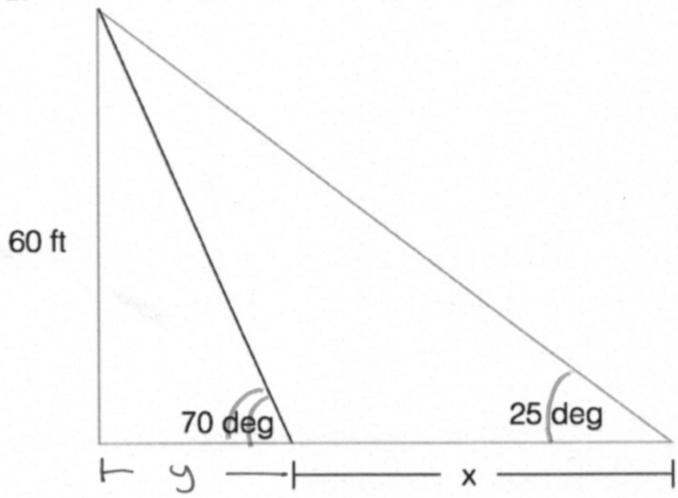
East Los Angeles College
Department of Mathematics
Math 241
Test 3

52

Solutions

Solve for x using right angle trigonometry.

1.



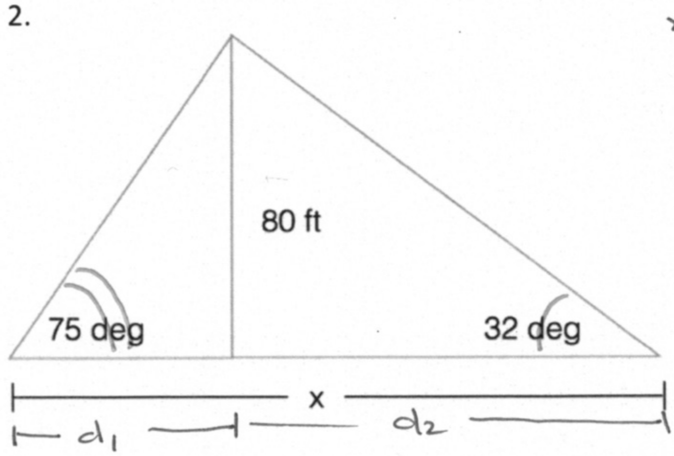
$\tan(25^\circ) = \dfrac{60}{x+y}$

$\tan(70^\circ) = \dfrac{60}{y}$

$y \approx 21.8$

but $\boxed{x \approx 106.9}$

2.



$x = d_1 + d_2$

$21.4 \qquad 128$

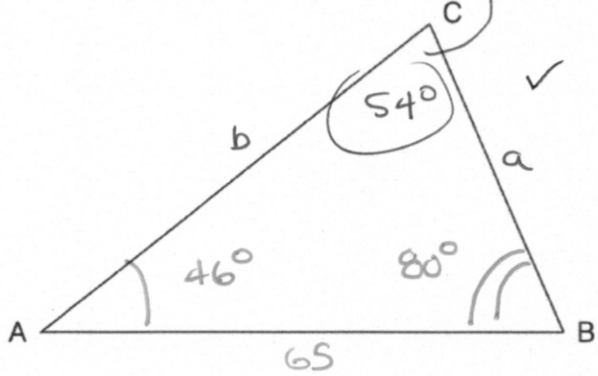
$\boxed{x \approx 149.4}$

8

Solve the following triangles. law of Sines

3.

180 - 80 - 46



$$\frac{\sin(80^\circ)}{b} = \frac{\sin(54)}{65}$$

$$b = \frac{65 \sin(80)}{\sin(54)}$$

$$b \approx 79.1$$

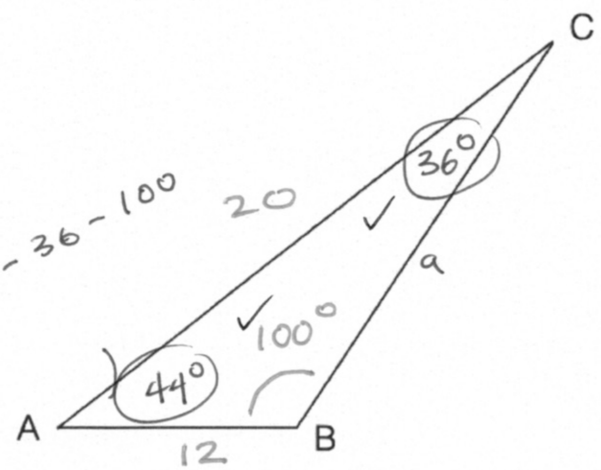
$$\frac{\sin(46^\circ)}{a} = \frac{\sin(54^\circ)}{65}$$

$$a = \frac{65 \sin(46^\circ)}{\sin(54^\circ)}$$

$$a \approx 57.8$$

4.

