50 ✓
~~30~~ ✓

East Los Angeles College
Department of Mathematics
Math 262
Test 1

Solutions

Evaluate the following limits.

1. $\lim\limits_{x\to-\infty} \frac{e^{2x}}{e^{2x}+1}$

$= \lim\limits_{x\to-\infty} \frac{1}{1+e^{-2x}}$ ✓

$= \frac{1}{\cancel{1+\infty}} = 0$ ✓

$e^{-2x} \to \infty$

as $x \to -\infty$

2. $\lim\limits_{x\to\infty} 10^{\frac{1}{x}}$

as $x \to \infty$ ✓

$\frac{1}{x} \to 0$

$10^{1/x} \to 10^0 = 1$

✓

3. $\lim\limits_{x\to 0} e^{\cos(x)}$

as $x \to 0$;

$\cos(x) \to \cos(0) = 1$ ✓

$e^{\cos(x)} \to e^{1} = e$

✓

4. $\lim\limits_{x\to-\infty} ln(e^{-x})$

as $x \to -\infty$ ✓

$e^{-x} \to \infty$

$\ln(e^{-x}) \to \infty$

✓

8 ✓

5. Prove $\frac{d}{dx}(sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$ for $-1 < x < 1$

let $y = \sin^{-1}(x)$, then $\sin(y) = x$ ✓

$\frac{d}{dx}[\sin(y)] = \frac{d}{dx}[x]$; $\cos(y) \cdot y' = 1$ ✓

$y' = \frac{1}{\cos(y)}$; $\cos^2(y) + \sin^2(y) = 1$ ✓

so $\cos^2(y) = 1 - \sin^2(y)$, $\cos(y) = \pm\sqrt{1-\sin^2(y)}$

$\cos(y) = \pm\sqrt{1-x^2}$; $\cos(y) = \sqrt{1-x^2}$ ✓

as $\cos(y) > 0$, $y' = \frac{1}{\sqrt{1-x^2}}$ ∎

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6. Let $y = e^{2x-x^2}$ determine intervals of increasing/decreasing, relative max, and relative min.

$y' = (2-2x)e^{2x-x^2}$ ✓

f is increasing on $(-\infty, 1)$ ✓

f is decreasing on $(1, \infty)$ ✓

Rel Max $= e$ at $x = 1$

✓

8 ✓

7. Let $y = x\ln(x)$ determine intervals of concavity and possible inflection points.

$y' = 1 + \ln x$; $y'' = \frac{1}{x}$

curve is concave up for $(0, \infty)$

note - curve is not concave down.

- No inflection points.

4

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8. Let $y = \tan^{-1}\left(\frac{1}{x}\right)$ determine the equation of the line tangent to the curve at $\left(1, \frac{\pi}{4}\right)$.

$$y' = \frac{1}{1 + \left(\frac{1}{x}\right)^2} \frac{d}{dx}\left(\frac{1}{x}\right)$$

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$$= \frac{1}{1 + \frac{1}{x^2}} \cdot \left(-\frac{1}{x^2}\right)$$

$$\boxed{y' = -\frac{1}{1+x^2}}$$

$$\boxed{y = -\frac{1}{2}x + \frac{1}{2} + \frac{\pi}{4}}$$

$$m = y'\Big|_{x=1} = -\frac{1}{2}$$

$$y - \frac{\pi}{4} = -\frac{1}{2}(x - 1)$$

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9. Let $y = x^{\cos(x)}$. Use logarithmic differentiation to differentiate.

$\ln y = \ln(x^{\cos x}) = \cos x \ \ln x$

$\ln y = \cos x \cdot \ln x \ ; \ \frac{d}{dx}(\ln y) = \frac{d}{dx}(\cos x \ \ln x)$

$\frac{1}{y} \cdot y' = \cos x \cdot \frac{1}{x} + \ln x \ (-\sin x)$

$y' = y\left[\frac{\cos x}{x} - \ln x \ \sin x\right]$

$y' = x^{\cos x}\left[\frac{\cos x}{x} - \ln x \ \sin x\right]$

Integrate the following functions.

