

East Los Angeles College  
Department of Mathematics  
Math 230  
Test 3

76  
solutions

Construct a truth table for the following compound statements and determine which statements are tautologies, self-contradictions, or neither.

1.  $(\sim p \vee q) \leftrightarrow \sim q$

| $(\sim p \vee q)$ | $\leftrightarrow$ | $\sim q$ |  |
|-------------------|-------------------|----------|--|
| T                 |                   | F        |  |
| F                 |                   | T        |  |
| T                 |                   | F        |  |
| T                 |                   | T        |  |

Neither

2.  $(p \wedge \sim q) \leftrightarrow \sim(p \rightarrow q)$

| $(p \wedge \sim q)$ | $\leftrightarrow$ | $\sim(p \rightarrow q)$ |  |
|---------------------|-------------------|-------------------------|--|
| F                   |                   | F                       |  |
| T                   |                   | T                       |  |
| F                   |                   | F                       |  |
| F                   |                   | F                       |  |

Tautology

3.  $(\sim p \vee q) \leftrightarrow (p \wedge \sim q)$

| $(\sim p \vee q)$ | $\leftrightarrow$ | $(p \wedge \sim q)$ |  |
|-------------------|-------------------|---------------------|--|
| T                 |                   | F                   |  |
| F                 |                   | T                   |  |
| T                 |                   | F                   |  |
| T                 |                   | F                   |  |

Self contradiction

15 -

Determine whether the statement is an implication by constructing a truth table.

4.  $p \rightarrow (p \vee \sim q)$

| $p$ | $\rightarrow$ | $(p \vee \sim q)$ |
|-----|---------------|-------------------|
| T   |               | T                 |
| T   |               | T                 |
| F   |               | F                 |
| F   |               | T                 |

implication

5.  $[(p \vee q) \wedge p] \rightarrow (p \vee q)$

|   |   |   |
|---|---|---|
| T | T | T |
| T | T | T |
| F | T | T |
| F | T | F |

implication

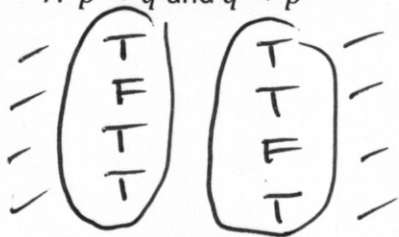
Determine which statements are equivalent by constructing a truth table.

6.  $p \rightarrow q$  and  $\sim p \vee q$

|   |   |
|---|---|
| T | T |
| F | F |
| T | T |
| F | T |

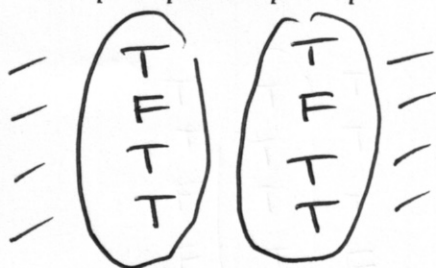
equivalent

7.  $p \rightarrow q$  and  $q \rightarrow p$



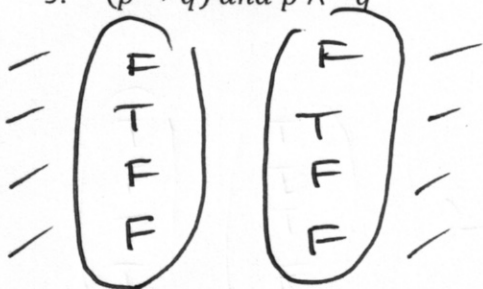
not equivalent

8.  $p \rightarrow q$  and  $\sim q \rightarrow \sim p$



equivalent

9.  $\sim(p \rightarrow q)$  and  $p \wedge \sim q$



equivalent

Use De Morgan's Laws to Determine whether the following statements are equivalent.  
Do not use Truth Tables.

10.  $\sim(p \vee q)$  and  $\sim p \vee \sim q$

$$\sim(p \vee q) \equiv \sim p \wedge \sim q \neq \sim p \vee \sim q$$

not equivalent

11.  $\sim(p \wedge q)$  and  $\sim p \vee q$

$$\sim(p \wedge q) \equiv \sim p \vee \sim q \neq \sim p \vee q$$

not equivalent

12.  $\sim(p \vee q)$  and  $\sim p \wedge \sim q$

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

equivalent

13.  $(\sim p \vee \sim q) \rightarrow r$  and  $\sim(p \wedge q) \rightarrow r$

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

So,  $(\sim p \vee \sim q) \rightarrow r \equiv \sim(p \wedge q) \rightarrow r$

equivalent

14.  $q \rightarrow \sim(p \wedge \sim r)$  and  $q \rightarrow (\sim p \vee r)$

$$\sim(p \wedge \sim r) \equiv \sim p \vee r$$

So,  $q \rightarrow \sim(p \wedge \sim r) \equiv q \rightarrow (\sim p \vee r)$

equivalent

15. What is your name?

is -



# Math 230 Test 3

①  $(\sim p \vee q) \leftrightarrow \sim q$

| p | q | $\sim p$ | $\vee$ | q | $(\sim p \vee q)$ | $\leftrightarrow$ | $\sim q$ |
|---|---|----------|--------|---|-------------------|-------------------|----------|
| T | T | F        | T      | T | T                 | F                 | F        |
| T | F | F        | F      | F | F                 | F                 | T        |
| F | T | T        | T      | T | T                 | F                 | F        |
| F | F | T        | T      | F | T                 | T                 | T        |

