = (4,-3)$



5. $A = (-4, -3)$ and $B = (2,6)$




6. $A = (3, -5)$ and $B = (5, -4)$

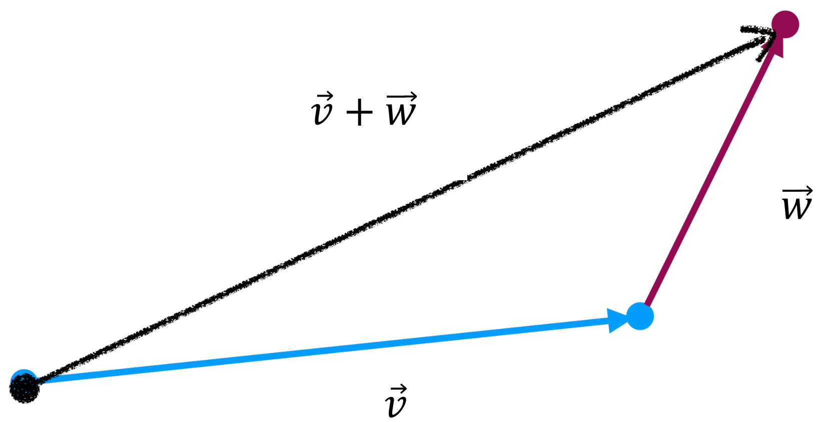


**Vector Properties**

Let $\vec{\boldsymbol{v}} =< a_1, b_1 >$ and $\vec{\boldsymbol{w}} =< a_2, b_2 >$ be two vectors in component form.
Then we have the following properties.

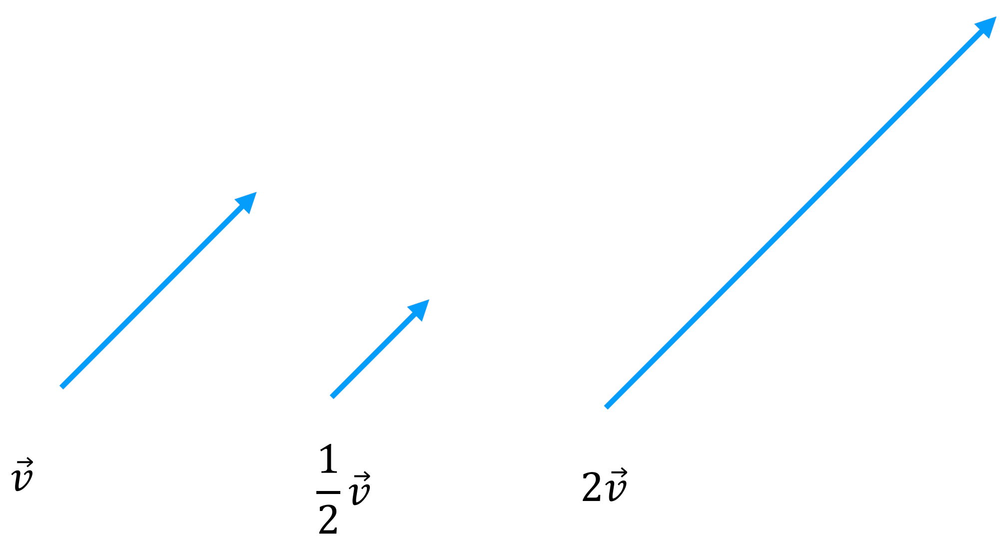
1. **Vector Addition** $\vec{v} + \vec{w} =< a_1 + a_2, b_1 + b_2 >$

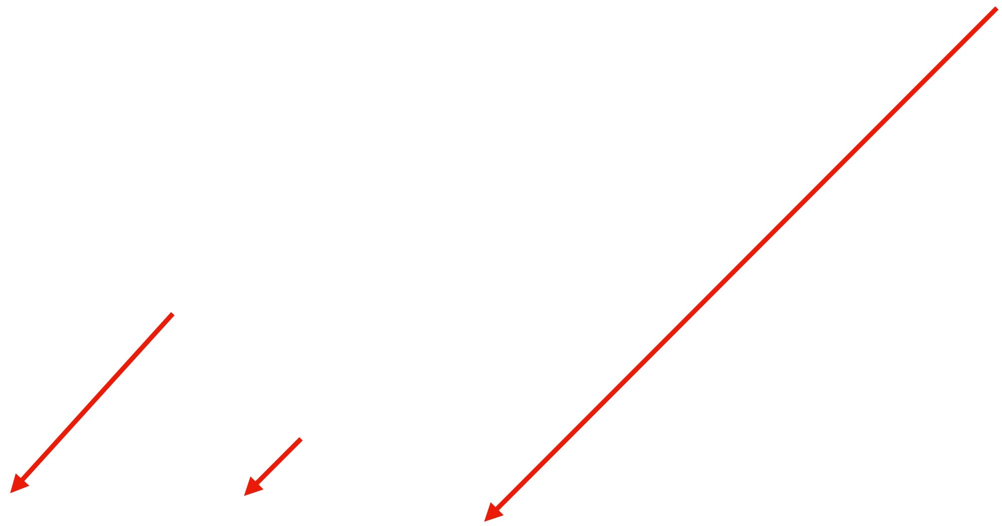
We can perform vector addition in free hand mode by using the terminal point with vector $\vec{v}$ with the initial point of vector $\vec{w}$ to create a vector sum called the resultant vector, $\vec{v} + \vec{w}$




