East Los Angeles College
Department of Mathematics
Math 262
Test 3 In class 125 points

SL ✓

Solutions

Determine whether the following integrals are convergent or divergent. Evaluate those that are convergent.

1. $\int_1^\infty \frac{1}{(2x+1)^3} dx$

$$\lim_{t\to\infty} \int_1^t \frac{1}{(2x+1)^3} dx$$

$u = 2x+1$

$$= \lim_{t\to\infty} \int \frac{1}{2} u^{-3} du \quad ✓$$

$$= \lim_{t\to\infty} -\frac{1}{4}\frac{1}{u^2} \quad ✓$$

$$= \lim_{t\to\infty} -\frac{1}{4}\frac{1}{(2x+1)^2}\Big|_1^t \quad ✓$$

$$\lim_{t\to\infty} -\frac{1}{4}\frac{1}{(2t+1)^2} + \frac{1}{36} \quad ✓$$

$$= \lim_{t\to\infty}\left(\frac{1}{36} - \frac{1}{4(2t+1)^2}\right)$$

$$= \frac{1}{36} \quad \text{conv} \quad ✓$$

2. $\int_{-\infty}^0 xe^{2x} dx$

$$= \lim_{t\to-\infty} \int_t^0 xe^{2x} dx \quad ✓$$

IBP

$$= \lim_{t\to-\infty} \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\Big|_{x=t}^{x=0} \quad ✓$$

10 ✓

✓ ✓

$$= \lim_{t\to-\infty}\left(-\frac{1}{4} - \frac{1}{2}te^{2t} + \frac{1}{4}e^{2t}\right)$$

$$-\frac{1}{4} \quad \text{conv} \quad ✓$$

3. $\int_{-\infty}^{\infty} x^3 e^{-x^4}\,dx \quad = \int_{-\infty}^{0} x^3 e^{-x^4}\,dx + \int_{0}^{\infty} x^3 e^{-x^4}\,dx$

$$I_1 \quad + \quad I_2$$

$$I = \int x^3 e^{-x^4}\,dx = -\frac{1}{4} e^{-x^4}$$

$$I_1 = \lim_{t\to-\infty} \int_t^0 x^3 e^{-x^4}\,dx = \lim_{t\to-\infty} -\frac{1}{4} e^{-x^4} \Big|_t^0$$

$$= \frac{1}{4} e^{-t^4} - \frac{1}{4} \;; \quad \text{as } t \to -\infty \;; \quad I_1 \to -\frac{1}{4}$$

$$I_2 = \lim_{t\to\infty} \int_0^t x^3 e^{-x^4}\,dx = \lim_{t\to\infty} \left(\frac{1}{4} - \frac{1}{4} e^{-t^4}\right)$$

$$\text{as } t \to \infty \;; \quad I_2 \to \frac{1}{4} \;; \quad -\frac{1}{4} + \frac{1}{4} = \boxed{0}$$

conv

4. $\int_0^1 \frac{\ln(x)}{\sqrt{x}}\,dx \quad = \lim_{t\to0^+} \int_t^1 \frac{\ln(x)}{\sqrt{x}}\,dx$ use IBP

$u = \ln x \;; \quad dv = \frac{1}{\sqrt{x}}\,dx$

$$\int_t^1 \frac{\ln(x)}{\sqrt{x}}\,dx = 2\sqrt{x}\ln(x) - 4\sqrt{x} \Big|_t^1$$

$$I = -4 - 2\sqrt{t}\ln(t) + 4\sqrt{t}$$

$$\text{as } t \to 0^+ \;; \quad I \to \boxed{-4}$$

note $\lim_{t\to0^+} \sqrt{t}\ln(t)$

$$\underset{H}{=} \lim_{t\to0^+} \frac{\ln(t)}{1/\sqrt{t}}$$

$$= \lim_{t\to0^+} -2\sqrt{t}$$

$$= 0$$

10

(Set up)

Determine the exact length of the curve.

5. $y = 1 - e^{-x}$ over $0 \leq x \leq 2$

$$S = \int_0^2 \sqrt{1+(y')^2}\, dx$$

$y' = e^{-x}$; ✓

$$1 + (y')^2 = 1 + (e^{-x})^2$$
$$= 1 + e^{-2x} ✓$$

ie, $S = \int_0^2 \sqrt{1+e^{-2x}}\, dx$ ✓

3 ✓

Determine the exact area of the surface by rotating the curve about the x-axis.

