S3 ✓

math 262 Test 2

2 ✓

① $\lim_{x \to \infty} \frac{4x+3}{2x-1}$ $\quad \frac{\infty}{\infty}$ ✓

$\underset{H}{=} \lim_{x\to\infty} \frac{4}{2} = \boxed{2}$ ✓

3 ✓

② $\lim_{x \to -\infty} x \sin\left(\frac{1}{x}\right)$ $\quad -\infty \cdot 0$

$= \lim_{x\to -\infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}}$ ✓ $\quad \frac{0}{0}$

$\underset{H}{=} \lim_{x\to -\infty} \frac{\cos\left(\frac{1}{x}\right)\cdot\left(-\frac{1}{x^2}\right)}{\left(-\frac{1}{x^2}\right)}$ ✓

$= \lim_{x\to -\infty} \cos\left(\frac{1}{x}\right) = \cos(0) = \boxed{1}$ ✓

5 ✓

③ $\int x^2 e^{-x}\,dx$; let $u = x^2$ ✓ $\quad dv = e^{-x}\,dx$

$uv - \int v\,du$ $\quad du = 2x\,dx$; $v = \int e^{-x}\,dx$

$= -x^2e^{-x} - \left[\int -e^{-x}\cdot 2x\,dx\right]$ $\quad = -e^{-x}$

$= -x^2e^{-x} + \int e^{-x}\,2x\,dx$ ✓

$= -x^2e^{-x} + \int 2x\,e^{-x}\,dx$; let $u = 2x$; $v = \int e^{-x}dx$

$du = 2dx$ $\quad v = -e^{-x}$

$uv - \int v\,du$

$$-x^2e^{-x} + \left[-2xe^{-x} - \int -e^{-x}\, 2\,dx\right]$$

$$-x^2e^{-x} - 2xe^{-x} + 2\int e^{-x}\,dx$$

$\Downarrow$ $-e^{-x}$

$$\boxed{-x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C}$$

(4) $\int \ln^2(x)\,dx$; $u = \ln^2(x)$; $dv = dx$

$du = 2\ln x \cdot \frac{1}{x}\,dx$ $\quad v = \int dx$

5

$uv - \int v\,du$ $\quad v = x$

$$x\ln^2(x) - \int \cancel{x}\, 2\ln x \cdot \frac{1}{\cancel{x}}\,dx$$

$$x\ln^2(x) - \int 2\ln x\,dx$$

$$x\ln^2(x) - 2\int \ln(x)\,dx \quad ; \quad u = \ln(x)$$

$\Downarrow$

$du = \frac{1}{x}\,dx$

$uv - \int v\,du$

$dv = dx$; $v = x$

$$x\ln^2(x) - 2\left[x\ln(x) - \int x \cdot \frac{1}{x}\,dx\right]$$

$$x\ln^2(x) - 2x\ln(x) + 2\cancel{\int dx}\cdot x + C$$

$$\boxed{x\ln^2(x) - 2x\ln(x) + 2x + C}$$

(5) $\int \underset{\Downarrow}{\cos^3 (x)} \sin^2 (x)\, dx$

$\cos^2 (x) \cdot \cos (x)$

$\Downarrow$

$1 - \sin^2 (x)$

6 ✓

$$\int (1 - \sin^2 (x)) \cos (x) \cdot \sin^2 (x)\, dx$$

let $u = \sin (x)$; $du = \cos (x)\, dx$

✓

$dx = \dfrac{du}{\cos (x)}$ ✓

$$\int (1 - u^2) \cancel{\cos (x)}\, u^2 \frac{du}{\cancel{\cos (x)}}$$

✓

$$\int (1 - u^2) u^2\, du = \int (u^2 - u^4)\, du$$

