

Math 262 Test 3 27✓

① $\int_0^{\infty} x e^{-x} dx$

$\lim_{t \rightarrow \infty} \int_0^t x e^{-x} dx$

u-sub

$u = x ; dv = \int e^{-x} dx$
 $du = dx$

$v = -e^{-x}$

$-x e^{-x} - \int -e^{-x} dx$

3✓

$-x e^{-x} + \int e^{-x} dx$

$-x e^{-x} - e^{-x} \Big|_0^t$

$-t e^{-t} - e^{-t} - (0 e^{-0} - e^{-0})$

$-t e^{-t} - e^{-t} + 1$ ✓

as $t \rightarrow \infty$

$\lim_{t \rightarrow \infty} (1 - e^{-t} - t e^{-t})$

$1 - \lim_{t \rightarrow \infty} e^{-t} - \lim_{t \rightarrow \infty} t e^{-t}$

$\infty \cdot 0$ ✓

$\boxed{1}$ ✓

$\lim_{t \rightarrow \infty} \frac{t}{e^t} \quad \frac{\infty}{\infty}$

$\lim_{t \rightarrow \infty} \frac{1}{e^t}$ ✓

$$(2) \int_0^3 \frac{1}{\sqrt{9-x^2}} dx \quad \begin{matrix} x \neq 3 \\ \text{VA} \end{matrix}$$

$$\lim_{t \rightarrow 3^-} \int_0^t \frac{1}{\sqrt{9-x^2}} dx$$

$$\begin{aligned} \int_0^t \frac{1}{\sqrt{9-x^2}} dx &= \sin^{-1}\left(\frac{x}{3}\right) \Big|_0^t \\ &= \sin^{-1}\left(\frac{t}{3}\right) - \cancel{\sin^{-1}\left(\frac{0}{3}\right)} \end{aligned}$$

$$\lim_{t \rightarrow 3^-} \sin^{-1}\left(\frac{t}{3}\right) = \sin^{-1}\left(\frac{3}{3}\right)$$

$$= \sin^{-1}(1) \quad \checkmark$$

$$= \left| \frac{\pi}{2} \right| \quad \checkmark$$

$$(3) \int_0^3 \sqrt{1+x^3} dx ; \quad \Delta x = \frac{3-0}{6}$$

$$\Delta x = 0.5 \quad \checkmark$$



$$\begin{aligned} s_6 &= \frac{\Delta x}{3} \left[f(0) + 4 f(0.5) + 2 f(1) \right. \\ &\quad \left. + 4 f(1.5) + 2 f(2) + 4 f(2.5) \right. \\ &\quad \left. + f(3) \right] \end{aligned}$$

$$S_6 = \frac{1}{3} \left[\sqrt{1+0^3} + 4\sqrt{1+0.5^2} + 2\sqrt{1+1^2} + 4\sqrt{1+1.5^2} + 2\sqrt{1+2^2} + 4\sqrt{1+2.5^2} + \sqrt{1+3^2} \right]$$

3 ✓

$$S_6 = \frac{1}{6} \left[1 + 4\sqrt{1+0.5^2} + 2\sqrt{2} + 4\sqrt{1+1.5^2} + 2\sqrt{5} + 4\sqrt{1+2.5^2} + \sqrt{10} \right]$$

$$\boxed{S_6 \approx 7.3398} \checkmark$$

④ $y = x^{3/2}$ over $1 \leq x \leq 2$

$$S = \int_1^2 \sqrt{1 + (y')^2} dx$$

$$y' = \frac{3}{2} x^{3/2-1} ; y' = \frac{3}{2} x^{1/2} \checkmark$$

$$(y')^2 = \left(\frac{3}{2} x^{1/2} \right)^2 ; (y')^2 = \frac{9}{4} x$$

$$S = \int_1^2 \sqrt{1 + \frac{9}{4} x} dx \quad \text{u-sub}$$

$$u = 1 + \frac{9}{4} x ; \frac{du}{dx} = \frac{9}{4} ; dx = \frac{4}{9} du$$

$$S = \int_{13/4}^{11/2} r u \frac{4}{9} du$$

$$= \frac{4}{9} \int_{13/4}^{11/2} u^{1/2} du \quad \checkmark$$

$$= \frac{4}{9} \frac{u^{3/2}}{3/2} \Big|_{13/4}^{11/2}$$

$$= \frac{4}{9} \cdot \frac{2}{3} u^{3/2} \Big|_{13/4}^{11/2} \quad \checkmark$$

$$= \frac{8}{27} \left(\frac{11}{2} \right)^{3/2} - \frac{8}{27} \left(\frac{13}{4} \right)^{3/2}$$

