East Los Angeles College
Department of Mathematics
Math 262
Test 1

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Solutions

Show all work for credit- No calculators

Determine the limits for the following.

1. $\lim_{x\to 0^+} \ln\left(\frac{1}{x}\right)$

as $x \to 0^+$

$\frac{1}{x} \to \infty$

$\ln\left(\frac{1}{x}\right) \to \boxed{\infty}$

2. $\lim_{x\to -1} \sin^{-1}\left(\frac{2x}{x-1}\right)$

as $x \to -1$

$\frac{2x}{x-1} \to 1$

$\sin^{-1}\left(\frac{2x}{x-1}\right) \to \sin^{-1}(1) = \boxed{\frac{\pi}{2}}$

3. $\lim_{x\to 0} \frac{\ln(1+x)}{\ln(1-x^2)}$

DNE

4. $\lim_{x\to 0} \frac{e^{x^3}-1}{x-\sin(x)}$

$\frac{0}{0}$

$\boxed{6}$

5. $\lim_{x\to\infty} xe^{-\sqrt{x}}$ $\quad \infty \cdot 0$

$\boxed{0}$

6. $\lim_{x\to \frac{\pi}{2}^+} \sec(x)\cot(x)$ $\quad -\infty \cdot 0$

$\boxed{1}$

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$$\frac{1}{1+x} \div \frac{-2x}{1-x^2}$$

$$\frac{1}{1+x} \cdot \frac{1-x^2}{-2x}$$

$$\frac{1}{\cancel{1+x}} \cdot \frac{\cancel{(1+x)}(1-x)}{-2x}$$

$$\frac{1-x}{-2x} = -\frac{1}{2x} + \frac{x}{2x}$$

$$\frac{1}{2} - \frac{1}{2x} \ ; \ \lim_{x \to 0} \left(\frac{1}{2} - \frac{1}{2x} \right)$$

$$\frac{1}{2} - \frac{1}{2} \boxed{\lim_{x \to 0} \left(\frac{1}{x} \right)} \quad \text{DNE}$$

(4) $\lim_{x \to 0} \dfrac{e^{x^3} - 1}{x - \sin(x)}$ $\quad \dfrac{0}{0}$

$\underset{H}{=} \lim_{x \to 0} \dfrac{e^{x^3} \cdot 3x^2}{1 - \cos x}$ $\quad \dfrac{0}{0}$

$\underset{H}{=} \lim_{x \to 0} \dfrac{e^{x^3} \cdot 6x + e^{x^3} \cdot 3x^2 \cdot 3x^2}{\sin x}$

<u>note</u> $\dfrac{6x\, e^{x^3} + 9x^4\, e^{x^3}}{\sin x}$

$\dfrac{3x\, e^{x^3} (2 + 3x^3)}{\sin x}$

or $\dfrac{6x\, e^{x^3} + 9x^4\, e^{x^3}}{\sin x}$ $\quad \dfrac{0}{0}$ $\quad e^{x^3}(6x + 9x^4)$

$\underset{H}{=} \lim_{x \to 0} \dfrac{e^{x^3}(6 + 36x^3) + e^{x^3} \cdot 3x^2 \cdot (6x + 9x^4)}{\cos x}$

$\dfrac{e^0 (6 + 36 \cdot 0) + e^0 \cdot 3 \cdot 0 \cdot (6 \cdot 0 + 9 \cdot 0)}{\cos 0}$

$\dfrac{1(6) + 1 \cdot 3 \cdot 0 \cdot 0}{1}$ $\quad \boxed{6}$

(5) $\lim_{x \to \infty} x e^{-\sqrt{x}}$ $\qquad \infty \cdot 0$

as $x \to \infty$ $\qquad = \lim_{x\to\infty} \frac{e^{-\sqrt{x}}}{1/x}$ $\qquad \frac{0}{0}$