neither

②

| p | q | $p \wedge \sim q$ | $p \vee q$ | $(p \wedge \sim q) \leftrightarrow \sim(p \vee q)$ |
|---|---|-------------------|------------|----------------------------------------------------|
| T | T | F                 | T          | F                                                  |
| T | F | T                 | T          | T                                                  |
| F | T | F                 | T          | F                                                  |
| F | F | F                 | F          | T                                                  |

tautologous

③

| p | q | $\sim p \vee q$ | $p \wedge \sim q$ | $(\sim p \vee q) \leftrightarrow (p \wedge \sim q)$ |
|---|---|-----------------|-------------------|-----------------------------------------------------|
| T | T | F               | F                 | F                                                   |
| T | F | F               | T                 | F                                                   |
| F | T | T               | F                 | F                                                   |
| F | F | T               | F                 | F                                                   |

Self-contradiction

(4)  $p \rightarrow (p \vee \sim q)$

| p | q | $p \vee \sim q$ | $p \rightarrow (p \vee \sim q)$ |
|---|---|-----------------|---------------------------------|
| T | T | T               | T                               |
| T | F | T               | T                               |
| F | T | F               | F                               |
| F | F | T               | T                               |

Implication

(5)

| p | q | $p \vee q$ | $(p \vee q) \wedge p$ | $(p \vee q) \wedge p \rightarrow (p \vee q)$ |
|---|---|------------|-----------------------|----------------------------------------------|
| T | T | T          | T                     | T                                            |
| T | F | T          | T                     | T                                            |
| F | T | T          | F                     | T                                            |
| F | F | F          | F                     | F                                            |

Implication

(6)  $p \rightarrow q \stackrel{?}{=} \sim p \vee q$

| p | q | $p \rightarrow q$ | $\sim p \vee q$ |
|---|---|-------------------|-----------------|
| T | T | T                 | T               |
| T | F | F                 | F               |
| F | T | T                 | T               |
| F | F | T                 | T               |

Äquivalent

7)

| P | Q | $P \rightarrow Q$ | $Q \rightarrow P$ |
|---|---|-------------------|-------------------|
| T | T | T                 | T                 |
| T | F | F                 | T                 |
| F | T | T                 | F                 |
| F | F | T                 | F                 |

not equivalent

8)

| P | Q | $P \rightarrow Q$ | $\sim Q \rightarrow \sim P$ |
|---|---|-------------------|-----------------------------|
| T | T | T                 | T                           |
| T | F | F                 | F                           |
| F | T | T                 | T                           |
| F | F | T                 | T                           |

equivalent

9)

| P | Q | $P \rightarrow Q$ | $\sim(P \rightarrow Q)$ | $P \wedge \sim Q$ |
|---|---|-------------------|-------------------------|-------------------|
| T | T | T                 | F                       | F                 |
| T | F | F                 | T                       | T                 |
| F | T | T                 | F                       | F                 |
| F | F | T                 | F                       | F                 |

equivalent

10)  $\sim(P \vee Q)$  and  $\sim P \vee \sim Q$

$\sim(P \vee Q) \equiv \sim P \wedge \sim Q \neq \sim P \vee \sim Q$

not equivalent



$$(11) \quad \sim(P \wedge Q) \stackrel{?}{=} \sim P \vee Q$$

$$\sim(P \wedge Q) \stackrel{\text{DM}}{=} \sim P \vee \sim Q \neq \sim P \vee Q$$

Not Equivalent

$$(12) \quad \sim(P \vee Q) \stackrel{?}{=} \sim P \wedge \sim Q$$

$$\sim(P \vee Q) \stackrel{\text{DM}}{=} \sim P \wedge \sim Q$$

Equivalent

$$(13) \quad (\sim P \vee \sim Q) \rightarrow R \stackrel{?}{=} \sim(P \wedge Q) \rightarrow R$$

(Start)

$$\sim(P \wedge Q) \stackrel{\text{DM}}{=} \sim P \vee \sim Q$$

$$\text{So, } (\sim P \vee \sim Q) \rightarrow R \equiv \sim(P \wedge Q) \rightarrow R$$

Equivalent

$$(14) \quad Q \rightarrow \sim(P \wedge \sim R) \stackrel{?}{=} Q \rightarrow (\sim P \vee R)$$

$$\sim(P \wedge \sim R) \stackrel{\text{DM}}{=} \sim P \vee R$$

$$\text{So, } Q \rightarrow \sim(P \wedge \sim R) \equiv Q \rightarrow (\sim P \vee R)$$

Equivalent