2. **Scalar Multiplication of a Vector** $c\vec{v} = c < a, b >=< ca, cb >$ where $c \neq 0$

Consider the free hand descriptions of the following vectors.





$-\vec{v}$ $\quad -\frac{1}{3}\vec{v}$ $\quad -3\vec{v}$

3. **Vector Subtraction** $\vec{v}-\vec{w}=<a_1-a_2, b_1-b_2>$

**Example** $\vec{v}=<2,-3>$ and $\vec{w}=<1,5>$

Determine the following vectors.

1. $\vec{v}+\vec{w}$
2. $\vec{v}-\vec{w}$
3. $3\vec{v}$
4. $-4\vec{w}$
5. $3\vec{v}-4\vec{w}$

Determine the magnitude of the following vectors.

6. $|\vec{v}+\vec{w}|$
7. $|\vec{v}-\vec{w}|$
8. $|3\vec{v}|$
9. $|-4\vec{w}|$
10. $|3\vec{v}-4\vec{w}|$

**Def- Zero Vector**
The vector $\overline{\mathbf{0}} = \langle 0,0 \rangle$ is called the **Zero Vector**.

**Vector Properties**

## Properties of Vectors

| Vector addition | Multiplication by a scalar |
|---|---|
| $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ | $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$ |
| $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$ | $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$ |
| $\mathbf{u} + \mathbf{0} = \mathbf{u}$ | $(cd)\mathbf{u} = c(d\mathbf{u}) = d(c\mathbf{u})$ |
| $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$ | $1\mathbf{u} = \mathbf{u}$ |
| Length of a vector | $0\mathbf{u} = \mathbf{0}$ |
| $\lvert c\mathbf{u} \rvert = \lvert c \rvert \lvert \mathbf{u} \rvert$ | $c\mathbf{0} = \mathbf{0}$ |

**Def-** Let vector $\overline{\boldsymbol{v}} = \langle a, b \rangle$.
The vector $\overline{\boldsymbol{v}}$ is a called the **Unit Vector**, if $|\overline{v}| = 1$

The following are examples of Unit vectors.

$$\overline{\boldsymbol{v}} = \langle \tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}} \rangle \text{ where } |\overline{v}| = 1$$

$$\overline{\boldsymbol{\imath}} = \langle 1,0 \rangle \text{ where } |\overline{\imath}| = 1$$

$$\overline{\boldsymbol{\jmath}} = \langle 0,1 \rangle\, \overline{\jmath} = \langle 0,1 \rangle \text{ where } |\overline{\jmath}| = 1$$

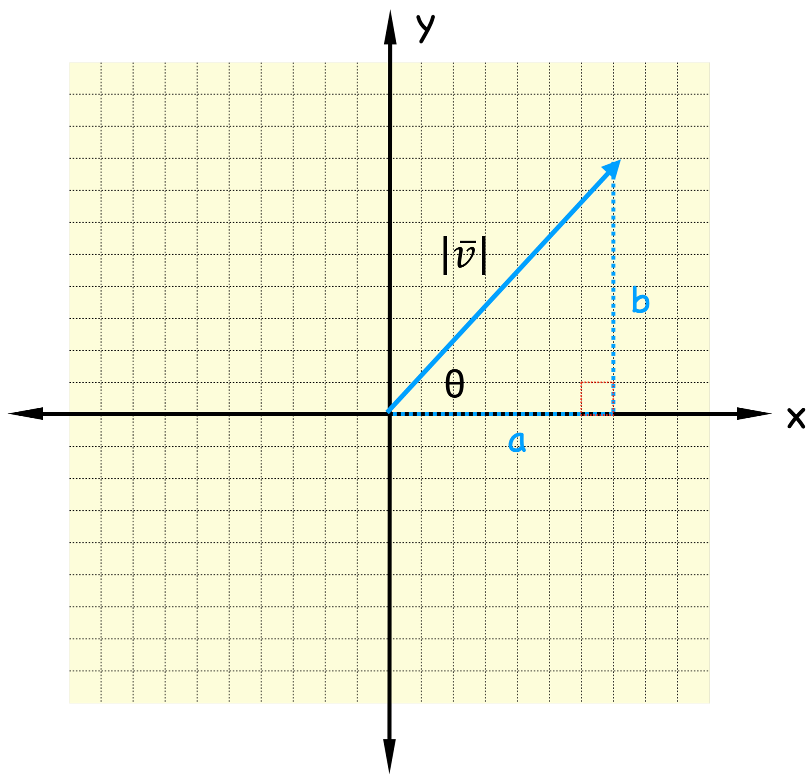
$$\overline{\boldsymbol{w}} = \langle \tfrac{3}{5}, -\tfrac{4}{5} \rangle \text{ where } |\overline{w}| = 1$$