$-\sqrt{x} \to -\infty$

$e^{-\sqrt{x}} \to 0$

$$\underset{H}{=} \lim_{x\to\infty} \frac{\frac{d}{dx}\left(e^{-\sqrt{x}}\right)}{\frac{d}{dx}\left(\frac{1}{x}\right)}$$

note $\frac{d}{dx}\left(e^{-\sqrt{x}}\right) = e^{-\sqrt{x}} \frac{d}{dx}\left(-\sqrt{x}\right)$

$-(x^{1/2})$

$-\frac{1}{2}x^{-1/2}$

$-\frac{1}{2\sqrt{x}}$

$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

$$\lim_{x\to\infty} \frac{e^{-\sqrt{x}}\left(-\frac{1}{2\sqrt{x}}\right)}{-\frac{1}{x^2}}$$

note: $e^{-\sqrt{x}}\left(-\frac{1}{2\sqrt{x}}\right) \div \left(-\frac{1}{x^2}\right)$

$e^{-\sqrt{x}}\left(-\frac{1}{2\sqrt{x}}\right) \cdot \left(-\frac{x^2}{1}\right)$

$$e^{-\sqrt{x}} \quad \frac{x^2}{2\sqrt{x}} = \frac{1}{2} e^{-\sqrt{x}} \quad x^{3/2}$$

$$= \frac{x^{3/2}}{2e^{\sqrt{x}}} \; ; \quad \lim_{x \to \infty} \frac{x^{3/2}}{2e^{\sqrt{x}}} \quad \frac{\infty}{\infty}$$

$$\overset{=}{H} \lim_{x \to \infty} \frac{\frac{3}{2} x^{1/2}}{2e^{\sqrt{x}} \frac{d}{dx}(x^{1/2})}$$

$$= \lim_{x \to \infty} \frac{\frac{3}{2}\sqrt{x}}{2e^{\sqrt{x}} \frac{1}{2\sqrt{x}}} \qquad \text{note} \quad \frac{3}{2}\sqrt{x} \div \frac{\cancel{2}e^{\sqrt{x}}}{\cancel{2}\sqrt{x}}$$

$$\frac{3}{2}\sqrt{x} \div \frac{e^{\sqrt{x}}}{\sqrt{x}} \qquad \frac{3\sqrt{x}}{2} \cdot \frac{\sqrt{x}}{e^{\sqrt{x}}}$$

$$\frac{3x}{2e^{\sqrt{x}}} \; ; \quad \lim_{x \to \infty} \frac{3x}{2e^{\sqrt{x}}} \quad \frac{\infty}{\infty}$$

$$\overset{=}{H} \lim_{x \to \infty} \frac{3}{2e^{\sqrt{x}} \cdot \frac{d}{dx}(x^{1/2})} = \lim_{x \to \infty} \frac{3}{2e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}}$$

$$\lim_{x \to \infty} \frac{3}{\frac{e^{\sqrt{x}}}{\sqrt{x}}} = \lim_{x \to \infty} \frac{3\sqrt{x}}{e^{\sqrt{x}}}$$

$$3 \lim_{x \to \infty} \frac{\sqrt{x}}{e^{\sqrt{x}}} \quad \frac{\infty}{\infty}$$

$\underset{H}{=} 3 \lim_{x\to\infty} \frac{\sqrt{x}}{e^{\sqrt{x}}} \underset{H}{=} 3 \lim_{x\to\infty} \frac{\cancel{\frac{1}{2\sqrt{x}}}}{e^{\sqrt{x}}\cancel{\frac{1}{2\sqrt{x}}}}$

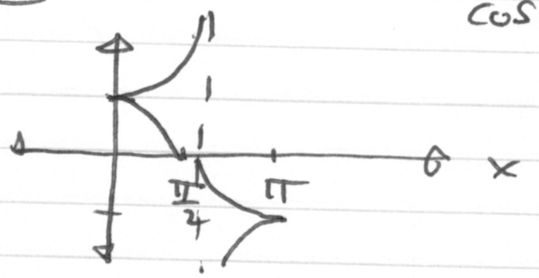
$= 3 \lim_{x\to\infty} e^{-\sqrt{x}} = 3\cdot 0$