6. $x = \frac{1}{3}(y^2+2)^{3/2}$ over $1 \leq \cancel{x} \leq 2$ supposed to be y $1 \leq y \leq 2$

$SA = \int 2\pi r\, ds$

y $\sqrt{(y^2+1)^2} = y^2+1$ ✓

8 ✓

$$SA = \int_1^2 2\pi y\,(y^2+1)\, dy = \cancel{2\pi}\Big[$$

$$= \int_1^2 2\pi(y^3+y)\, dy ✓ = 2\pi\left[\left[\tfrac{1}{4}y^4 + \tfrac{1}{2}y^2\right]\Big|_1^2\right] ✓$$

$$= 2\pi\left[\tfrac{1}{4}\cdot 2^4 + \tfrac{1}{2}\cdot \cancel{2}^2\right] - 2\pi\left[\tfrac{1}{4}\cdot 1^4 + \tfrac{1}{2}\cdot 1^2\right]$$ ✓

$$= 2\pi\left[4 + 2 - \tfrac{1}{4} - \tfrac{1}{2}\right]$$

$$= 2\pi\left(\tfrac{21}{4}\right) \quad \boxed{\tfrac{21\pi}{2}}$$ ✓

5 ✓

Determine the exact area of the surface by rotating the curve about the y-axis.

7. $y = \sqrt[3]{x}$ over $1 \le y \le 2$

$SA = \int 2\pi r \, dS$

$SA = \int 2\pi x \, dS$

$\sqrt{1+(x')^2}\, dy$

$x = y^3$

$SA = \int_1^2 2\pi y^3 \sqrt{1+9y^4}\, dy$

u-Sub ; $u = 1+9y^4$

$SA = \int_1^2 2\pi y^3 \sqrt{u}\, \frac{du}{36y^3}$

$SA = \int_1^2 \frac{\pi}{36} u^{1/2}\, du$

$\frac{\pi}{54} u^{3/2} \Big|_{u=10}^{u=145}$

$\frac{\pi}{54}\left[145\sqrt{145} + 10\sqrt{10}\right]$

8

Eliminate the parameter and find the Cartesian coordinate equation of the curve and sketch the curve.

8. $x = \sqrt{t+1}$ $\quad x^2 = t+1$; $\quad x^2 - 1 = t$

$y = \sqrt{t-1}$ $\quad y^2 = t-1$; $\quad y^2 + 1 = t$

$x^2 - 1 = y^2 + 1$

$x^2 - y^2 = 2$

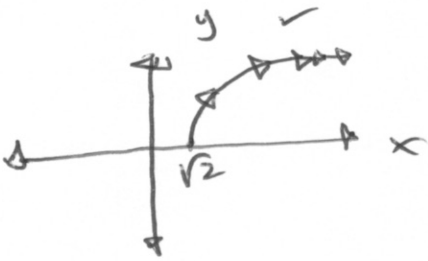
$\frac{x^2 - y^2}{2} = 1$

i.e., $\frac{x^2}{2} - \frac{y^2}{2} = 1$ Horizontal Hyperbola

note: $x = \sqrt{t+1} \ge 0$ for $t+1 \ge 0$ or $t \ge -1$

$y = \sqrt{t+1} \ge 0$ for $t+1 \ge 0$ or $t \ge 1$

i.e., (x, y) are in QI and $t \ge 1$



Sr

9. Use Simpson's rule to approximate the integral $\int_0^2 sin(e^{\pi x})dx$ with n=6. Setup, do not calculate.

$n = 6$

$\Delta x = \frac{2-0}{6} = \frac{1}{3}$ ✓

$x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6$ ✓

$0 \quad 1/3 \quad 2/3 \quad 1 \quad 4/3 \quad 5/3 \quad 2$

✓

$S_6 = \frac{1}{9}\left[\operatorname{Sin}(1) + 4\operatorname{Sin}(e^{\pi/3}) + 2\operatorname{Sin}(e^{2\pi/3})\right.$

