Poisson Probability Distribution Notes

We use this distribution when counting the number of arrivals in an interval for an experiment while using the knowledge of the average number of arrivals in an interval. This distribution is represented by the following formula.

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

x= the number of arrivals in an **interval**
μ= the average number of arrivals in the **interval**

The **interval** in this model is abstract as it can represent an interval or a location interval.

Office Hour Visits
Professor Snodgrass typically has 4 students visit in an office hour (60-minutes). In a random office hour, what is the probability:

1. No student's visit?
2. One student visits?
3. Two student visits?
4. Three student visits?
5. Four students visit?

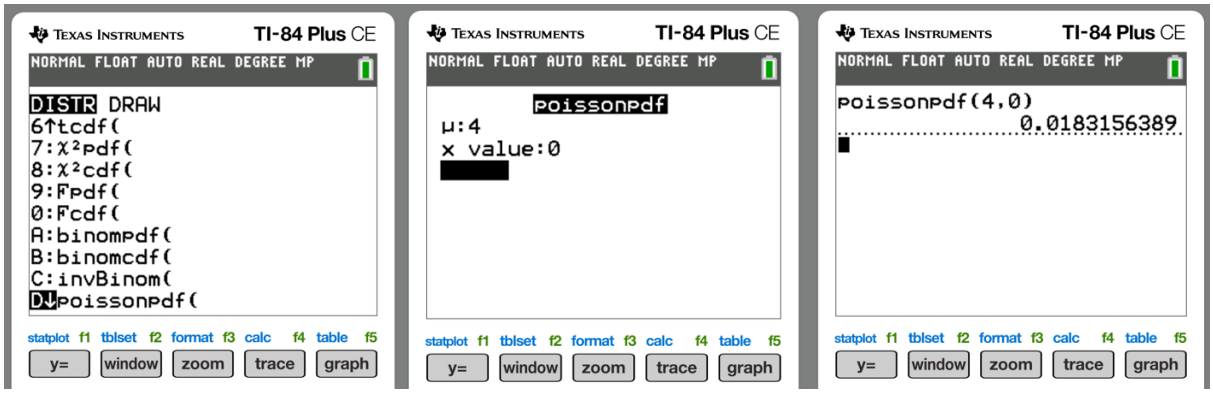
Let x = # of student arrivals in an office hour (**60-minutes**)
μ = 4 student arrivals in an office hour (**60-minutes**)

1. No student visit?
   $x = 0$

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

TI-83 or TI-84 Plus
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **poissonpdf** and click **enter**.
3. Enter the values for μ and x to complete the command **poissonpdf(μ, x)** and press **enter.**

poissonpdf(4,0)



$$p(0) \approx 0.018$$

2. One student visits?

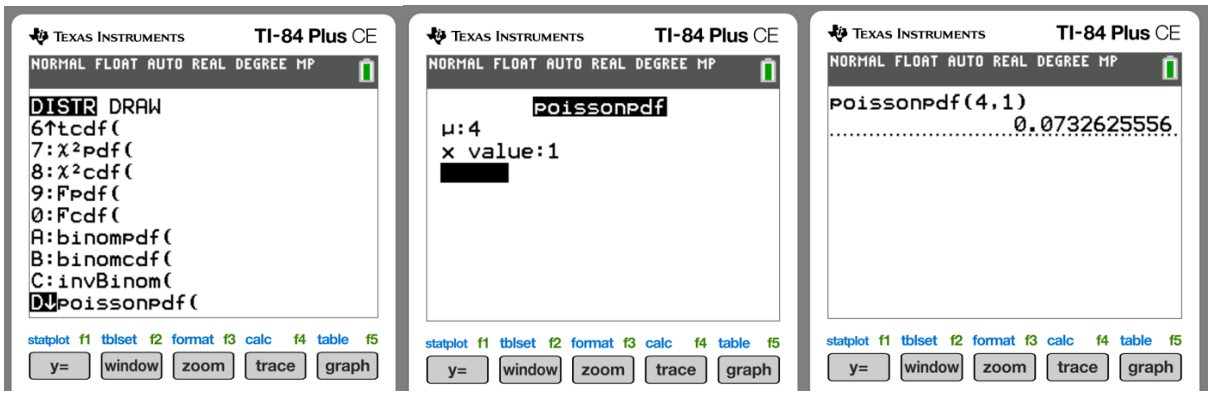
$x = 1$

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

TI-83 or TI-84 Plus

1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **poissonpdf** and click **enter**.
3. Enter the values for μ and x to complete the command **poissonpdf(μ, x)** and press **enter.**

poissonpdf(4,1)