✓

$$= \frac{1}{3} u^3 - \frac{1}{5} u^5 + C$$

$$= \boxed{\frac{1}{3} \sin^3 (x) - \frac{1}{5} \sin^5 (x) + C}$$

✓ ✓

(6) $\int \tan^3(x)\sec^5(x)\,dx$

$\Downarrow$

$\sec(x)\tan(x)\sec^4(x)\tan^2(x)$

$\Downarrow$

6 ✓

$\sec^2(x)-1$

$$\int \sec(x)\tan(x)\sec^4(x)\left(\sec^2(x)-1\right)dx$$

let $u=\sec(x)$, $du=\sec(x)\tan(x)\,dx$ ✓

$$dx=\frac{du}{\sec(x)\tan(x)} \quad ✓$$

$$\int \cancel{\sec(x)}\cancel{\tan(x)}\cdot u^4(u^2-1)\frac{du}{\cancel{\sec(x)}\cancel{\tan(x)}}$$ ✓

$$\int u^4(u^2-1)\,du=\int(u^6-u^4)\,du$$

$$\frac{1}{7}u^7-\frac{1}{5}u^5+C$$ ✓

$$\boxed{\frac{1}{7}\sec^7(x)-\frac{1}{5}\sec^5(x)+C}$$

✓ ✓

(7) $\int \sqrt{16+x^2}\,dx$

$a^2 = 16$; $a = 4$

$x = 4\tan(\theta)$

$dx = 4\sec^2(\theta)\,d\theta$

$\sqrt{16+x^2} = 4\sec(\theta)$

6 ✓

$$\int 4\sec(\theta)\cdot 4\sec^2(\theta)\,d\theta$$

$$\int 16\sec^3\theta\,d\theta = 16\int \sec^3\theta\,d\theta$$ ✓

previous example

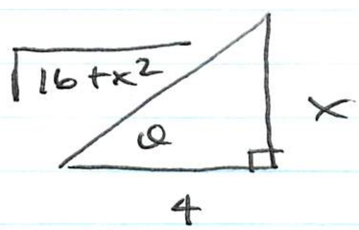
$$16\left[\frac{1}{2}\left[\sec(x)\tan(x) + \ln|\sec(x)+\tan(x)|\right]\right]$$

$$8\sec(x)\tan(x) + 8\ln|\sec(x)+\tan(x)| + C$$

✓ ✓

But,

$\tan(\theta) = \dfrac{x}{4}$; $\sec(x) = \dfrac{\sqrt{16+x^2}}{4}$



$$8\frac{\sqrt{16+x^2}}{4}\,\frac{x}{4} + 8\ln\left|\frac{\sqrt{16+x^2}}{4} + \frac{x}{4}\right| + C$$

$$\frac{x^2\sqrt{16+x^2}}{2} + 8\ln\left|\frac{\sqrt{16+x^2}}{4} + \frac{x}{4}\right| + C$$

(8) $\int x^3 \sqrt{1-x^2}\, dx$

let $x = \sin\varphi$;
$dx = \cos\varphi\, d\varphi$
$\sqrt{1-x^2} = \cos\varphi$

8

$\int \sin^3\varphi \cdot \cos\varphi \cdot \cos\varphi\, d\varphi$

$\int \sin^3\varphi \cos^2\varphi\, d\varphi$

$\sin\varphi \cdot \underbrace{\sin^2\varphi}_{1-\cos^2\varphi} \cdot \cos^2\varphi$

$\int \sin\varphi\, (1-\cos^2\varphi) \cos^2\varphi\, d\varphi$

let $u = \cos\varphi$; $\frac{du}{d\varphi} = -\sin\varphi$

$d\varphi = \frac{du}{-\sin\varphi}$

$\int \cancel{\sin\varphi}\, (1-u^2)\, u^2 \frac{du}{-\cancel{\sin\varphi}}$

$\int (u^2-1)\, u^2\, du$

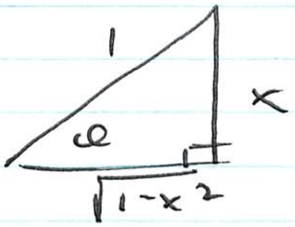
$\boxed{\frac{1}{5}\cos^5(x) - \frac{1}{3}\cos^3(x) + C}$

