Poisson Probability Distribution Notes

We use this distribution when counting the number of arrivals in an interval for an experiment while using the knowledge of the average number of arrivals in an interval. This distribution is represented by the following formula.

x= the number of arrivals in an **interval** μ = the average number of arrivals in the **interval**

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

The **interval** in this model is abstract as it can represent an interval or a location interval.

Office Hour Visits

Professor Snodgrass typically has 4 students visit in an office hour (60-minutes). In a random office hour, what is the probability:

- 1. No student's visit?
- 2. One student visits?
- 3. Two student visits?
- 4. Three student visits?
- 5. Four students visit?

Let x = # of student arrivals in an office hour (**60-minutes**) $\mu = 4$ student arrivals in an office hour (**60-minutes**)

1. No student visit? x = 0 $P(x) = \frac{\mu^{x} e^{-\mu}}{x!}$

TI-83 or TI-84 Plus

- 1. Press 2nd then vars to access DISTR (distributions) menu.
- 2. Select poissonpdf and click enter.

3. Enter the values for μ and x to complete the command **poissonpdf**(μ , x) and press **enter**.

poissonpdf(4,0)



 $p(0) \approx 0.018$

2. One student visits?

$$x = 1$$
 $P(x) = \frac{\mu^{x} e^{-\mu}}{x!}$

- 1. Press 2^{nd} then vars to access DISTR (distributions) menu.
- 2. Select **poissonpdf** and click **enter**.
- 3. Enter the values for μ and x to complete the command **poissonpdf**(μ , x) and press **enter**.

poissonpdf(4,1)



 $p(1) \approx 0.073$

3. Two students visits?

$$x = 2$$
 $P(x) = \frac{\mu^{x} e^{-\mu}}{x!}$

- 1. Press **2**nd then **vars** to access DISTR (distributions) menu.
- 2. Select **poissonpdf** and click **enter**.
- 3. Enter the values for μ and x to complete the command **poissonpdf**(μ , x) and press **enter**.

poissonpdf(4,2)



 $p(2) \approx 0.147$

4. Three students visits?
$$x = 3$$

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

- 1. Press **2**nd then **vars** to access DISTR (distributions) menu.
- 2. Select **poissonpdf** and click **enter**.
- 3. Enter the values for μ and x to complete the command **poissonpdf**(μ , x) and press enter.

poissonpdf(4,3)



 $p(3) \approx 0.195$

5. Four students visits?

$$x = 4$$
 $P(x) = \frac{\mu^{x} e^{-\mu}}{x!}$

- 1. Press $\mathbf{2^{nd}}$ then \mathbf{vars} to access DISTR (distributions) menu.
- 2. Select **poissonpdf** and click **enter**.
- 3. Enter the values for μ and x to complete the command **poissonpdf**(μ , x) and press **enter**.

poissonpdf(4,4)



 $p(4) \approx 0.195$

Of course, my favorite questions are:

6. At least one student visits?

TEXAS INSTRUMENTS TI-84 Plus CE	TEXAS INSTRUMENTS TI-84 Plus CE Normal Float auto real degree MP
DISTR DRAW 6ttcdf(7:X ² Pdf(8:X ² cdf(9:Fpdf(0:Fcdf(A:binompdf(B:binomcdf(C:invBinom(Poissonpdf(1-poissonpdf(4,0) 0.9816843611 ■
staplot f1 tblset f2 format f3 calc f4 table f5 y= window zoom trace graph	statplot f1 tblset f2 format f3 calc f4 table f5 y= window zoom trace graph

$$p(x \ge 1) = p(1) + p(2) + p(3) + \dots = 1 - p(0)$$

 $p(x \ge 1) \approx 0.982$

7. At least two students visit?

$$p(x \ge 2) = p(2) + p(3) + p(4) + \dots = 1 - p(0) - p(1)$$

TEXAS INSTRUMENTS TI-84 Plus CE	TEXAS INSTRUMENTS TI-84 Plus CE
NORMAL FLOAT AUTO REAL DEGREE MP	NORMAL FLOAT AUTO REAL DEGREE MP
DISTR DRAW 6ftcdf(7:%2pdf(8:%2cdf(9:Fpdf(0:Fcdf(A:binompdf(B:binomcdf(C:tereficer	1-PoissonPdf(4,0)-Poisson⊮ 0.9084218056
Depoissonedf(
statplotf1tblsetf2formatf3calcf4tablef5y=windowzoomtracegraph	staplot f1 tblset f2 format f3 calc f4 table f5 y= window zoom trace graph

 $p(x \ge 2) \approx 0.908$

8. More than three students visit?

$$p(x > 3) = p(4) + p(5) + p(6) + \dots = 1 - p(0) - p(1) - p(2) - p(3)$$

TEXAS INSTRUMENTS TI-84 Plus CE	TEXAS INSTRUMENTS TI-84 Plus CE
NORMAL FLOAT AUTO REAL DEGREE MP	NORMAL FLOAT AUTO REAL DEGREE MP
DISTR DRAW 61tcdf(7:X2pdf(8:X2cdf(9:Frdf(0:Frcdf(A:binomrdf(B:binomcdf(C:invBinom(DProissonrdf(1-poissonpdf(4,0)-poisson⊮ 0.5665298796
statplot f1 tblset f2 format f3 calc f4 table f5 y= window zoom trace graph	statplot f1 tblset f2 format f3 calc f4 table f5 y= window zoom trace graph

