East Los Angeles College
Department of Mathematics
Math 262
Test 1

61 -

Perform the following limits.

1. $\lim_{x\to\infty} e^{-x^3}$

as $x \to \infty$; $-x^3 \to -\infty$

$e^{-x^3} \to \boxed{0}$

2. $\lim_{x\to(\pi/2)^-} 1.01^{\tan(x)}$

as $x \to \frac{\pi}{2}^-$; $\tan(x) \to \infty$

$1.01^{\tan(x)} \to \boxed{\infty}$

3. $\lim_{x\to\infty} log\left(\frac{1}{x}\right)$

as $x \to \infty$; $\frac{1}{x} \to 0$

$\log\left(\frac{1}{x}\right) \to \boxed{-\infty}$

4. $\lim_{x\to 0} ln\left(\frac{x^2-4x+3}{x^2+10x+1}\right)$

as $x \to 0$;

$\ln\left(\frac{x^2-4x+3}{x^2+10x+1}\right) \to \boxed{\ln(3)}$

5. $\lim_{x\to\infty} tan^{-1}(10^x)$

as $x \to \infty$; $10^x \to \infty$

$\tan^{-1}(10^x) \to \boxed{\frac{\pi}{2}}$

6. $\lim_{x\to\infty} cos^{-1}\left(\frac{\sqrt{3}x+1}{1-2x}\right)$

as $x \to \infty$;

$\cos^{-1}\left(\frac{\sqrt{3}x+1}{1-2x}\right) \to \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

4

$\boxed{\frac{5\pi}{6}}$

12

7. Differentiate

$$f(x) = \frac{x\sqrt{x+1}}{(2x+3)^2}$$

$y = \frac{x\sqrt{x+1}}{(2x+3)^2}$; $\ln(y) = \ln\left(\frac{x\,(x+1)^{1/2}}{(2x+3)^2}\right)$

$y' = \frac{x\sqrt{x+1}}{(2x+3)^2}\left[\frac{1}{x} + \frac{1}{2(x+1)} - \frac{4}{2x+3}\right]$

✓ ✓ ✓ ✓

4✓

8. Determine intervals of increasing/decreasing.

$$f(x) = \frac{e^x}{x^2}$$

✓

$$f'(x) = \frac{e^x(x-2)}{x^3}$$ ✓ ✓

✓ ✓

f is increasing on $(-\infty, 0) \cup (2, \infty)$

f is decreasing on $(0, 2)$ ✓

6 ✓

9. Determine the intervals of concavity and any inflection points.

$$f(x) = ln(1 + x^2)$$

$f'(x) = \frac{2x}{1+x^2}$; $f''(x) = \frac{2(1-x^2)}{(1+x^2)^2}$

f is CD on $(-\infty, -1) \cup (1, \infty)$

f is CU on $(-1, 1)$

IP $(-1, f(-1))$

$(1, f(1))$

9

Differentiate the following.

10. $f(x) = 4xe^{-x}$

$f'(x) = 4e^{-x}(1-x)$ ✓ ✓ ✓

11. $f(x) = \frac{ln(x)}{x+1}$

✓ ✓

$$f'(x) = \frac{x+1 - x\ln(x)}{x(x+1)^2}$$

✓

12. $f(x) = tan^{-1}(2x)$

✓

$$f'(x) = \frac{2}{1+4x^2}$$

✓ ✓

13. $f(x) = \pi sin(e^{\sqrt{x}})$

✓

$$f'(x) = \frac{\pi e^{\sqrt{x}}}{2\sqrt{x}} \cos(e^{\sqrt{x}})$$

✓ ✓

12 ✓

Integrate the following.

