$52\checkmark$

~~50~~

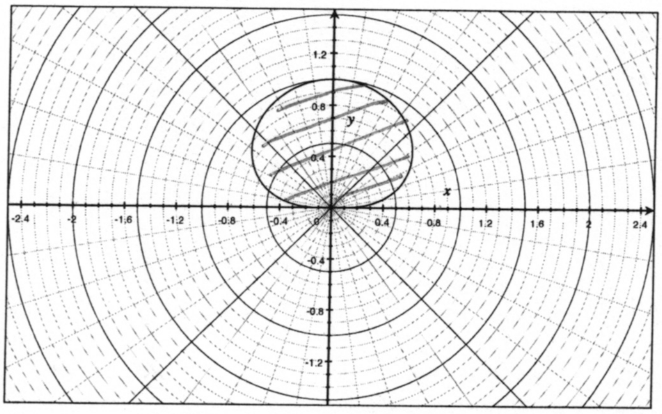
East Los Angeles College
Department of Mathematics
Math 262
Test 4

~~4~~ $\checkmark$

Solutions

1. Find the area of the region bounded by the given curve that lies in the sector $0 \leq \theta \leq \pi$

$r = \sqrt{sin(\theta)}$



$$A = \int \frac{1}{2} r^2 d\theta$$

$$A = \frac{1}{2}\int_0^{\pi} \left(\sqrt{\sin\theta}\right)^2 d\theta = \frac{1}{2}\int_0^{\pi} \sin\theta \, d\theta$$

$$= -\frac{1}{2}\cos\theta \Big|_{\theta=0}^{\theta=\pi} = -\frac{1}{2}\underbrace{\cancel{\cos\pi}}_{-1} + \frac{1}{2}\underbrace{\cancel{\cos 0}}_{1}$$

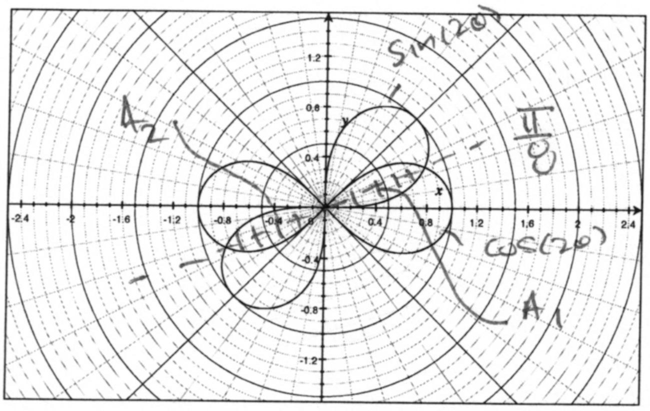
$$= -\frac{1}{2}(-1) + \frac{1}{2}(1) = \frac{1}{2} + \frac{1}{2} = \boxed{1}$$

$4\checkmark$

2. Find the area of the region inside both curves.

$r^2 = sin(2\theta)$

$r^2 = cos(2\theta)$



$r^2 = r^2$

$\sin(2\theta) = \cos(2\theta)$

$\tan(2\theta) = 1$

$\rightarrow \theta \quad 2\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

$\theta = \frac{\pi}{8}, \frac{5\pi}{8}$

$A = A_1 + A_2 \quad ; \quad A_1 = A_2$ ✓

$A = 2A_1 \quad ; \quad A_1 = \int_0^{\pi/8} \frac{1}{2} r^2 \, d\theta + \int_{\pi/8}^{\pi/4} \frac{1}{2} r^2 \, d\theta$ ✓

$\qquad\qquad\qquad\qquad\qquad \sin(2\theta) \qquad\qquad \cos(2\theta)$

$A_1 = \int_0^{\pi/8} \frac{1}{2} \sin(2\theta) \, d\theta + \int_{\pi/8}^{\pi/4} \frac{1}{2} \cos(2\theta) \, d\theta$

$= -\frac{1}{4} \cos(2\theta) \Big|_0^{\pi/8} + \frac{1}{4} \sin(2\theta) \Big|_{\pi/8}^{\pi/4}$ ✓ ✓