180 - 36 - 100



$$\frac{\sin(C)}{12} = \frac{\sin(100)}{20}$$

$$\sin(C) = \frac{12 \sin(100)}{20}$$

$$\sin(C) = 0.59$$

$$C = \sin^{-1}(0.59)$$

$C_1 \approx 36^\circ$; $C_2 \approx \cancel{144^\circ}$ not possible

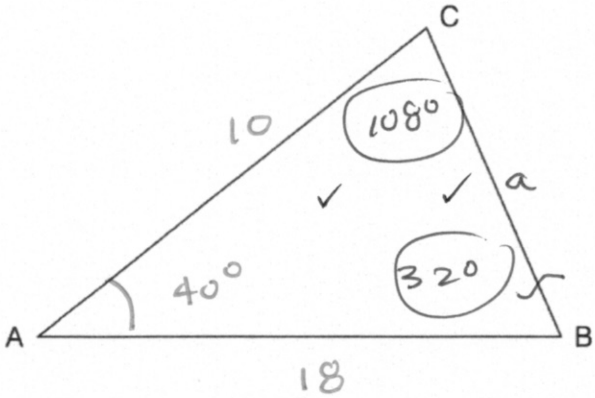
$$\frac{\sin(44)}{a} = \frac{\sin(100)}{20}$$

$$a = \frac{20 \sin(44)}{\sin(100)}$$

$$a \approx 14.1$$

8

5.



$\cos(C) \approx -0.31$

$C \approx 108°$

$180 - 108 - 40$

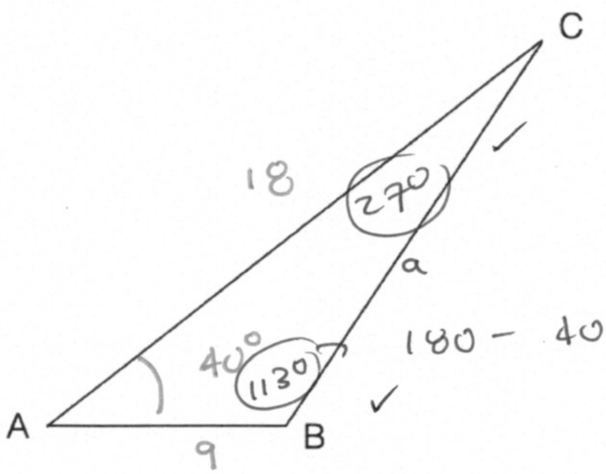
$a^2 = 10^2 + 18^2 - 2 \cdot 10 \cdot 18 \cdot \cos(40)$

$a^2 \approx 148.2$; $a \approx 12.2$

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos(C) = \frac{12.2^2 + 10^2 - 18^2}{2 \cdot 12.2 \cdot 10}$$

6.



$\cos(C) \approx 0.89$

$C \approx 27.5°$

$C \approx \cancel{28}$; $C \approx 27°$

$180 - 40 - 27$

$a^2 = 18^2 + 9^2 - 2 \cdot 18 \cdot 9 \cdot \cos(40°)$

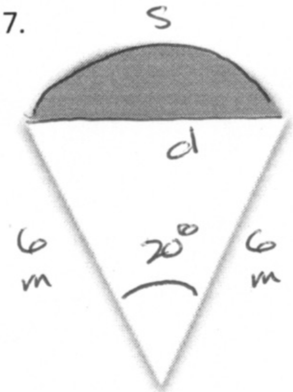
$a^2 = 156.8$; $a \approx 12.5$

$$\cos(C) = \frac{12.5^2 + 18^2 - 9^2}{2 \cdot 12.5 \cdot 18}$$

8 ✓

Determine the area and the perimeter of the shaded region that comes from a portion of a circle.

7.



