24 ✓ 30 ✓

29 ✓

29 ✓

East Los Angeles College
Department of Mathematics
Math 241
Test 3

Verify the following identities.

1. $\frac{sin(x)+cos(x)}{sec(x)+csc(x)} = sin(x)cos(x)$ See Scratch

✓ ✓
✓ ✓
✓

2. $\frac{1+cos(x)}{cos(x)} = \frac{tan^2(x)}{sec(x)-1}$ See Scratch

✓ ✓
✓ ✓

8
10 ✓

Express in terms of a single sine function.

3. $y = -\sqrt{3}sin(x) + cos(x)$

$y = 2 \sin\left(x - \frac{\pi}{6}\right)$

✓ ✓

Write the product as a sum.

4. $sin(2x)cos(3x)$

$\frac{1}{2}\sin(5x) - \frac{1}{2}\sin(x)$

5. $sin(x)sin(5x)$

$\frac{1}{2}\sin(5x) - \frac{1}{2}\sin(x)$

$\frac{1}{2}\cos(4x) - \frac{1}{2}\cos(6x)$

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6. Determine $tan\left(\frac{\theta}{2}\right)$ when $sec(\theta) = \frac{3}{4}$ and $270^o < \theta < 360^o$

$\tan\left(\frac{\theta}{2}\right) = -\frac{\sqrt{5}}{5}$

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7. Reduce the power to a single sine for $sin^6(x)$

$\frac{5}{16} - \frac{7}{16}\cos(2x) + \frac{3}{8}\cos(4x) - \frac{1}{16}\cos(6x)$

$\cos(7x)$

10

Determine the exact value for the following.

8. $cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\frac{\pi}{4}$ ✓

9. $sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$-\frac{\pi}{4}$ ✓

10. $tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\frac{\pi}{6}$ ✓

11. $tan^{-1}(-\sqrt{3})$

$-\frac{\pi}{3}$ ✓

12. $tan\left(sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$

$\frac{\pi}{4}$ ✓

$\frac{\pi}{3}$ ✓

$\frac{\pi}{6}$ ✓

1

13. $cos^{-1}\left(2sin\left(\frac{\pi}{3}\right)\right)$

Impossible

✓ ✓

14. $sin\left(sin^{-1}\left(\frac{1}{2}\right)+cos^{-1}\left(\frac{1}{2}\right)\right)$ = 1

✓ ✓

dv

Height of a Balloon- A 680-ft rope anchors a hot-air balloon.

15. Express the angle θ *as a function of the height h of the balloon.*

$\theta = \sin^{-1}\left(\frac{h}{680}\right)$ ✓

10 pts

16. Determine the angle θ *when the ballonn is* 600 *feet.*

$\theta \approx 61.90$ ✓

math 241 – Test 3

(1) $\dfrac{\sin(x)+\cos(x)}{\sec(x)+\csc(x)} = \sin(x)\cos(x)$

$$LHS = \frac{\sin(x)+\cos(x)}{\sec(x)+\csc(x)}$$

$$= \frac{\sin(x)+\cos(x)}{\frac{1}{\cos(x)}+\frac{1}{\sin(x)}}$$

$$= \frac{\sin(x)+\cos(x)}{\frac{\sin(x)+\cos(x)}{\sin(x)\cos(x)}}$$

$$= \sin(x)+\cos(x) \div \frac{\sin(x)+\cos(x)}{\sin(x)\cos(x)}$$

$$= \left(\sin(x)+\cos(x)\right) \cdot \frac{\sin(x)\cos(x)}{\left(\sin(x)+\cos(x)\right)}$$

$$= \sin(x)\cos(x) = RHS \ \square$$

(2) $\dfrac{1+\cos(x)}{\cos(x)} = \dfrac{\tan^2(x)}{\sec(x)-1}$

LHS RHS

$$RHS = \frac{\tan^2(x)}{\sec(x)-1} \cdot \frac{\sec(x)+1}{\sec(x)+1}$$

$$RHS = \frac{\tan^2(x)\,[\sec(x)+1]}{\sec^2(x)+1}$$

$$= \frac{\tan^2(x)\,[\sec(x)+1]}{\tan^2(x)}$$

$$= \sec(x)+1$$

$$= \frac{1}{\cos(x)} + 1$$

$$= \frac{1}{\cos(x)} + \frac{\cos(x)}{\cos(x)}$$

$$= \frac{\cos(x)+1}{\cos(x)} = LHS$$

(3) $y = \underset{A}{-\sqrt{3}}\sin(x) + \underset{B}{1}\cos(x)$

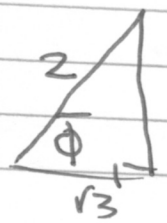
$$k = \sqrt{A^2+B^2} \quad ; \quad k = \sqrt{(-\sqrt{3})^2+1^2}$$

$$k = \sqrt{3+1} = 2$$

$$\tan\phi = \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

$$\bar{\phi} = \frac{\pi}{6}$$

S	A
T	C



$$\phi = -\frac{\pi}{6}$$

$$\boxed{y = 2\sin\left(x - \frac{\pi}{6}\right)}$$

(4) $\sin(2x)\cos(3x)$

$$= \frac{1}{2}\left[\sin(5x) + \sin(-x)\right]$$

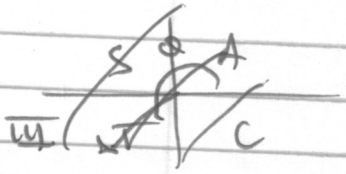
$$= \boxed{\frac{1}{2}\sin(5x) - \frac{1}{2}\sin(x)}$$