✓ $+ 4\operatorname{Sin}(e^{2\pi}) + 2\operatorname{Sin}(e^{4\pi/3})$ ✓ 5 ✓

$\left. + 4\operatorname{Sin}(e^{5\pi/3}) + \operatorname{Sin}(e^{2\pi})\right]$

10. Determine the equation of the line tangent to the curve at the indicated point.

$x = cos(t) + cos(2t)$
$y = sin(t) + sin(2t)$
$(-1,1)$

5 ✓

✓

$m = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos(t) + 2\cos(2t)}{-\operatorname{Sin}(t) - 2\operatorname{Sin}(2t)} \Bigg|_{t=\pi/2}$ ✓

✓

$m = 2$

$y - 1 = 2(x - -1)$

✓

$y - 1 = 2x + 2$

$y = 2x + 3$ 10 ✓

✓

math 262 Test 3 (class)

(1)

$$\int_1^\infty \frac{1}{(2x+1)^3}\,dx$$

$$= \lim_{t\to\infty} \int_1^t \frac{1}{(2x+1)^3}\,dx$$

u-sub

$u = 2x+1$; $du = 2\,dx$; $dx = \frac{du}{2}$

$$= \lim_{t\to\infty} \int_1^t \frac{1}{u^3}\,\frac{du}{2} = \frac{1}{2}\lim_{t\to\infty}\int_{x=1}^{x=t} u^{-3}\,du$$

$$= \frac{1}{2}\lim_{t\to\infty} \frac{u^{-2}}{-2} = -\frac{1}{4}\lim \frac{1}{u^2}\Bigg|_{x=1}^{x=t}$$

$$= -\frac{1}{4}\lim_{t\to\infty} \frac{1}{(2x+1)^2}\Bigg|_{x=1}^{x=t}$$

$$= -\frac{1}{4}\lim_{t\to\infty} \frac{1}{\cancel{36}\ (2t+1)^2} - \frac{1}{\cancel{9}}$$

$$= -\frac{1}{4}\lim_{t\to\infty}\left[\frac{1}{\cancel{(2t+1)^2}\ (2t+1)^2} - \frac{1}{\cancel{9}}\right]$$

$$= -\frac{1}{4}\left[-\frac{1}{\cancel{9}}\right] = \frac{1}{\cancel{144}}\ 3$$ convergence

$$\frac{1}{36}$$

(2) $\int_{-\infty}^{0} x e^{2x}\,dx$

$= \lim_{t \to -\infty} \int_{t}^{0} x e^{2x}\,dx$

use IBP

$I = \int x e^{2x}\,dx = uv - \int v\,du$

$u = x;\ dv = e^{2x}\,dx \qquad = \frac{1}{2} x e^{2x} - \int \frac{1}{2} e^{2x}\,dx$

$du = dx;\ v = \int e^{2x}\,dx \qquad = \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x}$

$= \frac{1}{2} e^{2x}$

$= \lim_{t \to -\infty} \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} \Big|_{t}^{0}$

$\lim_{t \to -\infty} \left(-\frac{1}{4} - \frac{1}{2} t e^{2t} + \frac{1}{4} e^{2t} \right)$

$-\frac{1}{4} - \frac{1}{2} \lim_{t \to -\infty} t e^{2t} + \frac{1}{4} \cancel{\lim_{t \to -\infty}} \overset{0}{e^{2t}}$

$-\infty \cdot 0$

fix

$\lim_{t \to -\infty} t e^{2t} = \lim_{t \to -\infty} \frac{t}{e^{-2t}} \qquad \frac{-\infty}{\infty}$

$\overset{H}{=} \lim_{t \to -\infty} \frac{1}{-2e^{-2t}} =$

ie, $I \to \left(-\frac{1}{4}\right)$ as $t \to -\infty$

Conv

$= \lim_{t \to -\infty} -\frac{1}{2} e^{2t} = 0$

(3) $\displaystyle\int_{-\infty}^{\infty} x^3 e^{-x^4} dx$

$$= \underbrace{\int_{-\infty}^{0} x^3 e^{-x^4} dx}_{I_1} + \underbrace{\int_{0}^{\infty} x^3 e^{-x^4} dx}_{I_2}$$

