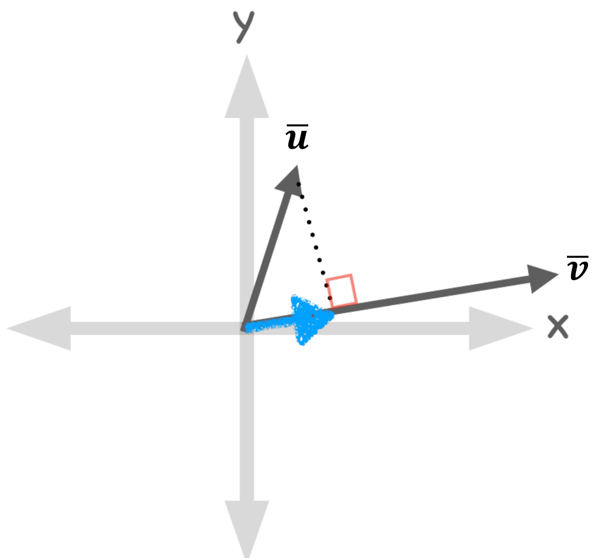
Projection Vector

Now that we know how to describe the "shadow" of a vector of $\bar{u}$ onto $\bar{v}$ known as the component of $\bar{u}$ onto $\bar{v}$ described below. We can move forward and determine the **projection vector of $\bar{u}$ onto $\bar{v}$.**

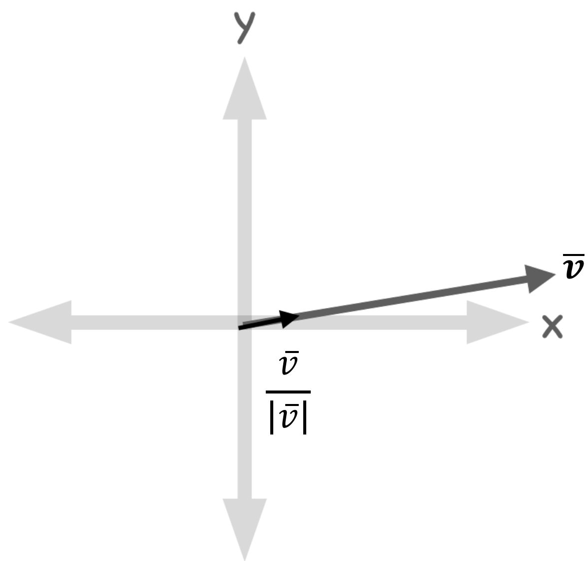


The projection vector $\bar{u}$ onto $\bar{v}$ is based on "unitizing" the vector $\bar{v}$ and then applying the component of $\bar{u}$ onto $\bar{v}$.

Unitizing

Let $\bar{v} = \langle a, b\rangle$ be a non-zero vector. We can unitize the vector by determine the magnitude of $\bar{v}$ and dividing the components by $|\bar{v}|$.

$\bar{v} = \langle \frac{a}{|\bar{v}|}, \frac{b}{|\bar{v}|}\rangle$ will be a **unit vector.**



Unitize the following vectors.

1. $\bar{v} = \langle 1,1 \rangle$
2. $\bar{v} = \langle 4,2 \rangle$
3. $\bar{v} = \langle -1,-3 \rangle$
4. $\bar{v} = \langle 4,0 \rangle$
5. $\bar{v} = \langle 0,-3 \rangle$

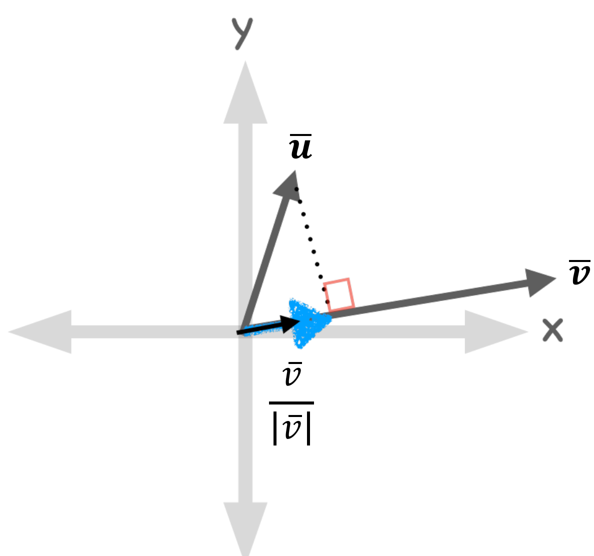
We can now proceed to determine the component of component of $\bar{u}$ onto $\bar{v}$ to determine the **projection vector of $\bar{u}$ onto $\bar{v}$.**