$= \boxed{0}$

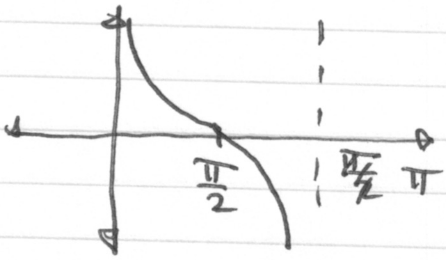
(6) $\lim_{x\to\frac{\pi}{2}^+} \sec(x)\cot(x)$ $\quad -\infty\cdot 0$

note $\sec(x) = \frac{1}{\cos(x)} \to -\infty$

as $x\to\frac{\pi}{2}^+$



$\cot(x) \to 0$ as $x\to\frac{\pi}{2}^+$



so, $\lim_{x\to\frac{\pi}{2}^+} \frac{\sec(x)}{\tan(x)}$ $\quad \frac{-\infty}{-\infty}$

$$\underset{H}{=} \lim_{x \to \frac{\pi}{2}^+} \frac{\sec(x)\tan(x)}{\sec^2(x)}$$

$$\lim_{x \to \frac{\pi}{2}^+} \frac{\tan(x)}{\sec(x)}$$

note $$\frac{\tan(x)}{\sec(x)} = \frac{\sin x / \cos x}{1 / \cos x}$$

$$\frac{\sin x}{\cos x} \div \frac{1}{\cos x}$$

$$\frac{\sin x}{\cos x} \cdot \cos x = \sin x$$

$$\lim_{x \to \frac{\pi}{2}^+} \sin x = \sin \frac{\pi}{2}$$

$$= 1$$

(7) $y - y_1 = m(x - x_1)$; $y = x^{\sin x}$

$\overset{x}{(\pi}, \overset{y}{1}),$

$y - 1 = m(x - \pi)$; $m = y'\big|_{x=\pi}$

$\cancel{y}$ $\ln y = \ln\left(x^{\sin x}\right)$

$= \sin x \ln x$

~~as~~ $\cancel{x/\theta}$ $\frac{d}{dx}(\ln y) = \frac{d}{dx}(\sin x \ln x)$

$\frac{1}{y} \cdot y' = \sin x \cdot \frac{1}{x} + \ln x \cos x$

$y' = y\left[\frac{\sin x}{x} + \ln x \cos x\right]$

$\cancel{y'\big|_{x=\pi} =}$

$y'\Big|_{x=\pi} = x^{\sin x}\left[\frac{\sin x}{x} + \ln x \cos x\right]$

$y' = \pi^{\sin \pi}\left[\frac{\sin \pi}{\pi} + \ln \pi \cos \pi\right]$

$= \pi^0\left[\frac{0}{\pi} + \ln \pi \cdot (-1)\right]$

$m = -\ln \pi$

$$y - 1 = -\ln\pi\,(x - \pi)$$

$$\boxed{y = -\ln\pi\, x + \pi\ln\pi + 1}$$

(e) $y = (x-2)\,e^{-x}$

$$y' = (x-2)\frac{d}{dx}\left(e^{-x}\right) + e^{-x}\frac{d}{dx}(x-2)$$

$$= (x-2)\,e^{-x}(-1) + e^{-x}\cdot 1$$