$= -\frac{1}{4} \cos\left(2 \cdot \frac{\pi}{8}\right) + \frac{1}{4} \cos(0)$

6 ✓

$+ \frac{1}{4} \sin\left(2 \cdot \frac{\pi}{4}\right) - \frac{1}{4} \sin\left(\frac{2\pi}{8}\right)$

$= -\frac{1}{4} \cos\cancel{\left(\frac{\pi}{4}\right)}^{\sqrt{2}/2} + \frac{1}{4} \cancel{\cos(0)}^{1}$

$+ \frac{1}{4} \sin\cancel{\left(\frac{\pi}{2}\right)}^{1} - \frac{1}{4} \cancel{\sin(\pi/4)}^{\sqrt{2}/2}$

$A = 2\left(\frac{1}{2} - \frac{\sqrt{2}}{4}\right)$

$\boxed{A = 1 - \frac{\sqrt{2}}{2}}$ ✓ ✓

$= -\frac{\sqrt{2}}{8} + \frac{1}{4} + \frac{1}{4} - \frac{\sqrt{2}}{8}$

$= -\frac{\sqrt{2}}{4} + \frac{1}{2} \, ;$

3. Find the exact length of the curve.

$r = 3sin(\theta)$ over $0 \le \theta \le \frac{\pi}{3}$

A✓

$$S = \int_0^{\pi/3} \sqrt{r^2 + (r')^2}\, d\theta$$

$r = 3\sin\theta$; $r' = 3\cos\theta$

$$\sqrt{r^2+(r')^2} = \sqrt{(3\sin\theta)^2 + (3\cos\theta)^2} = \sqrt{9\sin^2\theta + 9\cos^2\theta}$$

$$= \sqrt{9} = 3$$

✓

$$S = \int_0^{\pi/3} 3 \cdot d\theta = 3\int_0^{\pi/3} d\theta = 3\theta \Big|_{\theta=0}^{\theta=\pi/3}$$ ✓

✓

$$= 3\cdot\frac{\pi}{3} - 3\cdot 0 = \boxed{\pi}$$

✓ 4✓

Determine whether the sequences converge or diverge. Justify your answer, no guessing.

4. $a_n = ln(n+1) - ln(n)$

$a_n = \ln\left(\frac{n+1}{n}\right)$

$\lim a_n = \lim \ln\left(1+\frac{1}{n}\right)$ ✓ ✓

$n \to \infty$, $a_n \to \ln(1) = 0$

$a_n \to 0$ ✓

converges

5. $a_n = sin\left(\frac{3}{n}\right)$

as $n \to \infty$; $\frac{3}{n} \to 0$

$a_n = \sin\left(\frac{3}{n}\right) \to \sin(0) = 0$

✓ Converges

✓

✓

6. $a_n = sin\left(\frac{n}{3}\right)$

DNE ✓

$\frac{n}{3} \to \infty$ as $n \to \infty$

$\sin\left(\frac{n}{3}\right) \to \pm 1$ oscillates

Diverges ✓

✓

7. $a_n = (-1)^n \frac{n^3}{n^3+2n^2+1}$

$\lim \left| (-1)^n \frac{n^3}{n^3+2n^2+1} \right|$ ✓

$= \left| \frac{n^3}{n^3+2n^2+1} \right| = \frac{n^3}{n^3+2n^2+1}$ ✓

$= \frac{1}{1+\frac{2}{n}+\frac{1}{n^3}} \to 1$ as $n \to \infty$ ✓

ie, $a_n \to \pm 1$ as $n \to \infty$

DNE Diverges.

8. $a_n = \frac{(-1)^n}{e^n}$

$|a_n| = \left| \frac{(-1)^n}{e^n} \right| = \frac{1}{e^n} \to 0$

✓ ✓ ✓

as $n \to \infty$; 15 ✓

therefore $a_n \to 0$ as $n \to \infty$ 15 ✓

Converges

Determine whether the series converge or diverge. If it's convergent, find its sum. Justify your answer, no guessing.