$$proj_{\bar{v}.}(\bar{\boldsymbol{u}}) = comp\ of\ \bar{u}\ along\ \bar{v} \cdot \frac{\bar{v}}{|\bar{v}|}$$

$$proj_{\bar{v}.}(\bar{\boldsymbol{u}}) = |\bar{\boldsymbol{u}}| cos(\theta) \cdot \frac{\bar{v}}{|\bar{v}|}$$

$$proj_{\bar{v}.}(\bar{\boldsymbol{u}}) = |\bar{\boldsymbol{u}}| \frac{\bar{\boldsymbol{u}} \cdot \bar{\boldsymbol{v}}}{|\bar{\boldsymbol{u}}||\bar{v}|} \cdot \frac{\bar{v}}{|\bar{v}|}$$

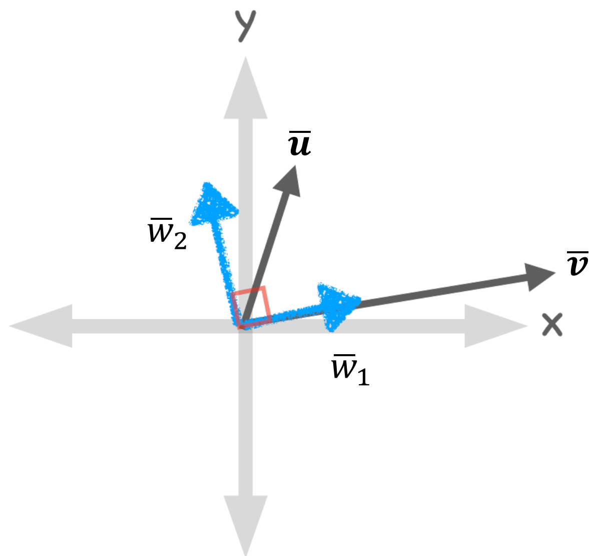
$$\boldsymbol{proj_{\bar{v}.}}(\bar{\boldsymbol{u}}) = \left(\frac{\bar{\boldsymbol{u}} \cdot \bar{\boldsymbol{v}}}{|\bar{\boldsymbol{v}}|^2}\right) \bar{\boldsymbol{v}}$$



Determine the projection vector of $\bar{u}$ onto $\bar{v}$.

6. $\bar{u} = \langle 2, -1 \rangle$ and $\bar{v} = \langle 1,3 \rangle$
7. $\bar{u} = \langle 0,4 \rangle$ and $\bar{v} = \langle 1, -1 \rangle$
8. $\bar{u} = \langle -5,1 \rangle$ and $\bar{v} = \langle 6, -6 \rangle$
9. $\bar{u} = \langle 0,5 \rangle$ and $\bar{v} = \langle -2,3 \rangle$
10. $\bar{u} = \langle 3, -4 \rangle$ and $\bar{v} = \langle 4, -1 \rangle$

We can **resolve** the projection vector of $\bar{u}$ onto $\bar{v}$ into two **orthogonal vectors** $\bar{\boldsymbol{w}}_{\mathbf{1}}$ and $\bar{\boldsymbol{w}}_{\mathbf{2}}$.



Where $\bar{\boldsymbol{w}}_{\mathbf{1}} = \boldsymbol{proj}_{\bar{v}.}(\bar{\boldsymbol{u}})$ and $\bar{\boldsymbol{w}}_{\mathbf{2}} = \bar{\boldsymbol{u}} - \boldsymbol{proj}_{\bar{v}.}(\bar{\boldsymbol{u}})$

Determine the projection vector of $\bar{\boldsymbol{u}}$ onto $\bar{\boldsymbol{v}}$ and resolve the two orthogonal vectors $\bar{\boldsymbol{w}}_{\mathbf{1}}$ and $\bar{\boldsymbol{w}}_{\mathbf{2}}$.

11. $\bar{u} = \langle 2, -1 \rangle$ and $\bar{v} = \langle 1,3 \rangle$
12. $\bar{u} = \langle 0,4 \rangle$ and $\bar{v} = \langle 1, -1 \rangle$
13. $\bar{u} = \langle -5,1 \rangle$ and $\bar{v} = \langle 6, -6 \rangle$
14. $\bar{u} = \langle 0,5 \rangle$ and $\bar{v} = \langle -2,3 \rangle$
15. $\bar{u} = \langle 3, -4 \rangle$ and $\bar{v} = \langle 4, -1 \rangle$