10. $\int \frac{1}{1-3x}\,dx$; $u = 1-3x$; $dx = \frac{du}{-3}$

$\int \frac{1}{u} \frac{du}{-3} = -\frac{1}{3}\ln|u| + C$

$= -\frac{1}{3}\ln|1-3x| + C$

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11. $\int \frac{5}{x^2+25} dx$ $= 5 \int \frac{1}{x^2+25} dx$; $a^2 = 25$

$a = \pm \sqrt{25}$

$a = 5$

$5 \cdot \frac{1}{5} \tan^{-1}\left(\frac{x}{5}\right) + C$

$\boxed{\tan^{-1}\left(\frac{x}{5}\right) + C}$

12. $\int \tan(x)\, dx$ $= \int \frac{\sin(x)}{\cos(x)} dx$; $u = \cos(x)$

$du = -\sin(x)\, dx$

$dx = \frac{du}{-\sin(x)}$

$= \int \frac{\sin(x)}{u} \frac{du}{-\sin(x)} = -\int \frac{1}{u} du$

$= -\ln|u| + C = -\ln|\cos(x)| + C$

$= \boxed{\ln|\sec(x)| + C}$

13. $\int \frac{\cos(lnx)}{x} dx$; $u = \ln x$; $\frac{du}{dx} = \frac{1}{x}$

$dx = x\, du$

$= \int \frac{\cos(u)}{\cancel{x}} \cancel{x}\, du$

$\int \cos(u)\, du$

$-\sin(u) + C$

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$\boxed{-\sin(\ln x) + C}$

14. $\int \frac{x+1}{x^2+2x} dx$ $= \cancel{\int \frac{x}{x^2+}}$ $\int \frac{x+1}{u} \frac{du}{2x+2}$

$u = x^2 + 2x$

$\frac{du}{dx} = 2x + 2$

$dx = \frac{du}{2x+2}$

$\frac{1}{2} \int \frac{1}{u} du$

$\frac{1}{2} \ln |u| + C$

$\frac{1}{2} \ln |x^2 + 2x| + C$

15. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

$\int \frac{e^u}{\cancel{\sqrt{x}}} 2\cancel{\sqrt{x}} \, du$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x} \, du$