<u>note</u> $I = \int x^3 e^{-x^4} dx$

u-sub

$u = -x^4$

$du = -4x^3 dx$

$dx = \dfrac{du}{-4x^3}$

$$= \int x^3 e^u \frac{du}{-4x^3}$$

$$= -\frac{1}{4}\int e^u du = -\frac{1}{4} e^u$$

$$= -\frac{1}{4} e^{-x^4}$$

$$I_1 = \lim_{t \to -\infty} \int_t^0 x^3 e^{-x^4} dx$$

$$= \lim_{t \to -\infty} -\frac{1}{4} e^{-x^4} \Big|_t^0$$

$$= \lim_{t \to -\infty} -\frac{1}{4} e^0 + \frac{1}{4} e^{-t^4}$$

$$= \lim_{t \to -\infty} \left(\frac{1}{4} e^{-t^4} - \frac{1}{4}\right)$$

as $t \to -\infty$ $\quad e^{-t^4} \to 0$

$-t^4 \to -\infty$

; $I \to -\frac{1}{4}$

(4) $\int_0^1 \frac{\ln(x)}{\sqrt{x}}\,dx = \lim_{t\to 0^+} \int_t^1 \frac{\ln(x)}{\sqrt{x}}\,dx$

$\int_t^1 \frac{\ln(x)}{\sqrt{x}}\,dx$ use IBP

$u = \ln(x)$; $dv = \int \frac{1}{\sqrt{x}}\,dx$

$du = \frac{1}{x}\,dx$ $\quad v = \int x^{-1/2}\,dx$

$dx = x\,du$ $\quad v = \frac{x^{1/2}}{1/2} = 2\sqrt{x}$

$uv - \int v\,du$

$2\sqrt{x}\ln(x) - \int 2\sqrt{x}\cdot\frac{1}{x}\,dx$

$2\sqrt{x}\ln(x) - 2\int \frac{1}{\sqrt{x}}\,dx$

$2\sqrt{x}\ln(x) - 2\int x^{-1/2}\,dx$

$2\sqrt{x}\ln(x) - 2\frac{x^{1/2}}{1/2}$

$2\sqrt{x}\ln(x) - 4\sqrt{x}\,\Big|_t^1$

$2\overset{0}{\cancel{\ln(1)}} - 4\sqrt{1} - \left(2\sqrt{t}\ln(t) - 4\sqrt{t}\right)$

$I = -4 - 2\sqrt{t}\ln(t) + 4\sqrt{t}$

as $t\to 0^+$; $\lim_{t\to 0^+}\left(-4 - 2\lim_{t\to 0^+}\left(\sqrt{t}\ln(t) + 4\sqrt{t}\right)\right)$

$$I_2 = \int_0^\infty x^3 e^{-x^4}\,dx$$

$$= \lim_{t\to\infty} \int_0^t x^3 e^{-x^4}\,dx$$

$$= \lim_{t\to\infty} -\frac{1}{4} e^{-x^4} \Big|_{x=0}^{x=t}$$

$$= \lim_{t\to\infty} -\frac{1}{4} e^{-t^4} + \frac{1}{4}$$

$$= \lim_{t\to\infty} \frac{1}{4} - \frac{1}{4} e^{-t^4} = \frac{1}{4}$$

ie, $I = I_1 + I_2 = -\frac{1}{4} + \frac{1}{4} = 0$

conv

as $t \to 0^+$

$$\lim_{t \to 0^+} \left(-4 - 2\sqrt{t}\ \ln(t) + 4\sqrt{t}\right)$$

$$-4 - 2\lim_{t \to 0^+} \sqrt{t}\ \ln(t) + 4\cancel{\lim_{t \to 0^+}} \sqrt{t}^{\,0}$$