9. $\sum_{n=1}^{\infty} \frac{10^n}{(-9)^{n-1}}$ $= \underset{a}{10} + \underset{ar}{\frac{10^2}{-9}} + \frac{10^3}{(-9)^2} + \ldots$

$a = 10$; $ar = \frac{10^2}{-9}$; $r = \frac{10}{-9}$

$|r| = \left|\frac{10}{-9}\right| > 1$ Diverses ✓

Geometric Series ; ✓

Diverges.

10. $\sum_{n=1}^{\infty} \left(\frac{3}{\sqrt{7}}\right)^n$ $= \frac{3}{\sqrt{7}} + \frac{3^2}{(\sqrt{7})^2} + \ldots$

$a = \frac{3}{\sqrt{7}}$; $ar = \frac{3^2}{(\sqrt{7})^2}$

$r = \frac{3}{\sqrt{7}}$; $\left|\frac{3}{\sqrt{7}}\right| > 1$

Geometric Series Diverges.

✓ ✓

12✓
8✓

11. $\sum_{n=1}^{\infty} ln\left(\frac{n^2+1}{2n^2+1}\right)$

$a_n = \ln\left(\frac{n^2+1}{2n^2+1}\right)$

$= \ln\left(\frac{1+1/n^2}{2+1/n^2}\right) \to \ln(1/2) \neq 0$ ✓

as $n \to \infty$ ✓

Diverges By the Divergence test.

12. $\sum_{n=1}^{\infty}(\sqrt{n+1} - \sqrt{n})$

a_n

$S_k = \cancel{\sqrt{2}} - \sqrt{1} + \cancel{\sqrt{3}} - \cancel{\sqrt{2}} + \sqrt{4} - \cancel{\sqrt{3}}$

$+ \ldots + \cancel{\sqrt{k}} - \sqrt{k-1}$

$+ \cancel{\sqrt{k+1}} - \cancel{\sqrt{k}}$

$+ \sqrt{k+2} - \cancel{\sqrt{k+1}}$

✓ ✓

$S_k = -\sqrt{1} + \sqrt{k+2} \to \infty$

as $k \to \infty$

Divergent telescoping Series.

Determine whether you have a convergent or divergent p-series. Justify your answer, no guessing.

13. $\sum_{n=1}^{\infty} \frac{1}{n^4}$ $\quad p = 4 > 1$

convergent p-Series

14. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2}}$ $\quad p = 2/3 < 1$

Divergent p-Series

15. $\sum_{n=1}^{\infty} \frac{1}{n^2\sqrt{n}}$; $\sum \frac{1}{n^2 \cdot n^{1/2}} = \sum \frac{1}{n^{5/2}}$ $\quad p = 5/2 > 1$

convergent p-Series.

16. $\sum_{n=1}^{\infty} \frac{n}{e^n}$ Hint- This is a decreasing, positive, sequence. Converges.

let $f(x) = \frac{x}{e^x}$

conv $\int_1^\infty \frac{x}{e^x}\,dx = \lim_{t\to\infty} \int_1^t x e^{-x}\,dx$; $I = \int x e^{-x}\,dx$

IBP

11

8

note $I = \int x e^{-x}\,dx$

$u = x$; $dv = e^{-x}\,dx$

$du = dx$ $\quad v = \int e^{-x}\,dx = -e^{-x}$

$uv - \int v\,du = -xe^{-x} - \int -e^{-x}\,dx = -xe^{-x} + \int e^{-x}\,dx$

$= -xe^{-x} - e^{-x} \Big|_1^t = -te^{-t} - e^{-t} - (-1e^{-1} - e^{-1})$

$= \left(\frac{1}{e} + \frac{1}{e}\right) - \frac{t}{e^t} - \frac{1}{e^t}$;

$I = \frac{2}{e} - \frac{t}{e^t} - \frac{1}{e^t}$; $\quad$ But $\lim_{t\to\infty} \frac{t}{e^t}$ $\quad \frac{\infty}{\infty}$

$I \to \frac{2}{e}$ as $t \to \infty$ $\quad \overset{H}{=} \lim_{t\to\infty} \frac{1}{e^t} = 0$

a/q

so $\sum \frac{n}{e^n} < \infty$

conv