14. $\int (e^{2x} + e^{-3x})dx$

$\int e^{2x} dx + \int e^{-3x} dx$

$u = 2x$ $\quad$ $u = -3x$

$dx = \frac{du}{2}$ $\quad$ $dx = \frac{du}{-3}$

$\frac{1}{2} e^{u} + C \quad \frac{1}{3} e^{u} + C$

$\boxed{\frac{1}{2} e^{2x} - \frac{1}{3} e^{-3x} + C}$

15. $\int \left(2 + \frac{1}{x}\right)\left(3 + \frac{1}{x}\right) dx$

$\int \left(6 + \frac{5}{x} + \frac{1}{x^2}\right) dx$

$\boxed{6x + 5 \ln|x| - \frac{1}{x} + C}$

16. $\int \frac{1}{5x+4} dx$

$u = 5x + 4 \; ; \; dx = \frac{du}{5}$

$\int \frac{1}{u} \frac{du}{5}$

$\frac{1}{5} \ln|u| + C$

$\boxed{\frac{1}{5} \ln|5x + 4| + C}$

17. $\int \frac{1}{1+16x^2} dx$

$u = 4x$

$\int \frac{1}{1+u^2} \frac{du}{4}$

$\frac{1}{4} \int \frac{1}{1+u^2} du$

$\frac{1}{4} \tan^{-1}(u) + C$

$\boxed{\frac{1}{4} \tan^{-1}(4x) + C}$

12 ✓

18. $\int \frac{x-5}{x^2+25}\,dx$

19. $\int \frac{1}{\sqrt{25-x^2}}\,dx$ $= \boxed{\sin^{-1}\left(\frac{x}{5}\right)+C}$

5^2

$\int \frac{x}{x^2+25}\,dx - 5\int \frac{1}{x^2+25}\,dx$

$u = x^2+25$

$dx = \frac{du}{2x}$

$\frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right)+C$

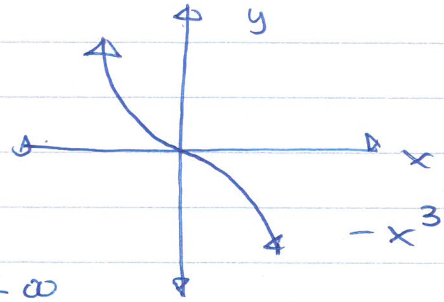
6 ✓

$\boxed{\frac{1}{2}\ln(x^2+25) + \tan^{-1}\left(\frac{x}{5}\right)+C}$

✓ ✓ ✓

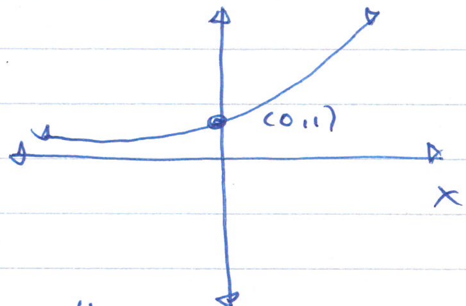
math 262 Test 1

(1) $\lim_{x \to \infty} e^{-x^3}$

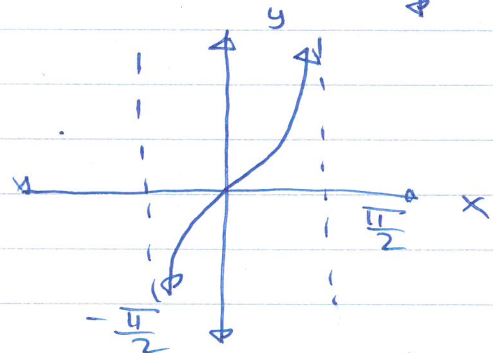


as $x \to \infty$, $-x^3 \to -\infty$

$e^{-x^3} \to \boxed{0}$



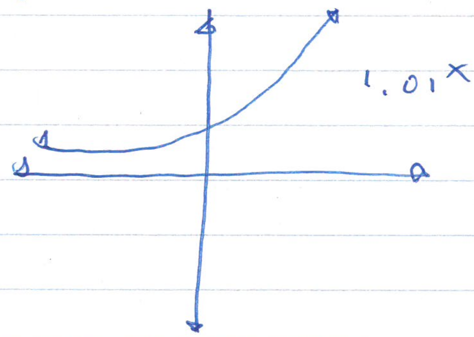
(2) $\lim_{x \to \frac{\pi}{2}^-} 1.01^{\tan(x)}$



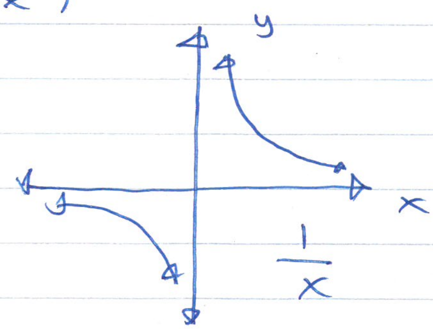
as $x \to \frac{\pi}{2}^-$

$\tan(x) \to \infty$,

$1.01^{\tan(x)} \to \boxed{\infty}$



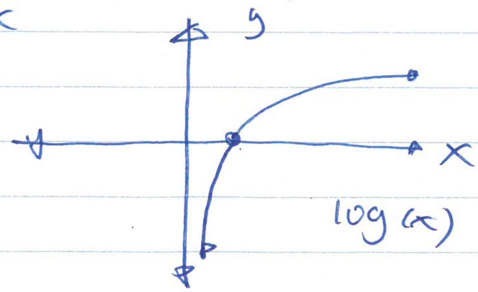
(3) $\lim_{x \to \infty} \log\left(\frac{1}{x}\right)$