$0 \cdot -\infty$

fix

$$\lim_{t \to 0^+} \frac{\ln(t)}{1/\sqrt{t}} \qquad \frac{-\infty}{\infty}$$

$$\underset{H}{=} \lim_{t \to 0^+} \frac{\frac{1}{t}}{-\frac{1}{2}t^{-3/2}} = \lim_{t \to 0^+} \frac{1}{t} \div \left(-\frac{1}{2t^{3/2}}\right)$$

$$= \lim_{t \to 0^+} \frac{1}{t} \cdot \frac{-2t^{3/2}}{1} = -2 \cancel{\lim_{t \to 0^+}} \sqrt{t}^{\,0}$$

ie, $I \to \boxed{-4}$ convergence

(5) $S = \int_0^2 \sqrt{1 + (y')^2}\, dx$

$y = 1 - e^{-x}$; $\frac{dy}{dx} = -e^{-x}(-1) = e^{-x}$

$(y')^2 = (e^{-x})^2 = e^{-2x}$; $1 + (y')^2 = 1 + e^{-2x}$

ie, $S = \int_0^2 \sqrt{1 + e^{-2x}}\, dx$

(6) $x = \frac{1}{3}(y^2+2)^{3/2}$ over $1 \le x \le 2$

rotate about x-axis

$SA = \int 2\pi r\, ds$

$y \qquad \sqrt{1+(x')^2}\, dy$

$x' = \cancel{\frac{1}{3}} \cdot \cancel{\frac{3}{2}} (y^2+2)^{1/2} \cdot \cancel{2}y$

$x' = y\sqrt{y^2+2}$; $(x')^2 = y^2(y^2+2)$

$\sqrt{1 + (x')^2} = \sqrt{1 + y^2(y^2+2)} = \sqrt{1 + y^4 + 2y^2}$

$= \sqrt{y^4 + 2y^2 + 1} = \sqrt{(y^2+1)^2}$

$= y^2 + 1$;

$SA = \int 2\pi y\,(y^2+1)\, dy = 2\pi \int (y^3 + y)\, dy$

$= 2\pi \left[\frac{1}{4}y^4 + \frac{1}{2}y^2 \right]_{1}^{2} =$

$$2\pi\left[\frac{1}{4}2^4+\frac{1}{2}2^2\right]-2\pi\left[\frac{1}{4}1^4+\frac{1}{2}1^2\right]$$

$$2\pi\left[\frac{1}{4}\cdot 16+\frac{1}{2}\cdot 4-\frac{1}{4}-\frac{1}{2}\right]$$

$$2\pi\left[4+2-\frac{1}{4}-\frac{1}{2}\right]$$

$$2\pi\left(6-\frac{1}{4}-\frac{1}{2}\right)$$

$$2\pi\left(\frac{6\cdot 4}{4}-\frac{1}{4}-\frac{2}{4}\right)$$

$$2\pi\left(\frac{24-1-2}{4}\right)$$

$$\cancel{2}\pi\cdot\frac{21}{\cancel{4}\,2}\ ; \quad \boxed{\frac{21\pi}{2}}$$

(7) $y = \sqrt[3]{x} = x^{1/3}$; $1 \le y \le 2$

$SA = \int 2\pi r \, dS$ revolve around y-axis

$x \quad \sqrt{1+(x')^2} \; dy$

y^3

note $y = x^{1/3}$ or $y^3 = x$

$SA = \int_1^2 2\pi y^3 \sqrt{1+(x')^2} \; dy$; $x = y^3$

$\sqrt{1+9y^4}$ $\qquad x' = 3y^2$

$1+(x')^2 = 1+(3y^2)^2$

$SA = \int_1^2 2\pi \, y^3 \sqrt{1+9y^4} \; dy$ $\qquad = 1+9y^4$

u-sub

$u = 1+9y^4$

$\frac{du}{dy} = 36y^3$; $dy = \frac{du}{36y^3}$

$SA = \int_1^2 \pi \cancel{y^3} \sqrt{u} \; \frac{du}{36\cancel{y^3}}$

$= \frac{\pi}{36} \int_1^2 u^{1/2} \, du$

$= \frac{\pi}{36} \frac{u^{3/2}}{3/2} = \frac{\pi}{\cancel{36}} \cdot \frac{\cancel{2}}{3} u^{3/2} = \frac{\pi}{54} u^{3/2}$

$= \frac{\pi}{54} u^{3/2} \Big|_{u=1+9\cdot1^4}^{u=1+9\cdot2^4} = \frac{\pi}{54} u^{3/2} \Big|_{u=10}^{u=145}$

$\frac{\pi}{54} 145^{3/2} - \frac{\pi}{54} 10^{3/2} = \frac{\pi}{54}\left[145 \cdot \sqrt{145} + 10\sqrt{10}\right]$