6 ✓

$$= \frac{8}{27} \left[\frac{11}{2} \sqrt{\frac{11}{2}} - \frac{13}{4} \sqrt{\frac{13}{4}} \right]$$

$$= \frac{8}{27} \left[\frac{11}{2} \frac{\sqrt{22}}{2} - \frac{13}{4} \frac{\sqrt{13}}{2} \right]$$

$$\boxed{S = \frac{8}{27} \left[\frac{11}{4} \sqrt{22} - \frac{13}{8} \sqrt{13} \right]}$$

✓

(5) $y = x^2$ wov $0 \leq x \leq 1$

$$s = \int 2\pi r ds$$

φ
 y

$$\sqrt{1 + (y')^2} dx \quad ; \quad y' = 2x$$

3 ✓

$$SA = \int_0^1 2\pi x^2 \sqrt{1 + 4x^2} dx$$

$$(y')^2 = 4x^2$$

$$SA = 2\pi \int_0^1 x^2 \sqrt{1 + 4x^2} dx$$

(6) $x = e^t$
 $y = t$
 $t = 1$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{1}{e^t} \bigg|_{t=1}$$

$$y - y_1 = m(x - x_1)$$

$$\begin{matrix} \varphi & \varphi & \varphi \\ 1 & 1/e & e \end{matrix}$$

$$= \left(\frac{1}{e} \right) \checkmark$$

$$y - 1 = \frac{1}{e}(x - e) \quad ; \quad y - 1 = \frac{1}{e}x - 1$$

$$y = \frac{1}{e}x$$

✓

$$(7) \quad x = t^2$$

$$y = t^3 - 4t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad ; \quad \frac{dy}{dx} = \frac{3t^2 - 4}{2t} \quad \checkmark$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(y')}{dx/dt} = \frac{3t}{2} - \frac{2}{t}$$

$$= \frac{\frac{d}{dt} \left(\frac{3t^2 - 4}{2t} \right)}{2t}$$

$$\frac{2t \frac{d}{dt} (3t^2 - 4) - (3t^2 - 4) \frac{d}{dt} (2t)}{(2t)^2}$$

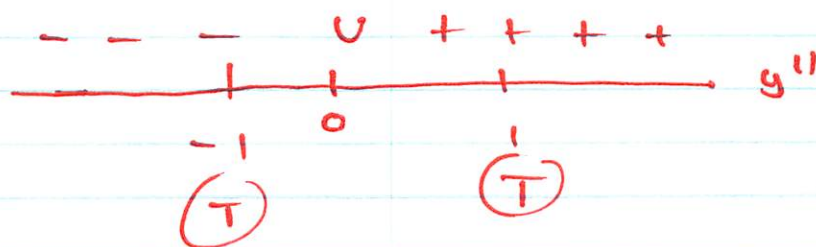
$$\frac{12t^2 - (3t^2 - 4) \cdot 2}{4t^2}$$

$$\frac{12t^2 - 6t^2 + 8}{4t^2}$$

$$\frac{6t^2 + 8}{4t^2}$$

$$= \frac{6t^2 + 8}{4t^2} \div 2t$$

$$y'' = \frac{6t^2 + 8}{8t^3} \quad \checkmark$$



The curve is CU for $t > 0$. ✓

(2) $x = 2t$
 $y = 1 + t^{3/4}$
 $0 \leq t \leq 4$

$$SA = \int_0^4 2\pi r ds$$

$\begin{matrix} 4 & 4 \\ y & \sqrt{(x')^2 + (y')^2} \end{matrix}$

Sorry it was just an arc length question!

3 ✓

$$S = \int_a^b \sqrt{(x')^2 + (y')^2} dt$$

$$\frac{dx}{dt} = 2 ; \quad \frac{dy}{dt} = \frac{3}{4} t^{-1/4} \quad \checkmark$$

$$(x')^2 + (y')^2 = 2^2 + \left(\frac{3}{4} t^{-1/4}\right)^2$$

$$= 4 + \frac{9}{16} t^{-1/2}$$

$$= 4 + \frac{9}{16\sqrt{t}}$$

$$SA = \int_0^4 \sqrt{4 + \frac{9}{16\sqrt{t}}} dt$$

✓