We can write any vector $\bar{v} = \langle a, b \rangle$ with unit vectors!

**Vectors In Terms of $\bar{\imath}$ and $\bar{\jmath}$**

$$\bar{\boldsymbol{v}} = \langle a, b \rangle = a\bar{\imath} + b\bar{\jmath}$$

**Horizontal and Vertical Components of a Vector** $\bar{\boldsymbol{v}} = \langle \boldsymbol{a}, \boldsymbol{b} \rangle = \boldsymbol{a}\bar{\boldsymbol{\imath}} + \boldsymbol{b}\bar{\boldsymbol{\jmath}}$




Let $a$ represent the horizontal component and $b$ represent the vertical component.

Then $\boldsymbol{a} = |\bar{v}|\boldsymbol{cos}(\boldsymbol{\theta})$ and $\boldsymbol{b} = |\bar{v}|\boldsymbol{sin}(\boldsymbol{\theta})$ such that $\bar{\boldsymbol{v}} = \langle \boldsymbol{a}, \boldsymbol{b} \rangle = |\bar{v}|\boldsymbol{cos}(\boldsymbol{\theta})\bar{\boldsymbol{\imath}} + |\bar{v}|\boldsymbol{sin}(\boldsymbol{\theta})\bar{\boldsymbol{\jmath}}$

$$\boldsymbol{tan}(\boldsymbol{\theta}) = \frac{\boldsymbol{b}}{\boldsymbol{a}}$$

Write the following vectors in terms of $\bar{\imath}$ and $\bar{\jmath}$, and determine its magnitude, and direction $\theta$ in whole degrees.

1. $\bar{\boldsymbol{v}} = \langle 3,8 \rangle$

2. $\bar{\boldsymbol{v}} = \langle -2,1 \rangle$

3. $\bar{\boldsymbol{v}} = \langle 6,-4 \rangle$

4. $\bar{\boldsymbol{v}} = \langle 0,2 \rangle$

5. $\bar{\boldsymbol{v}} = \langle -5,-2 \rangle$

6. $\bar{\boldsymbol{v}} = \langle 1,-1 \rangle$

7. $\bar{\boldsymbol{v}} = \langle 6,-4 \rangle$

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Determine the **horizontal** and **vertical components** of the vector and write in terms of $\bar{\imath}$ and $\bar{\jmath}$.

8. $|\bar{v}| = 4$ and $\theta = 30^o$

9. $|\bar{v}| = 25$ and $\theta = 100^o$

10. $|\bar{v}| = \sqrt{3}$ and $\theta = 135^o$

11. $|\bar{v}| = 200$ and $\theta = -60^o$

12. $|\bar{v}| = 1$ and $\theta = 225^o$

13. $|\bar{v}| = 2\sqrt{5}$ and $\theta = -120^o$

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Determine the **resultant vector**, its **magnitude**, and **direction** in whole degrees.

14. $\bar{\boldsymbol{v}} = 4\bar{\imath} - 2\bar{\jmath}$ and $\bar{w} = 2\bar{\imath} + \bar{\jmath}$

15. $\bar{\boldsymbol{v}} = 3\bar{\imath} + 5\bar{\jmath}$ and $\bar{w} = -6\bar{\imath} + 2\bar{\jmath}$

16. $\bar{\boldsymbol{v}} = -\bar{\imath} + 6\bar{\jmath}$ and $\bar{w} = -4\bar{\jmath}$

17. $\bar{\boldsymbol{v}} = \bar{\imath} + 4\bar{\jmath}$ and $\bar{w} = -5\bar{\jmath}$ and $\bar{u} = 8\bar{\imath} - \bar{\jmath}$

18. $\bar{\boldsymbol{v}} = -2\bar{\imath} - 6\bar{\jmath}$ and $\bar{w} = 4\bar{\imath} + \bar{\jmath}$ and $\bar{u} = 3\bar{\imath} - 2\bar{\jmath}$