$2 \int e^u du$

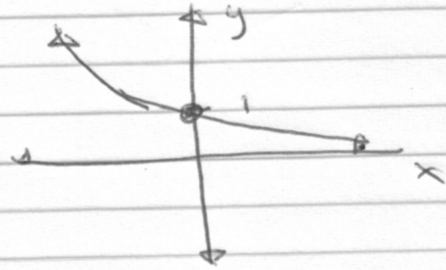
$2e^u + C$

$2e^{\sqrt{x}} + C$

8

Math 262 Test 1

(1) $\lim_{x\to-\infty} \dfrac{e^{2x}/e^{2x}}{(e^{2x}+1)/e^{2x}}$



$\lim_{x\to-\infty} \dfrac{1}{1+\frac{1}{e^{2x}}}$

$\lim_{x\to-\infty} \dfrac{1}{1+e^{-2x}} = \cancel{4}$ as $e^{-2x} \to \infty$

$\cancel{4}$ $\dfrac{1}{1+0}$ $\dfrac{1}{1}$ 1

(2) $\lim_{x\to\infty} 10^{1/x}$

as $x \to \infty$

$\frac{1}{x} \to 0$

$10^{1/x} \to 10^0 = \boxed{1}$

(3) $\lim_{x\to 0} e^{\cos(x)}$

as $x \to 0$

$\cos(x) \to 1$

$e^{\cos(x)} \to e^1$

e

(4) $\lim_{x\to-\infty} \ln(e^{-x})$

as $x \to -\infty$

$e^{-x} \to \infty$

$\ln(e^{-x}) \to \boxed{\infty}$

(5) See test

(6) $y = e^{2x-x^2}$; $y' = e^{2x-x^2}\frac{d}{dx}(2x-x^2)$

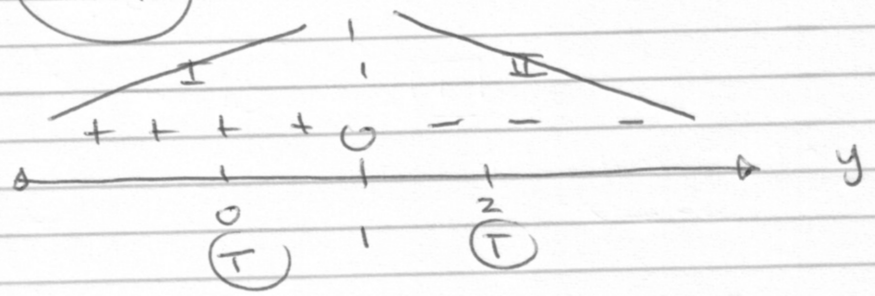
$$y' = e^{2x-x^2}(2-2x)$$

$$\boxed{y' = (2-2x)e^{2x-x^2}}$$

Sign analysis on y'

$$(2-2x)e^{2x-x^2} = 0$$

$2-2x=0$	$e^{2x-x^2} = 0$
$2x = 2$	This never happens!
$x = 1$	



The curve is increasing on $(-\infty, 1)$
decreasing on $(1, \infty)$

Rel max at $x=1$; $y = e^{2\cdot 1 - 1^2}$

$$y = e^{2-1}$$

$$y = e$$

(7) $y = x \ln x$

$$y' = \frac{d}{dx}(x \ln x)$$

$$= x \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(x)$$

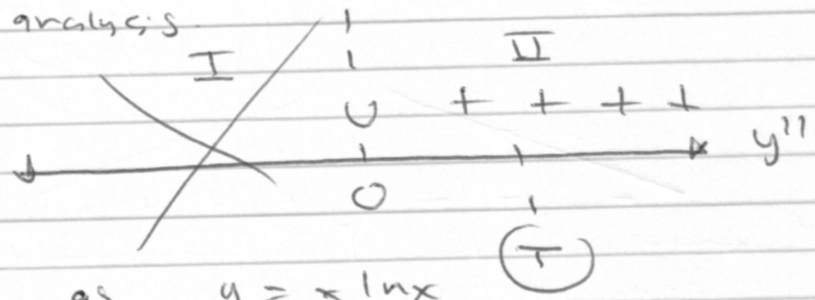
$$= x \cdot \frac{1}{x} + \ln x \cdot 1$$

$$\boxed{y' = 1 + \ln x} \; ; \quad y'' = \frac{d}{dx}(y')$$

$$y'' = \frac{d}{dx}(1 + \ln x)$$

$$\boxed{y'' = \frac{1}{x}}$$

sign analysis



$\frac{1}{x} = 0$

never happens!

But $x = 0$

y'' und

as $y = x \ln x$

$x > 0$

concave up $(0, \infty)$

(8) $y = \tan^{-1}\left(\frac{1}{x}\right)$

$$y' = \frac{1}{1 + \left(\frac{1}{x}\right)^2} \frac{d}{dx}\left(\frac{1}{x}\right)$$