 $p(x > 3) \approx 0.567$

9. No more than four students visit?

$$p(x \le 4) = p(0) + p(1) + p(2) + p(3) + p(4)$$

TI-84 Plus CE	TEXAS INSTRUMENTS TI-84 Plus CE
NORMAL FLOAT AUTO REAL DEGREE MP	NORMAL FLOAT AUTO REAL DEGREE MP
DISTE DRAW 61tcdf(7:X ² Pdf(8:X ² cdf(9:Fpdf(0:Fcdf(R:binompdf(B:binomcdf(C:invBinom(Poissonpdf(PoissonPdf(4,0)+PoissonPd≯ 0.6288369352
statplotf1tblsetf2formatf3calcf4tablef5y=windowzoomtracegraph	statplot f1tblsetf2formatf3calcf4tablef5y=windowzoomtracegraph

 $p(x \le 4) \approx 0.629$

10. Between two and five students visit?

$$p(2 \le x \le 5) = p(2) + p(3) + p(4) + p(5)$$

TEXAS INSTRUMENTS TI-84 Plus CE	TEXAS INSTRUMENTS TI-84 Plus CE
NORMAL FLOAT AUTO REAL DEGREE MP	NORMAL FLOAT AUTO REAL DEGREE MP
DISTR DRAW 61tcdf(7:%2Pdf(8:%2cdf(9:Fpdf(0:Fcdf(A:binompdf(B:binomcdf(C:invBinom(PoissonPdf(4,2)+PoissonPd) 0.6935521926
statplot f1 tblset f2 format f3 calc f4 table f5 y= window zoom trace graph	statplot f1 tblset f2 format f3 calc f4 table f5

 $p(2 \le x \le 5) \approx 0.694$

Interval

In the next 38 minutes of an office hour, what's the probability that:

4 visitors	_ μ visitors
60 min	38 min
TEXAS INSTRUMENTS	TI-84 Plus CE
4*38/60	2,5333333333
statplot f1 tblset f2 form y= window zoo	at f3 caic f4 table f5 om trace graph

 $\mu = rac{4*38}{60} pprox 2.533$ visitors per 38 minutes

11. No more than two students visit?

$$p(x \le 2) = p(0) + p(1) + p(2)$$



 $p(x \le 2) \approx 0.535$

12. Less than two students visit?

TI-84 Plus CE	TEXAS INSTRUMENTS TI-84 Plus CE
NORMAL FLOAT AUTO REAL DEGREE MP	NORMAL FLOAT AUTO REAL DEGREE MP
DISTR DRAW 6ttcdf(7:%2pdf(8:%2cdf(9:Fpdf(0:Fcdf(8:binompdf(B:binomcdf(C:invBinom(DPpoissonpdf(PoissonPdf(2.533.0)+Poiss) 0.2805922779 ■
statplot f1 tblset f2 format f3 calc f4 table f5 y= window zoom trace graph	statplot f1 tblset f2 format f3 calc f4 table f5 y= window zoom trace graph

p(x<2)=p(0)+p(1)

 $p(x < 2) \approx 0.281$

13. At least three students visit?

$$p(x \ge 3) = p(3) + p(4) + p(5) + \dots = 1 - p(0) - p(1) - p(2)$$



 $p(x \ge 3) \approx 0.465$

In the next 72 minutes of an office hour, what's the probability that:

$\frac{4 \ visitors}{60 \ min} =$	$=\frac{\mu \ visitors}{72 \ min}$
TEXAS INSTRUMENTS	TI-84 Plus CE
NORMAL FLOAT AUTO RE	AL DEGREE MP
4*72/60	4.0
	4.8.
statplot f1 tblset f2 forma y= window zoo	t f3 calc f4 table f5 m trace graph

$$\mu=rac{4*72}{60}=4.8$$
 visitors per 72 minutes

14. More than one student visit?

$$p(x > 1) = p(2) + p(3) + p(5) + \dots = 1 - p(0) - p(1)$$



 $p(x > 1) \approx 0.719$

15. No more than one student visits?

TEXAS INSTRUMENTS TI-84 Plus CE	TEXAS INSTRUMENTS TI-84 Plus CE
DISTR DRAW 6^tcdf(7:%2Pdf(8:%2cdf(9:Fpdf(0:Fcdf(A:binompdf(B:binomcdf(C:invBinom(DUpoissonpdf(PoissonPdf(2.533,0)+Poiss≯ 0.2805922779
statplot f1 tblset f2 format f3 calc f4 table f5 y= window zoom trace graph	statplotf1tblsetf2f3calcf4tablef5y=windowzoomtracegraph

 $p(x \leq 1) = p(0) + p(1)$

 $p(x \le 1) \approx 0.281$

This Poisson Porbability Distribution also has a way to compute its variance and standard deviation. In fact, these are the short cut formulas.

Mean μ

Variance $\sigma^2 = \mu$

Standard Deviation $\sigma = \sqrt{\mu}$