as $x \to \infty$

$\frac{1}{x} \to 0^+$

$\log\left(\frac{1}{x}\right) \to \boxed{-\infty}$



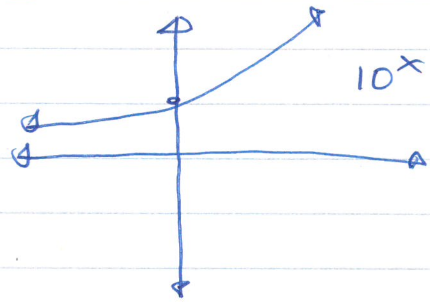
(4) $\lim_{x \to 0} \ln\left(\dfrac{x^2 - 4x + 3}{x^2 + 10x + 1}\right)$

as $x \to 0$; $\dfrac{x^2 - 4x + 3}{x^2 + 10x + 1} \to \dfrac{0^2 - 4 \cdot 0 + 3}{0^2 + 10 \cdot 0 + 1}$

$\dfrac{3}{1}$ 3

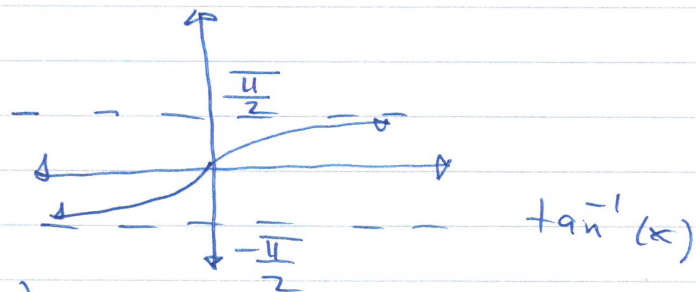
$\ln\left(\dfrac{x^2 - 4x + 3}{x^2 + 10x + 1}\right) \to \ln(3)$ continuous at $x = 0$

(5) $\lim_{x \to \infty} \tan^{-1}(10^x)$



as $x \to \infty$; $10^x \to \infty$

$\tan^{-1}(10^x) \to \boxed{\dfrac{\pi}{2}}$



(6) $\lim_{x \to \infty} \cos^{-1}\left(\dfrac{\sqrt{3}x + 1}{1 - 2x}\right)$

$\lim_{x \to \infty} \dfrac{\sqrt{3}x + 1}{1 - 2x}$ $\quad \dfrac{\infty}{\infty}$

$\lim_{x \to \infty} \dfrac{\sqrt{3}x/x + 1/x}{\frac{1}{x} - 2x/x} = \lim_{x \to \infty} \dfrac{\sqrt{3} + 1/x}{\frac{1}{x} - 2}$

$= \dfrac{\sqrt{3}}{-2}$ or $-\dfrac{\sqrt{3}}{2}$

ie, as $x \to \infty$; $\dfrac{\sqrt{3}x + 1}{1 - 2x} \to -\dfrac{\sqrt{3}}{2}$

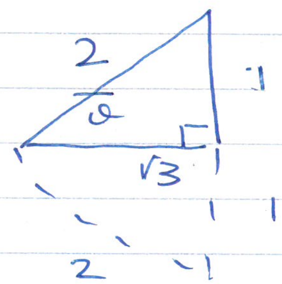
so, $\cos^{-1}\left(\frac{\sqrt{3}x+1}{1-2x}\right) \to \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

θ

what is this?