$P = s + d$

$P = r\theta + 2$

$P = 6 \cdot 20^\circ \cdot \frac{\pi^\circ}{180^\circ} + 2$

$P = \frac{2\pi}{3} + 2$

$\boxed{P = 4.09 \text{ m}}$ 4✓

8✓

$d^2 = 6^2 + 6^2 - 2 \cdot 6 \cdot 6 \cos(20^\circ)$

$d^2 = 4.34$; $d = \sqrt{4.34}$

$d \approx 2$

$A = A_{sector} - A_{\triangle}$

$A = \frac{1}{2} r^2 \theta - \sqrt{7(7-6)(7-6)(7-2)}$

$A = \frac{1}{2} 6^2 20^\circ \frac{\pi}{180^\circ} - \sqrt{7 \cdot 5}$

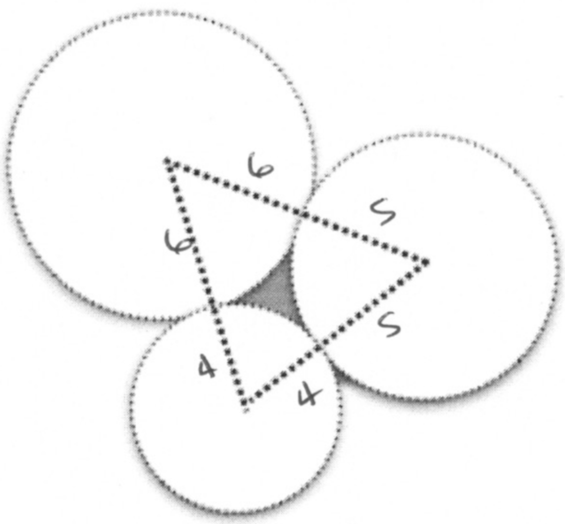
3.14

$A = 2\pi - \sqrt{35}$; $A = 6.28 - 5.92$

$\boxed{A \approx 0.36 \text{ m}^2}$

4✓

8. The circles are barely touching one another and have the following radii's- 4, cm, 5 cm, and 6 cm from smallest to largest. Determine the shaded area.



Prove the following Identities

9. $\dfrac{\cos(x)\sec(x)}{\tan(x)} = \cot(x)$

LHS = RHS

$LHS = \dfrac{\cos(x)\sec(x)}{\tan(x)}$

$= \dfrac{\cos(x)\cdot\dfrac{1}{\cos(x)}}{\dfrac{\sin(x)}{\cos(x)}}$

$= \dfrac{1}{\dfrac{\sin(x)}{\cos(x)}}$

$= 1 \div \dfrac{\sin(x)}{\cos(x)}$

$LHS = 1 \cdot \dfrac{\cos(x)}{\sin(x)}$

$LHS = \cot(x)$

LHS = RHS ∎

S4✓

10. $(1 - cos^2(x))(1 + cot^2(x)) = 1$

LHS RHS

$LHS = 1 - \cos^2(x) + \cot^2(x) - \cos^2(x)\cot^2(x)$

$LHS = 1 - \cos^2(x) + \cot^2(x) - \cos^2(x)\cdot\cancel{\dfrac{\cos^2(x)}{\sin^2(x)}}\ \cot^2(x))$

$LHS = 1 - \cos^2(x) + \cot^2(x)\,(1 - \cos^2(x))$

$LHS = \sin^2(x) + \cot^2(x)\cdot\sin^2(x)$

$LHS = \sin^2(x)\,(1 + \cot^2(x))$

$LHS = \sin^2(x)\cdot\csc^2(x)$

$LHS = \sin^2(x)\cdot\dfrac{1}{\sin^2(x)}$

$LHS = 1$

LHS = RHS ∎

S 4✓

8✓

10✓

11. $tan^2(x) - sin^2(x) = tan^2(x)sin^2(x)$

LHS = RHS

$$\begin{aligned}
LHS &= \tan^2(x) - \sin^2(x) \\
&= \frac{\sin^2(x)}{\cos^2(x)} - \sin^2(x) \\
&= \sin^2(x)\left[\frac{1}{\cos^2(x)} - 1\right] \\
&= \sin^2(x)\left[\frac{1}{\cos^2(x)} - \frac{\cos^2(x)}{\cos^2(x)}\right] \\
&= \frac{\sin^2(x)}{\cos^2(x)}\left[1 - \cos^2(x)\right] \\
&= \tan^2(x) \cdot \sin^2(x) \\
&= RHS
\end{aligned}$$

5 ✓

12. $\frac{1-\cos(x)}{\sin(x)} + \frac{\sin(x)}{1-\cos(x)} = 2\csc(x)$

LHS RHS

$$\begin{aligned}
LHS &= \frac{1-\cos(x)}{\sin(x)} + \frac{\sin(x)}{1-\cos(x)} \\
&= \frac{(1-\cos(x))(1-\cos(x)) + \sin^2(x)}{\sin(x)(1-\cos(x))} \\
&= \frac{1 - 2\cos(x) + \cos^2(x) + \sin^2(x)}{\sin(x)(1-\cos(x))} \\
&= \frac{2 - 2\cos(x)}{\sin(x)(1-\cos(x))} \\
&= \frac{2\cancel{(1-\cos(x))}}{\sin(x)\cancel{(1-\cos(x))}} \\
&= \frac{2}{\sin(x)}
\end{aligned}$$