(5) $\sin(x)\sin(5x)$

$$\frac{1}{2}\left[\cos(-4x) - \cos(6x)\right]$$

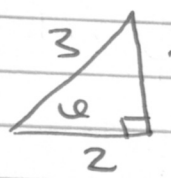
$$\frac{1}{2}\left[\cos(4x) - \cos(6x)\right]$$

$$\boxed{\frac{1}{2}\cos(4x) - \frac{1}{2}\cos(6x)}$$



(6) $\sec\theta = \frac{3}{2}$, θ in Q IV

$\rightarrow \cos\theta = \frac{2}{3}$



$2^2 + ?^2 = 3^2$

$4 + ?^2 = 9 \;\; ; \;\; ?^2 = 5$

$270 < \theta < 360$

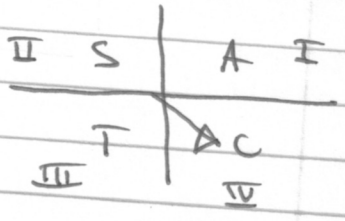
$\frac{270}{2} < \frac{\theta}{2} < \frac{360}{2}$

$135 < \frac{\theta}{2} < 180$

$\tan\left(\frac{\theta}{2}\right) = \frac{1 - \cos\theta}{\sin\theta}$

$= \sqrt{5} - 1$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta} \quad ; \quad \theta \in \text{Q IV}$$



$$\cos\theta = \frac{2}{3}$$

$$\sin\theta = -\frac{\sqrt{5}}{3}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\frac{2}{3}}{-\sqrt{5}/3} = \frac{\frac{3}{3}-\frac{2}{3}}{-\sqrt{5}/3}$$

$$= \frac{1/3}{-\sqrt{5}/3} = -\frac{1}{\sqrt{5}} = \boxed{-\frac{\sqrt{5}}{5}}$$

⑦ Reduce Powers

$$\sin^6(x) = \sin^2(x)\cdot\sin^2(x)\cdot\sin^2(x)$$

$$= \frac{1}{2}[1-\cos(2x)]\,\frac{1}{2}[1-\cos(2x)]\,\frac{1}{2}[1-\cos(2x)]$$

$$= \frac{1}{8}(1-\cos(2x))(1-\cos(2x))(1-\cos(2x))$$

$$= \frac{1}{8}[1-2\cos(2x)+\cos^2(2x)][1-\cos(2x)]$$

$$\cos^2\theta = \frac{1}{2}(1+\cos(4x))$$

$$\left(\frac{1}{8}-\frac{1}{4}\cos(2x)+\frac{1}{16}(1+\cos(4x))\right)(1-\cos(2x))$$

$$\left(\frac{1}{8}-\frac{1}{4}\cos(2x)+\frac{1}{16}+\frac{1}{16}\cos(4x)\right)(1-\cos(2x))$$

$$\left(\frac{3}{16} - \frac{1}{4}\cos(2x) + \frac{1}{16}\cos(4x)\right)\left(1 - \cos(2x)\right)$$

$$\frac{3}{16} - \frac{1}{4}\cos(2x) + \frac{1}{16}\cos(4x)$$

$$-\frac{3}{16}\cos(2x) + \frac{1}{4}\cos^2(2x) - \frac{1}{16}\cos(4x)\cos(2x)$$

$$\frac{3}{16} - \frac{7}{16}\cos(2x) + \frac{1}{16}\cos(4x) - \frac{1}{16}\cos(4x)\cos(2x)$$

$$+\frac{1}{4}\cos^2(2x)$$

$$\frac{1}{2}\left[1 + \cos(4x)\right]$$

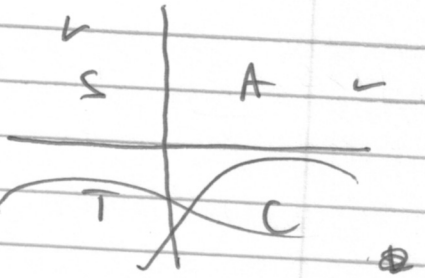
$$\frac{3}{16} - \frac{7}{16}\cos(2x) + \frac{1}{16}\cos(4x) - \frac{1}{16}\cos(4x)\cos(2x)$$

$$+\frac{1}{8} + \frac{1}{8}\cos(4x)$$

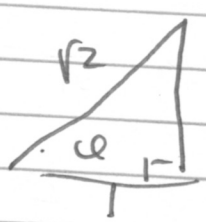
~~$\frac{3}{16} - \frac{7}{16}\cos(2x)$~~

$$\frac{5}{16} - \frac{7}{16}\cos(2x) + \frac{3}{8}\cos(4x) - \frac{1}{16}\cos(4x)\cos(2x)$$