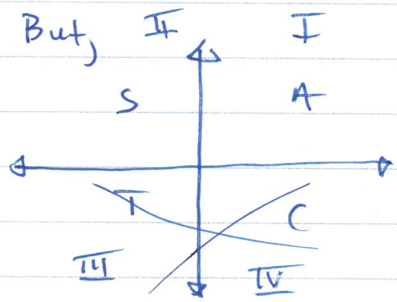
<u>note</u> $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \theta$

or $\cos\theta = \frac{\sqrt{3}}{2}$ $\frac{\text{adj}}{\text{hyp}}$



$\bar{\theta} = \frac{\pi}{6}$

But,



$\theta = \pi - \frac{\pi}{6}$

$\theta = \frac{6\pi}{6} - \frac{\pi}{6}$

$\theta = \frac{5\pi}{6}$

ie, $\cos^{-1}\left(\frac{\sqrt{3}x+1}{1-2x}\right) \to \boxed{\frac{5\pi}{6}}$

(7) $y = \frac{x\sqrt{x+1}}{(2x+3)^2}$

$\ln(y) = \ln\left(\frac{x(x+1)^{1/2}}{(2x+3)^2}\right)$

$\ln(y) = \ln\left(x(x+1)^{1/2}\right) - \ln\left((2x+3)^2\right)$

$\ln(y) = \ln(x) + \ln\left((x+1)^{1/2}\right) - 2\ln(2x+3)$

$$\ln(y) = \ln(x) + \tfrac{1}{2}\ln(x+1) - 2\ln(2x+3)$$

$$\frac{d}{dx}\left[\ln(y)\right] = \frac{d}{dx}\left[\ln(x) + \tfrac{1}{2}\ln(x+1) - 2\ln(2x+3)\right]$$

$$\frac{1}{y}\cdot y' = \frac{1}{x} + \frac{1}{2(x+1)} - \frac{2}{2x+3}\cdot 2$$

$$\frac{y'}{y} = \frac{1}{x} + \frac{1}{2(x+1)} - \frac{4}{2x+3}$$

$$y' = y\left[\frac{1}{x} + \frac{1}{2(x+1)} - \frac{4}{2x+3}\right]$$

$$\boxed{y' = \frac{x\sqrt{x+1}}{(2x+3)^2}\left[\frac{1}{x} + \frac{1}{2(x+1)} - \frac{4}{2x+3}\right]}$$

8) $y = \dfrac{e^x}{x^2}$ note $x \neq 0$

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(\frac{e^x}{x^2}\right) = \frac{x^2 \underbrace{\frac{d}{dx}(e^x)}_{e^x} - e^x \underbrace{\frac{d}{dx}(x^2)}_{2x}}{x^4}$$

$$y' = \frac{x^2 e^x - 2x e^x}{x^4} \quad ; \quad y' = \frac{e^x x(x-2)}{x^4}$$

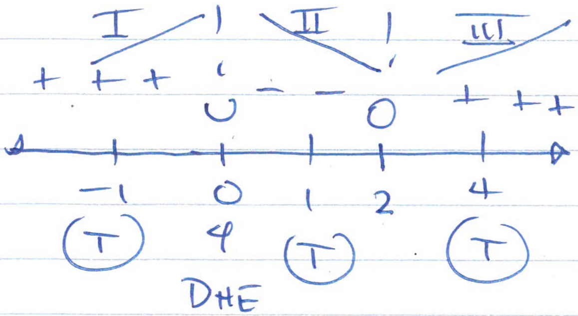
$$\boxed{y' = \frac{e^x(x-2)}{x^3}}$$

Sign analysis!

$e^x(x-2) = 0$

$e^x = 0$ never happens	$x-2=0$ $x=2$

$x=0$ can't happen! not in the domain.



$$y' = \frac{e^x (x-2)}{x}$$

f is increasing on $(-\infty, 0) \cup (2, \infty)$

f is decreasing on $(0, 2)$

(9) $y = x^2 - 8 \ln(x)$

$$y' = \frac{d}{dx}\left(x^2 - 8 \ln(x)\right)$$

$$= \frac{d}{dx}(x^2) - 8 \frac{d}{dx}\left(\ln(x)\right)$$

$$= 2x - \frac{8}{x} \quad ; \quad y' = \frac{2x \cdot x}{x} - \frac{8}{x}$$

$$y' = \frac{2x^2}{x} - \frac{8}{x} \quad ; \quad y' = \frac{2x^2 - 8}{x}$$

$$y' = \frac{2(x^2-4)}{x} \quad ;$$

$$y'' = \frac{d}{dx}\left[\frac{2(x^2-4)}{x}\right] = 2 \frac{d}{dx}\left[\frac{x^2-4}{x}\right]$$

$$= 2\left[\frac{x \cancelto{2x}{\frac{d}{dx}(x^2-4)} - (x^2-4) \cancelto{1}{\frac{d}{dx}(x)}}{x^2}\right]$$