$\int (u^4 - u^2)\, du$

$\frac{1}{5}u^5 - \frac{1}{3}u^3 + C$

But,

$$\sin(\theta) = \frac{x}{1} \quad ; \quad \cos(\theta) = \frac{\sqrt{1-x^2}}{1}$$



or $\cos(\theta) = \sqrt{1-x^2}$

$$\frac{1}{5}\left[\sqrt{1-x^2}\right]^5 - \frac{1}{3}\left[\sqrt{1-x^2}\right]^3 + C$$

$$\boxed{\frac{1}{5}(1-x^2)^{5/2} - \frac{1}{3}(1-x^2)^{3/2} + C}$$

(9) $\displaystyle\int \frac{1}{x^2-7x+10}\,dx$

$\Downarrow$

$(x-5)(x-2)$

5 ✓ $\displaystyle\int \frac{1}{(x-5)(x-2)}\,dx = \int \left(\frac{\cancel{A}^{1/3}}{x-5} + \frac{\cancel{B}^{-1/3}}{x-2}\right) dx$

$$\frac{1}{(x-5)(x-2)} = \frac{\cancel{A}^{\frac{1}{3}}}{x-5} + \frac{\cancel{B}^{-\frac{1}{3}}}{x-2}$$

$\rightarrow 1 = A(x-2) + B(x-5)$ ✓

$x=2$; $1 = B(2-5)$; $1 = -3B$

$B = -1/3$

$x=5$; $1 = A(5-2)$; $1 = 3A$

$A = 1/3$

✓ ✓

$$\int \left(\frac{1/3}{x-5} - \frac{1/3}{x-2}\right) dx$$

$$\frac{1}{3}\int \frac{1}{x-5}\,dx - \frac{1}{3}\int \frac{1}{x-2}\,dx$$

$$\boxed{\frac{1}{3}\ln|x-5| - \frac{1}{3}\ln|x-2| + C}$$

✓ ✓

(10) $\displaystyle\int \frac{18}{(x+3)(x^2+9)}\,dx$

$$= \int \left(\frac{A}{x+3} + \frac{Bx+C}{x^2+9} \right) dx$$

$$\frac{18}{(x+3)(x^2+9)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+9}$$

$$\to 18 = A(x^2+9) + (Bx+C)\underbrace{(x+3)}_{\text{linear}}$$

7✓ ✓

let $x=-3$;

$$18 = A(\underbrace{x^2}_{-3}+9) + \cancel{(Bx+C)}\underbrace{(x+3)}_{-3}\ 0$$

$18 = A((-3)^2+9)$; $18 = A(9+9)$

$18 = A \cdot 18$; $\boxed{A=1}$

$18 = x^2+9 + (Bx+C)(x+3)$

$18 = x^2+9 + (Bx^2 + Cx + 3Bx + 3C)$

$18 = x^2+9 + Bx^2 + Cx + 3Bx + 3C$

$18 = (1+B)x^2 + (C+3B)x + 3C+9$

$\to$ $1+B=0$ or $B=-1$ $\quad 3C+9=18$; $C=3$

$$\int \left(\frac{1}{x+3} + \frac{-x+3}{x^2+9} \right) dx$$

$$\int \left(\frac{1}{x+3} - \frac{x-3}{x^2+9} \right) dx$$

$$\int \frac{1}{x+3}\,dx - \int \frac{x-3}{x^2+9}\,dx$$

$$\int \frac{1}{x+3}\,dx - \int \frac{x}{x^2+9}\,dx + \int \frac{3}{x^2+9}\,dx$$

$$\int \frac{1}{x+3}\,dx - \int \frac{x}{x^2+9}\,dx + 3\int \frac{1}{x^2+9}\,dx$$

u-sub

$\ln|x+3|$

$u = x^2+9$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \frac{\cancel{x}}{u} \frac{du}{2\cancel{x}}$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u| + C$

φ

$\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$

$$\ln|x+3| - \frac{1}{2}\ln|x^2+9| + \tan^{-1}\left(\frac{x}{3}\right) + C$$