(8) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \theta$

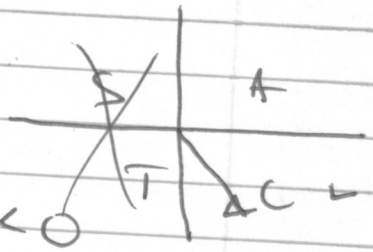


let $\cos\theta = \frac{1}{\sqrt{2}} > 0$

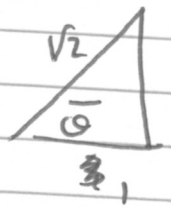


$\theta = \boxed{\frac{\pi}{4}}$

(9) $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \theta$

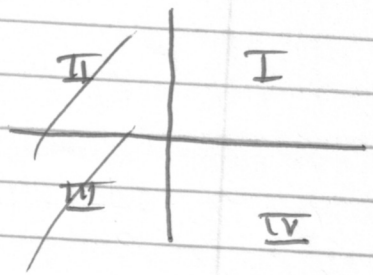


$\sin\theta = -\frac{1}{\sqrt{2}}$; $\sin\theta < 0$

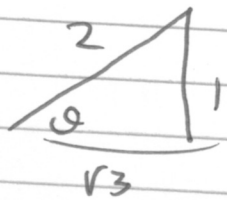


$\bar{\theta} = \frac{\pi}{4}$; $\theta = \boxed{-\frac{\pi}{4}}$

(10) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \theta$



$\tan\theta = \frac{1}{\sqrt{3}}$; $\tan\theta > 0$



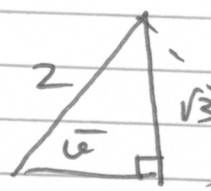
$\theta = \boxed{\frac{\pi}{6}}$

(11) $\tan^{-1}(-\sqrt{3}) = \theta$ $\boxed{-\frac{\pi}{3}}$

$\tan\theta = -\sqrt{3}$; $\tan\theta < 0$

II	I
S	A
	$t>0$
III	IV
T	C $t<0$

$\tan\bar{\theta} = \frac{\sqrt{3}}{1}$



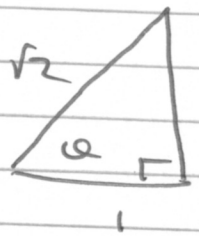
$\bar{\theta} = \frac{\pi}{3}$

$\theta = -\frac{\pi}{3}$

(12) $\tan\left(\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$

let $\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$; $\sin\theta = \frac{1}{\sqrt{2}} > 0$

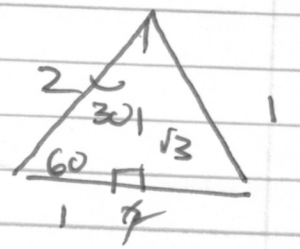
II	I
S	A $S>0$
T	C $S<0$
III	IV



$\theta = \frac{\pi}{4}$

ie, $\tan\left(\frac{\pi}{4}\right) = \boxed{1}$

(13) $\cos^{-1}\left[2\sin\left(\frac{\pi}{2}\right)\right]$



$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos^{-1}\left[2\cdot\frac{\sqrt{3}}{2}\right] = \cos^{-1}(\sqrt{3}) = \theta$

$\rightarrow \cos\theta = \sqrt{3}$ [impossible]

Φ

≈ 1.7

(14) $\sin\left[\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)\right]$

α β

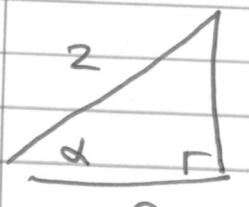
$\alpha = \sin^{-1}\left(\frac{1}{2}\right) \rightarrow \sin\alpha = \frac{1}{2}$

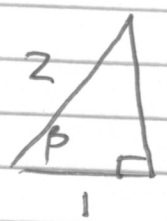
$\beta = \cos^{-1}\left(\frac{1}{2}\right) \rightarrow \cos\beta = \frac{1}{2}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

Φ Φ Φ Φ

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{\sqrt{3}}{2}$ $\frac{\sqrt{3}}{2}$

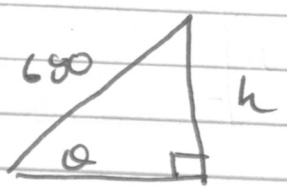




$\frac{1}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}$

$\frac{1}{4} + \frac{3}{4} = 1$ ✓

(15)



$$\sin\theta = \frac{h}{600}$$

$$\theta = \sin^{-1}\left(\frac{h}{600}\right)$$

(16) $h = 600$;

$$\theta = \sin^{-1}\left(\frac{600}{600}\right)$$

$$\theta \simeq 61.9^\circ$$