$$= 2\left[\frac{x \cdot 2x - (x^2-4) \cdot 1}{x^2}\right]$$

(9) $f(x) = \ln(1+x^2)$

$$f'(x) = \frac{d}{dx}\left[\ln(1+x^2)\right]$$

$$= \frac{1}{1+x^2}\,\cancel{\frac{d}{dx}(1+x^2)}^{\,2x}$$

$$f'(x) = \frac{2x}{1+x^2}$$

$$f''(x) = \frac{d}{dx}\left[\frac{2x}{1+x^2}\right]$$

$$= \frac{(1+x^2)\cancel{\frac{d}{dx}(2x)}^{\,2} - 2x\,\cancel{\frac{d}{dx}(1+x^2)}^{\,2x}}{(1+x^2)^2}$$

$$= \frac{(1+x^2)\cdot 2 - 4x^2}{(1+x^2)^2}$$

$$= \frac{2+2x^2-4x^2}{(1+x^2)^2}$$

$$= \frac{2-2x^2}{(1+x^2)^2} \; ; \quad \boxed{f''(x) = \frac{2(1-x^2)}{(1+x^2)^2}}$$

numerator

$2(1-x^2) = 0$

$1-x^2 = 0$

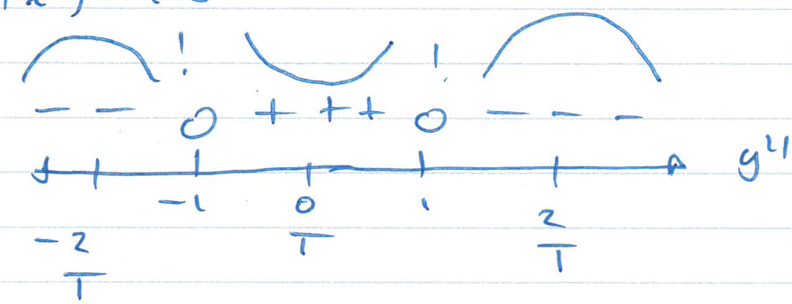
$x^2 = 1$

$x = \pm 1$

Denominator

$(1+x^2)^2 \neq 0$

Sign analysis



f is concave down $(-\infty,-1) \cup (1,\infty)$

f is concave up $(-1,1)$

$$y'' = 2\left(\frac{2x^2 - x^2 + 4}{x^2}\right)$$

$$y'' = 2\left(\frac{x^2+4}{x^2}\right)$$

Sign analysis:

note Domain

$x > 0$

$x^2+4=0$ never happens	$x^2=0$ $x=0$ ← can't happen domain $x>0$

$$\underset{0}{|} \quad + \; \underset{T}{+} \; + \; + \; + \quad y''$$

f is concave up $(0, \infty)$

(10) $f(x) = 4xe^{-x}$

$$\frac{d}{dx}\left(4xe^{-x}\right) = 4x\cancel{\frac{d}{dx}\left(e^{-x}\right)}^{\,e^{-x}\cancel{\frac{d}{dx}(-x)}^{-1}} + e^{-x}\cancel{\frac{d}{dx}(4x)}^{4}$$

$$= 4xe^{-x}(-1) + 4e^{-x}$$

$$= -4xe^{-x} + 4e^{-x} \;; \quad f'(x) = 4e^{-x} - 4xe^{-x}$$

or

$$f'(x) = 4e^{-x}(1-x)$$

(11) $f(x) = \dfrac{\ln(x)}{x+1}$ $\qquad \dfrac{1}{x}$

$$f'(x) = \frac{(x+1)\frac{d}{dx}(\ln(x)) - \ln(x)\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$f'(x) = \frac{(x+1)\cdot\frac{1}{x} - \ln(x)}{(x+1)^2}$$