$$= e^{-x} - e^{-x}(x-2)$$

$$= e^{-x}\left[1 - (x-2)\right]$$

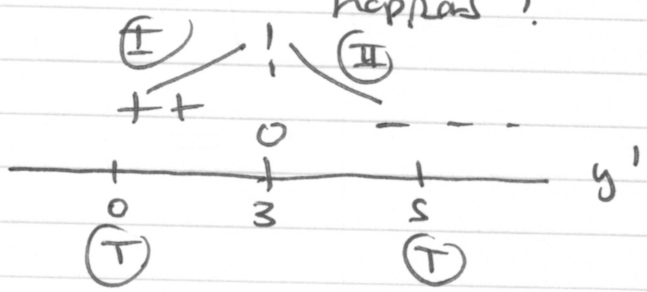
$$= e^{-x}\left[1 - x + 2\right]$$

$$y' = e^{-x}\left[3 - x\right] \;;$$

(a) $y' = 0 \;;\; e^{-x}(3-x) = 0$

$e^{-x} = 0$ | $3 - x = 0$

never happens ! | $x = 3$



increasing $(-\infty, 3)$

Decreasing $(3, \infty)$

Rel max $= y' \Big|_{x=3} = e^{-3}$

(9) $y = \dfrac{\ln x}{\sqrt{x}}$; $x > 0$

$$y' = \frac{\sqrt{x}\,\overset{\frac{1}{x}}{\cancel{\frac{d}{dx}(\ln x)}} - \ln x\,\overset{\frac{1}{2\sqrt{x}}}{\cancel{\frac{d}{dx}(\sqrt{x})}}}{(\sqrt{x})^2}$$

$$\frac{\dfrac{\sqrt{x}}{x} - \dfrac{\ln x}{2\sqrt{x}}}{x}$$

$$\frac{\dfrac{2\sqrt{x}}{2x} - \dfrac{\ln x\,\sqrt{x}}{2\sqrt{x}\,\sqrt{x}}}{x}$$

$$\frac{2\sqrt{x} - \sqrt{x}\ln x}{2x} \div x$$

$$y' = \frac{2\sqrt{x} - \sqrt{x}\ln x}{2x^2} \ ; \ y' = \frac{\sqrt{x}\,(2 - \ln x)}{2x^2}$$

$$y' = \frac{2 - \ln x}{2x^{3/2}}$$

$$y'' = \frac{2x^{3/2} \frac{d}{dx}(2-\ln x) - (2-\ln x)\frac{d}{dx}(2x^{3/2})}{(2x^{3/2})^2}$$

$$y'' = \frac{2x^{3/2}\left(-\frac{1}{x}\right) - (2-\ln x)\,3x^{1/2}}{4x^3}$$

$$= \frac{-2x^{1/2} - 3x^{1/2}(2-\ln x)}{4x^3}$$

$$= \frac{-2x^{1/2} - 6x^{1/2} + 3x^{1/2}\ln x}{4x^3}$$

$$= \frac{-8x^{1/2} + 3x^{1/2}\ln x}{4x^3}$$

$$\frac{-\sqrt{x}\,(8 - 3\ln x)}{4x^3}$$

$$y'' = -\frac{8-3\ln x}{4x^{5/2}}$$

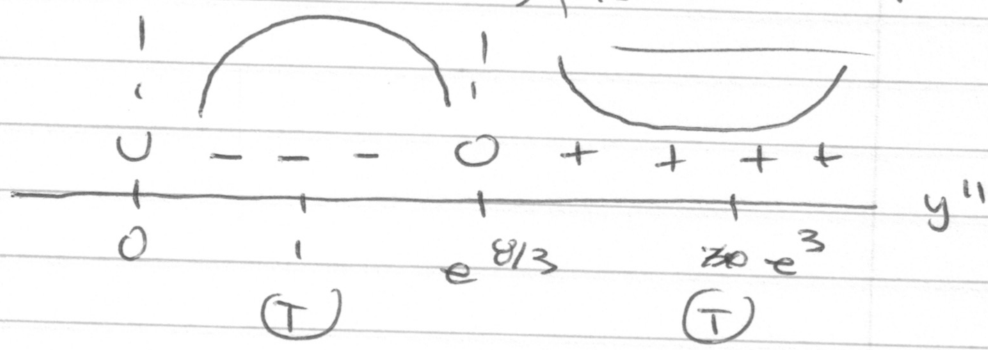
Sign analysis

$y'' = 0$; $-\dfrac{8 - 3\ln x}{4x^{5/2}} = 0$

$\rightarrow$ $8 - 3\ln x = 0$; $3\ln x = 8$

$\ln x = \dfrac{8}{3}$

$e^{\ln x} = e^{8/3}$; $\boxed{x = e^{8/3}}$



$y'' = -\dfrac{8 - 3\ln x}{4x^{5/2}}$; $y'' = -\dfrac{8 - 3\ln e^3}{4(e^3)^{5/2}}$