$$= \frac{1}{1 + \frac{1}{x^2}} \cdot -\frac{1}{x^2}$$

$$y' = \frac{1}{1+\frac{1}{x^2}}\left(-\frac{1}{x^2}\right)$$

$$y' = \frac{-1}{x^2+1} \quad ; \quad \boxed{y' = -\frac{1}{1+x^2}}$$

$$m = y'\Big|_{x=1} \quad ; \quad m = -\frac{1}{1+1^2}$$

$$\boxed{m = -\frac{1}{2}}$$

$$y - y_1 = m(x - x_1) \qquad \overset{x\ \ y}{\left(1, \frac{\pi}{4}\right)},$$

$$y - \frac{\pi}{4} = -\frac{1}{2}(x-1)$$

$$y - \frac{\pi}{4} = -\frac{1}{2}x + \frac{1}{2}$$

$$\boxed{y = -\frac{1}{2}x + \frac{1}{2} + \frac{\pi}{4}}$$

(9) $y = x^{\cos x}$

$$\ln y = \ln\left(x^{\cos x}\right)$$

$$\ln y = \cos x \ \ln x$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\cos x \ \ln x)$$

$$\frac{1}{y} \cdot y' = \cos x \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(\cos x)$$

$$\frac{y'}{y} = \cos x \cdot \frac{1}{x} + \ln x\,(-\sin x)$$

$$\frac{y'}{y} = \frac{\cos x}{x} - \sin x \ln x$$

$$y' = y\left[\frac{\cos x}{x} - \sin x \ln x\right]$$

$$\boxed{y' = x^{\cos x}\left[\frac{\cos x}{x} - \sin x \ln x\right]}$$

(10) $\displaystyle\int \frac{1}{1-3x}\,dx$ $\qquad u = 1-3x$

$\qquad \dfrac{du}{dx} = -3 \;;\; dx = \dfrac{du}{-3}$

$$\int \frac{1}{u}\,\frac{du}{-3} \qquad -\frac{1}{3}\int \frac{1}{u}\,du$$

$$-\frac{1}{3}\ln|u| + C$$

$$\boxed{-\frac{1}{3}\ln|1-3x| + C}$$

(11) $\displaystyle\int \frac{5}{x^2+25}\,dx = 5\int \frac{1}{x^2+25}\,dx$ $\qquad a^2 = 25$

$a = \pm\sqrt{25}$

$a = \pm 5$

$a = 5$

$$\frac{5}{5}\tan^{-1}\left(\frac{x}{5}\right) + C$$

$$\boxed{\tan^{-1}\left(\frac{x}{5}\right) + C}$$

(12) $$\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx$$

$$u = \cos(x) \; ; \; \frac{du}{dx} = -\sin(x)$$

$$= \int \frac{\cancel{\sin(x)}}{u} \, \frac{du}{-\cancel{\sin}(x)} = -\int \frac{1}{u}\,du$$

$$= -\ln|u| + C = \boxed{-\ln|\cos(x)| + C}$$

(13) $$\int \frac{\cos(\ln x)}{x}\,dx \qquad u = \ln x$$

$$\frac{du}{dx} = \frac{1}{x} \; ; \; dx = x\,du$$

$$\int \frac{\cos(u)}{\cancel{x}} \cancel{x}\,du$$

$$\int \cos(u)\,du = -\sin(u) + C$$

$$= \boxed{-\sin(\ln x) + C}$$

(14) $$\int \frac{x+1}{x^2+2x}\,dx \; ; \; u = x^2 + 2x$$

$$\frac{du}{dx} = 2x + 2$$

$$\int \frac{x+1}{u} \, \frac{du}{2x+2} \qquad dx = \frac{du}{2x+2}$$

$$\int \frac{x+\cancel{1}}{u} \cdot \frac{du}{2(x+\cancel{1})} = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u|$$

$$\boxed{\frac{1}{2}\ln|x^2+2x| + C}$$

(15) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$; $u = \sqrt{x}$

$\dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$

$\displaystyle\int \frac{e^{u}}{\cancel{\sqrt{x}}}\, 2\cancel{\sqrt{x}}\, du$ $\qquad dx = 2\sqrt{x}\, du$

$2\displaystyle\int e^{u}\,du \qquad 2e^{u} + C$

$\boxed{2e^{\sqrt{x}} + C}$