$\cancel{\frac{\pi}{27}\left[16 - 3\sqrt{2} + 1 - 10\sqrt{10}\right]}$

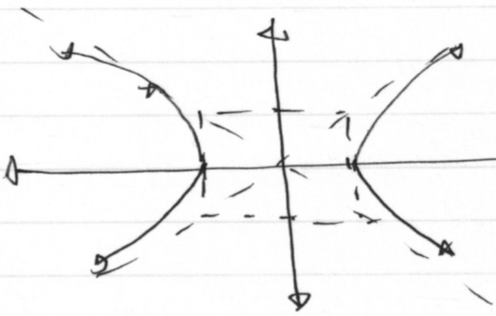
(8) $x = \sqrt{t+1}$; $x^2 = t+1$; $x^2 - 1 = t$

$y = \sqrt{t-1}$ $\quad y^2 = t-1$ $\quad y^2 + 1 = t$

$$x^2 - 1 = y^2 + 1$$

$\boxed{x^2 - y^2 = 2}$ or $\cancel{4}$

$$\frac{x^2}{2} - \frac{y^2}{2} = 1$$ Hyperbola (Horizontal)



But ; $t+1 \geq 0$

$t \geq -1$

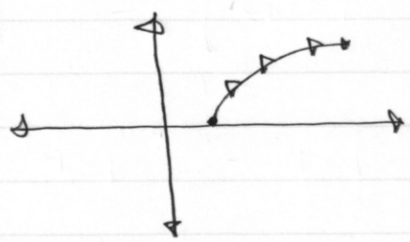
and $t-1 \geq 0$

$t \geq 1$

i.e, $t \geq 1$

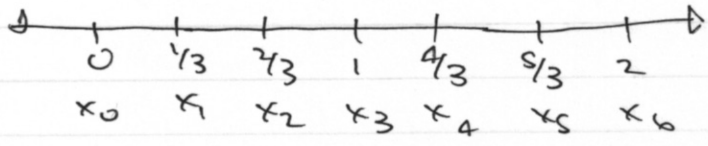
so $x \geq \sqrt{2}$

$y \geq 0$



(9) $\int_0^2 \sin(e^{\pi x})\,dx$; $n=6$

$$\Delta x = \frac{2-0}{6} = \frac{2}{6} = \frac{1}{3}$$



$$S_6 = \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6\right]$$

$$S_6 = \frac{1/3}{3}\left[\sin(e^{\pi\cdot 0}) + 4\sin(e^{\pi\cdot 1/3}) + 2\sin(e^{\pi\cdot 2/3})\right.$$
$$+ 4\sin(e^{\pi\cdot 1}) + 2\sin(e^{\pi\cdot 4/3})$$
$$\left. + 4\sin(e^{\pi 5/3}) + \sin(e^{2\pi})\right]$$

$$\boxed{\begin{aligned} S_6 = \frac{1}{9}\Big[&\sin(1) + 4\sin(e^{\pi/3}) + 2\sin(e^{2\pi/3}) \\ &+ 4\sin(e^{\pi}) + 2\sin(e^{4\pi/3}) \\ &+ 4\sin(e^{5\pi/3}) + \sin(e^{2\pi})\Big]\end{aligned}}$$

(10) $x = \cos(t) + \cos(2t)$

$y = \sin(t) + \sin(2t)$

$(-1, 1)$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos(t) + 2\cos(2t)}{-\sin(t) - 2\sin(2t)}$$

But, $(-1, 1)$

$\cos(t) + \cos(2t) = -1$; $t = \pi/2$

$\sin(t) + \sin(2t) = 1$

$$\text{ie,} \quad m = \frac{\overset{0}{\cancel{\cos(\pi/2)}} + 2\overset{-1}{\cancel{\cos(2\pi/2)}}}{-\underset{1}{\cancel{\sin(\pi/2)}} - 2\underset{0}{\cancel{\sin(2\cdot\pi/2)}}} = \frac{-2}{-1} = 2$$

$y - y_1 = m(x - x_1)$

$y - 1 = 2(x - -1)$; $y - 1 = 2(x + 1)$

$y - 1 = 2x + 2$

$+1 \qquad +1$

$\boxed{y = 2x + 3}$