1. Convergence and Divergence Tests for Series

Test	When to Use	Conclusions
Divergence Test	for any series $\sum_{n=0}^{\infty} a_n$	Diverges if $\lim_{n\to\infty} \lvert a_n\rvert \neq 0$.
Integral Test	$\sum_{n=0}^{\infty} a_n$ with $a_n \geq 0$ and a_n decreasing	$\int_1^{\infty} f(x)dx$ and $\sum_{n=0}^{\infty} a_n$ both converge/diverge where $f(n) = a_n$.
Comparison Test	$\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ if $0 \leq a_n \leq b_n$	$\sum_{n=0}^{\infty} b_n$ converges $\Longrightarrow \sum_{n=0}^{\infty} a_n$ converges. $\sum_{n=0}^{\infty} a_n$ diverges $\Longrightarrow \sum_{n=0}^{\infty} b_n$ diverges.
Limiting Comparison Test	$\sum_{n=0}^{\infty} a_n, (a_n > 0)$. Choose $\sum_{n=0}^{\infty} b_n$, $(b_n > 0)$ if $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ with $0 < L < \infty$	$\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ both converge/diverge
	if $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$	$\sum_{n=0}^{\infty} b_n$ converges $\Longrightarrow \sum_{n=0}^{\infty} a_n$ converges.
	if $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$	$\sum_{n=0}^{\infty} b_n$ diverges $\Longrightarrow \sum_{n=0}^{\infty} a_n$ diverges.
Convergent test for alternating Series	$\sum_{n=0}^{\infty} (-1)^n a_n \quad (a_n > 0)$	converges if $\lim_{n\to\infty} a_n = 0$ and a_n is decreasing
Absolute Convergence	for any series $\sum_{n=0}^{\infty} a_n$	If $\sum_{n=0}^{\infty} \lvert a_n\rvert$ converges, then $\sum_{n=0}^{\infty} a_n$ converges, (definition of absolutely convergent series.)
Conditional Convergence	for any series $\sum_{n=0}^{\infty} a_n$	if $\sum_{n=0}^{\infty} \lvert a_n\rvert$ diverges but $\sum_{n=0}^{\infty} a_n$ converges. $\sum_{n=0}^{\infty} a_n$ conditionally converges
 Ratio Test: Root Test:	For any series $\sum_{n=0}^{\infty} a_n$, Calculate $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = L$ Calculate $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L$	there are 3 cases: if $L < 1$, then $\sum_{n=0}^{\infty} \lvert a_n\rvert$ converges ; if $L > 1$, then $\sum_{n=0}^{\infty} \lvert a_n\rvert$ diverges; if $L = 1$, no conclusion can be made.

2. Important Series to Remember

Series	How do they look	Conclusions
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	Converges to $\frac{a}{1-r}$ if $\lvert r\rvert < 1$ and diverges if $\lvert r\rvert \geq 1$
p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges if $p > 1$ and diverges if $p \leq 1$

Common Power Series Expansions

Geometric series

The geometric series and its derivatives have Maclaurin series

$$\begin{aligned}\frac{1}{1-x} &= \sum_{n=0}^{\infty} x^n \\ \frac{1}{(1-x)^2} &= \sum_{n=1}^{\infty} n x^{n-1} \\ \frac{1}{(1-x)^3} &= \sum_{n=2}^{\infty} \frac{(n-1)n}{2} x^{n-2}.\end{aligned}$$

All are convergent for $|x| < 1$. These are special cases of the bi

Exponential function

The exponential function e^x (with base e) has Maclaurin series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots.$$

It converges for all x.

Natural logarithm

Main article: Mercator series

The natural logarithm (with base e) has Maclaurin series

$$\begin{aligned}\ln(1-x) &= -\sum_{n=1}^{\infty} \frac{x^n}{n} = -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots, \\ \ln(1+x) &= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots.\end{aligned}$$

They converge for $|x| < 1$. (In addition, the series for $\ln(1 - x)$

Trigonometric functions

The usual trigonometric functions and their inverses have the following Maclaurin series:

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \quad \text{for all } x$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \quad \text{for all } x$$

$$\tan x = \sum_{n=1}^{\infty} \frac{B_{2n} (-4)^n \left(1 - 4^n\right)}{(2n)!} x^{2n-1} = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots \quad \text{for } |x| < \frac{\pi}{2}$$

$$\sec x = \sum_{n=0}^{\infty} \frac{(-1)^n E_{2n}}{(2n)!} x^{2n} = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \cdots \quad \text{for } |x| < \frac{\pi}{2}$$

$$\arcsin x = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2 (2n+1)} x^{2n+1} = x + \frac{x^3}{6} + \frac{3x^5}{40} + \cdots \quad \text{for } |x| \le 1$$

$$\arccos x = \frac{\pi}{2} - \arcsin x = \frac{\pi}{2} - \sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2 (2n+1)} x^{2n+1} = \frac{\pi}{2} - x - \frac{x^3}{6} - \frac{3x^5}{40} - \cdots \quad \text{for } |x| \le 1$$

$$\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots \quad \text{for } |x| \le 1,\ x \ne \pm i$$

Hyperbolic functions

The hyperbolic functions have Maclaurin series closely related to the series for the corresponding trigonometric functions:

$$\sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \quad \text{for all } x$$

$$\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots \quad \text{for all } x$$

$$\tanh x = \sum_{n=1}^{\infty} \frac{B_{2n} 4^n \left(4^n - 1\right)}{(2n)!} x^{2n-1} = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \cdots \quad \text{for } |x| < \frac{\pi}{2}$$

$$\operatorname{arsinh} x = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{4^n (n!)^2 (2n+1)} x^{2n+1} = x - \frac{x^3}{6} + \frac{3x^5}{40} - \cdots \quad \text{for } |x| \le 1$$

$$\operatorname{artanh} x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} = x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots \quad \text{for } |x| \le 1,\ x \ne \pm 1$$

The numbers B_k appearing in the series for $\tanh x$ are the Bernoulli numbers.