$\therefore$ LHS = 2csc(x)

LHS = RHS

5 ✓

10 ✓

Formulas and Identities

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Periodic Formulas

If n is an integer.

$$\sin(\theta + 2\pi n) = \sin\theta \qquad \csc(\theta + 2\pi n) = \csc\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta \qquad \sec(\theta + 2\pi n) = \sec\theta$$

$$\tan(\theta + \pi n) = \tan\theta \qquad \cot(\theta + \pi n) = \cot\theta$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas

$$\sin^2\theta = \frac{1}{2}\left(1 - \cos(2\theta)\right)$$

$$\cos^2\theta = \frac{1}{2}\left(1 + \cos(2\theta)\right)$$

$$\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha + \beta) + \sin(\alpha - \beta)\right]$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha + \beta) - \sin(\alpha - \beta)\right]$$

Sum to Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

Math 241 Test 1

(1) $\tan(25^\circ) = \dfrac{60}{x+y}$; $\tan(70) = \dfrac{60}{y}$

so ; $y = \dfrac{60}{\tan(70)}$

$x+y = \dfrac{60}{\tan(25^\circ)}$

$-y \qquad -y$

$y \approx 21.8$

$x = \dfrac{60}{\tan(25^\circ)} - y$

$x = 128.7 - 21.8$; $\boxed{x = 106.9}$

(2) $x = d_1 + d_2$

$\tan(75) = \dfrac{80}{d_1}$; $\tan(32) = \dfrac{80}{d_2}$

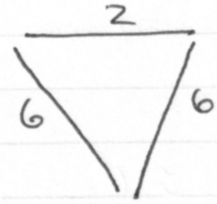
$d_1 = \dfrac{80}{\tan(75)}$ $\qquad d_2 = \dfrac{80}{\tan(32)}$

$d_1 \approx 21.4$ $\qquad d_2 \approx 128$

$x = 21.4 + 128$

$\boxed{x = 149.4}$

(7) Area $= A_{\text{sector}} - A_{\text{triangle}}$



$s = \dfrac{6+6+2}{2}$

$s = 7$

$A = \frac{1}{2} r^2 \theta$

$9 \quad \theta$

$6 \qquad \dfrac{20^\circ \cdot \pi}{180^\circ}$

$\sqrt{7\,(\cancel{7}-6)\,(7-6)\,(7-2)}$

$\sqrt{7 \cdot 1 \cdot 1 \cdot 5}$

$\sqrt{35}$

$A = \dfrac{6 \cdot \cancel{20^\circ} \cdot \overset{3.14}{\pi}}{\cancel{180^\circ}} + \sqrt{35}$

$A = \dfrac{6\pi}{9} + \sqrt{35}$; $A = \dfrac{2\overset{3.14}{\pi}}{3} + \sqrt{35}$

$A = \dfrac{2 \cdot 3.14}{3} + \sqrt{35}$; $A \approx 8.0$ m^2

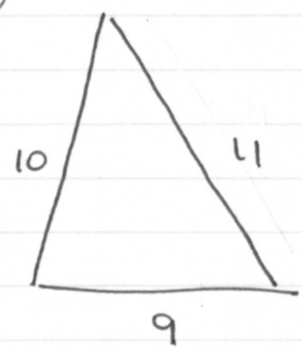
perimeter =  +

$\dfrac{2\pi r}{2} + d$

$P = \pi r + d$

$P = 3.14 \cdot 6 + 2$; $\boxed{P = 20.84 \ m^2}$

(e)



$$s = \frac{10+11+9}{2}$$

$$s = 15$$

$$A = A_{Triangle} - A_{Sector} - A_{Sector} - A_{Sector}$$

$$= \sqrt{15(15-10)(15-9)(15-11)}$$

$$+ \quad \frac{1}{2}\underset{\underset{6}{4}}{r}^2\theta \quad + \quad \frac{1}{2}\underset{\underset{5}{4}}{r}^2\theta \quad + \quad \frac{1}{2}\underset{\underset{4}{4}}{r}^2\theta$$

i.e., $A = \sqrt{15\cdot 5\cdot 6\cdot 4}$