$$f'(x) = \frac{\frac{x+1}{x} - \ln(x)\cdot\frac{x}{x}}{(x+1)^2}$$

$$f'(x) = \frac{x+1 - x\ln(x)}{x} \div (x+1)^2$$

$$f'(x) = \frac{x+1 - x\ln(x)}{x} \cdot \frac{1}{(x+1)^2}$$

$$\boxed{f'(x) = \frac{x+1 - x\ln(x)}{x(x+1)^2}}$$

(12) $f(x) = \tan^{-1}(2x)$

$$f'(x) = \frac{d}{dx}\left[\tan^{-1}(2x)\right]$$

$$= \frac{1}{1+(2x)^2}\,\frac{d}{dx}(2x) \quad 2$$

$$f'(x) = \frac{1}{1+4x^2} \cdot 2$$

$$\boxed{f'(x) = \frac{2}{1+4x^2}}$$

(13) $f(x) = \pi \sin(e^{\sqrt{x}})$

$$f'(x) = \frac{d}{dx}\left[\pi \sin(e^{x^{1/2}})\right]$$

$$= \pi \frac{d}{dx}\left[\sin(e^{x^{1/2}})\right]$$

$$= \pi \cos(e^{x^{1/2}}) \frac{d}{dx}(e^{x^{1/2}})$$

$$e^{x^{1/2}} \cancel{\frac{d}{dx}(x^{1/2})} \quad \frac{1}{2\sqrt{x}}$$

$$f'(x) = \pi \cos(e^{\sqrt{x}})\, e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$\boxed{f'(x) = \frac{\pi e^{\sqrt{x}}}{2\sqrt{x}} \cos(e^{\sqrt{x}})}$$

(14) $\int (e^{2x} + e^{-3x})\, dx$

$= \int e^{2x}\, dx + \int e^{-3x}\, dx$

let $u = 2x$

$\frac{du}{dx} = 2$;

$dx = \frac{du}{2}$

let $u = -3x$

$\frac{du}{dx} = -3$; $dx = \frac{du}{-3}$

$\int e^{u} \frac{du}{2}$

$\frac{1}{2}\int e^{u}\, du$

$\frac{1}{2} e^{u} + C$

$\int e^{u} \frac{du}{-3}$

$-\frac{1}{3}\int e^{u}\, du$

$-\frac{1}{3} e^{u} + C$

$$\boxed{\frac{1}{2} e^{2x} - \frac{1}{3} e^{-3x} + C}$$

(15) $\int (2 + \frac{1}{x})(3 + \frac{1}{x})\, dx$

mult - foil

$6 + \frac{2}{x} + \frac{3}{x} + \frac{1}{x^2}$

$6 + \frac{5}{x} + \frac{1}{x^2}$

$\int \left(6 + \frac{5}{x} + \frac{1}{x^2}\right) dx = 6\int dx + 5\int \frac{1}{x}\, dx + \int \frac{1}{x^2}\, dx$

$$\boxed{6x + 5\ln|x| - \frac{1}{x} + C}$$

(16) $\displaystyle\int \frac{1}{5x+4}\,dx = \int \frac{1}{u}\,\frac{du}{5}$

$u = 5x+4$

$\dfrac{du}{dx} = 5 \;;\quad dx = \dfrac{du}{5}$

$\dfrac{1}{5}\displaystyle\int \frac{1}{u}\,du$

$\dfrac{1}{5}\ln|u| + C$

$$\boxed{\frac{1}{5}\ln|5x+4| + C}$$

(17) $\displaystyle\int \frac{1}{1+16x^2}\,dx$

4

$(4x)^2$

$\displaystyle\int \frac{1}{1+(4x)^2}\,dx$ let $u = 4x$

$\dfrac{du}{dx} = 4 \;;\quad dx = \dfrac{du}{4}$

$\displaystyle\int \frac{1}{1+u^2}\,\frac{du}{4}$

$\dfrac{1}{4}\displaystyle\int \frac{1}{1+u^2}\,du = \frac{1}{4}\tan^{-1}(u) + C$

$4x$

$$\boxed{\frac{1}{4}\tan^{-1}(4x) + C}$$

(18) $\displaystyle\int \frac{x-5}{x^2+25}\,dx = \int \frac{x}{x^2+25}\,dx - \int \frac{5}{x^2+25}\,dx$

$\displaystyle\int \frac{x}{x^2+25}\,dx - 5\int \frac{1}{x^2+25}\,dx$

$$\int \frac{x}{x^2+25}\,dx - 5\int \frac{1}{x^2+25}\,dx$$

$25 = 5^2$

$u = x^2+25$

$\frac{du}{dx} = 2x \;;\; dx = \frac{du}{2x}$

$\frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right) + C$

$$\int \frac{\cancel{x}}{u}\cdot\frac{du}{2\cancel{x}}$$

$$\boxed{\frac{1}{2}\ln(x^2+25) + - \tan^{-1}\left(\frac{x}{5}\right) + C}$$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u|$

$\frac{1}{2}\ln(x^2+25) + C$

(19) $\int \frac{1}{\sqrt{25-x^2}}\,dx = \int \frac{1}{\sqrt{5^2-x^2}}\,dx$

$25 = 5^2$

note $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$

$$\boxed{\sin^{-1}\left(\frac{x}{5}\right) + C}$$