$y'' = -\dfrac{8}{4}$ $\qquad$ $y'' = -\dfrac{8 - 3\cdot 3}{4e^{15/2}}$

$y'' = -2$ $\qquad$ $= -\dfrac{8-9}{4e^{15/2}}$ $\quad$ $\dfrac{1}{4e^{15/2}}$

(10) $$\int \frac{5}{1+9x^2}\,dx$$

let $u = 3x$

$$\frac{du}{dx} = 3 \; ; \; dx = \frac{du}{3}$$

$$= \int \frac{5}{1+u^2}\,\frac{du}{3} = \frac{5}{3}\int \frac{1}{1+u^2}\,du$$

$$= \frac{5}{3}\tan^{-1}(u) + C$$

$$= \boxed{\frac{5}{3}\tan^{-1}(3x) + C}$$

(11) $$\int \frac{3}{2x-7}\,dx \; ; \; u = 2x-7$$

$$\frac{du}{dx} = 2 \; ;$$

$$dx = \frac{du}{2}$$

$$= \int \frac{3}{u}\,\frac{du}{2}$$

$$= \frac{3}{2}\int \frac{1}{u}\,du = \frac{3}{2}\ln|u| + C$$

$$\boxed{\frac{3}{2}\ln|2x-7| + C}$$

(12) $\int \frac{x}{1+x^4}\, dx$

let $u = x^2$

$\frac{du}{dx} = 2x \; ; \quad dx = \frac{du}{2x}$

$= \int \frac{x}{1+u^2} \frac{du}{2x} = \frac{1}{2} \int \frac{1}{1+u^2}\, du$

$\frac{1}{2} \tan^{-1}(u) + C$

$\boxed{\frac{1}{2} \tan^{-1}(x^2) + C}$

(13) $\int e^{-2\pi x}\, dx$

$u = -2\pi x$

$\frac{du}{dx} = -2\pi \; ; \quad dx = \frac{du}{-2\pi}$

$\int e^u \frac{du}{-2\pi} = -\frac{1}{2\pi} \int e^u\, du$

$-\frac{1}{2\pi} e^u + C$ $\boxed{-\frac{1}{2\pi} e^{-2\pi x} + C}$

(14) $\int \frac{1}{\sqrt{4-25x^2}} \, dx$

note $\sqrt{4-25x^2} = \sqrt{4\left(1-\frac{25x^2}{4}\right)}$

$= \sqrt{4}\sqrt{1-\left(\frac{5x}{2}\right)^2}$

$= 2\sqrt{1-\left(\frac{5x}{2}\right)^2}$

ie, $\int \frac{1}{2\sqrt{1-\left(\frac{5x}{2}\right)^2}} \, dx$; $u = \frac{5x}{2}$

$\frac{du}{dx} = \frac{5}{2}$

$\int \frac{1}{\cancel{2}\sqrt{1-u^2}} \frac{\cancel{2}}{5} \, du$ $\qquad dx = \frac{2}{5} \, du$

$\frac{1}{5}\int \frac{1}{\sqrt{1-u^2}} \, du = \frac{1}{5} \sin^{-1}(u) + C$

$\boxed{\frac{1}{5} \sin^{-1}\left(\frac{5x}{2}\right) + C}$

(15) $\displaystyle\int \frac{\sinh(\ln x)}{x}\,dx$

$u = \ln x$

$\dfrac{du}{dx} = \dfrac{1}{x}$; $dx = x\,du$

$\displaystyle\int \frac{\sinh(u)}{\cancel{x}}\,\cancel{x}\,du$

$\displaystyle\int \sinh(u)\,du = \underline{\cosh u + C}$

$= \boxed{\cosh(\ln x) + C}$