$$p(1) \approx 0.073$$

3. Two students visits?
   $x = 2$

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

TI-83 or TI-84 Plus

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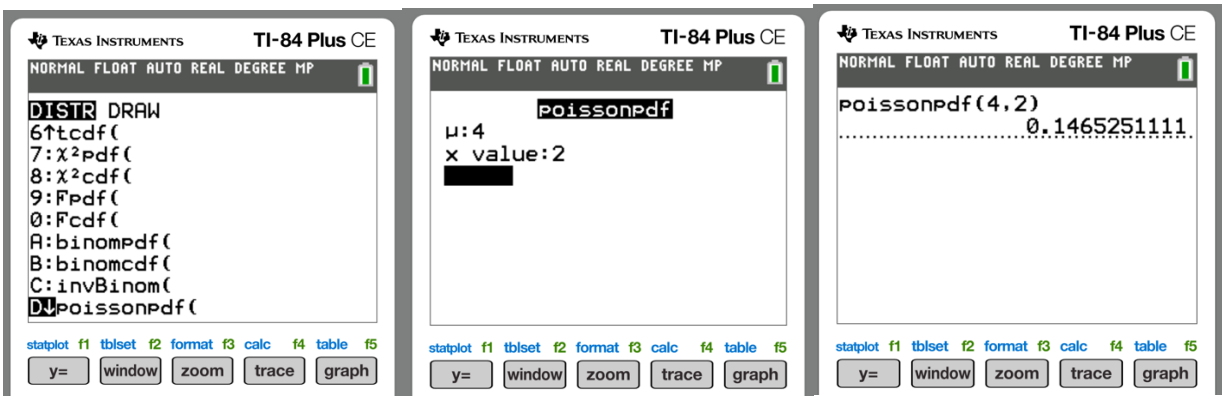
2. Select **poissonpdf** and click **enter**.
3. Enter the values for μ and x to complete the command **poissonpdf(μ, x)** and press **enter.**

poissonpdf(4,2)



$$p(2) \approx 0.147$$

4. Three students visits?

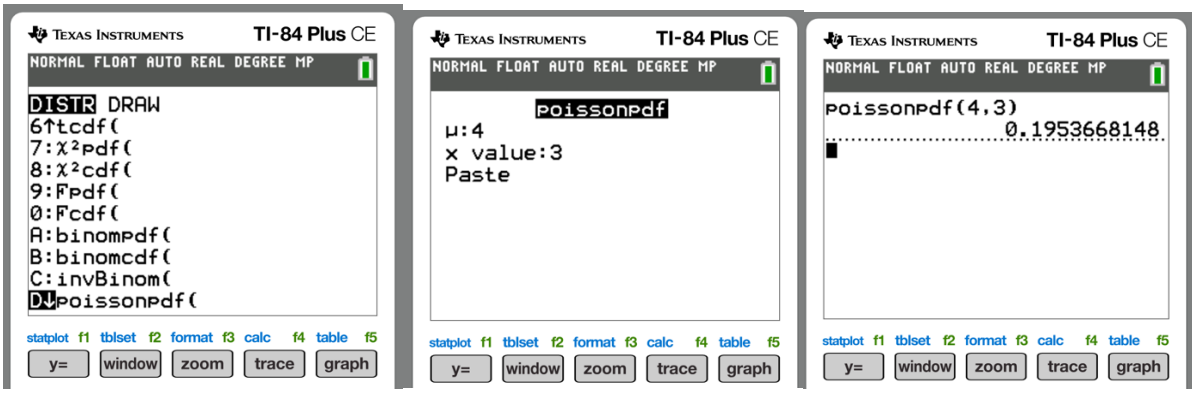
$x = 3$

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

TI-83 or TI-84 Plus

1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **poissonpdf** and click **enter**.
3. Enter the values for μ and x to complete the command **poissonpdf(μ, x)** and press **enter.**

poissonpdf(4,3)



$$p(3) \approx 0.195$$

5. Four students visits?

$x = 4$

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

TI-83 or TI-84 Plus

1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **poissonpdf** and click **enter**.
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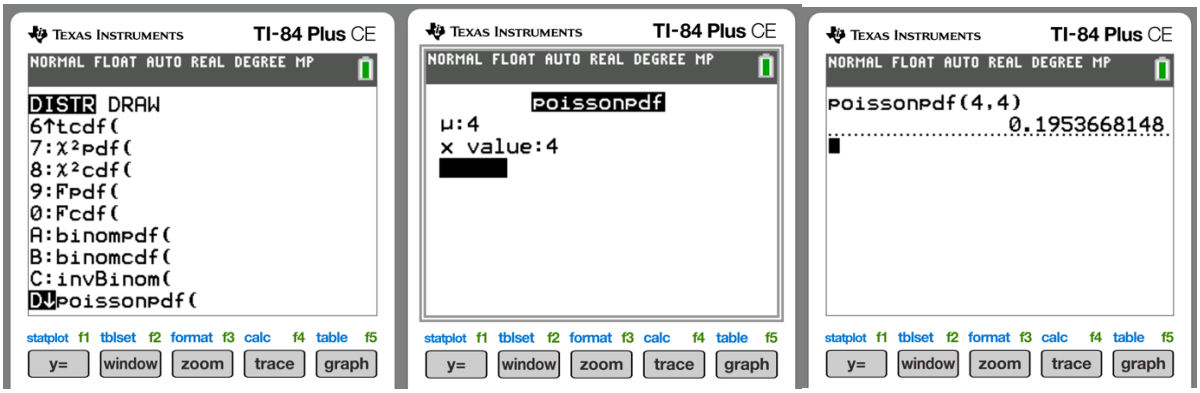
poissonpdf(4,4)

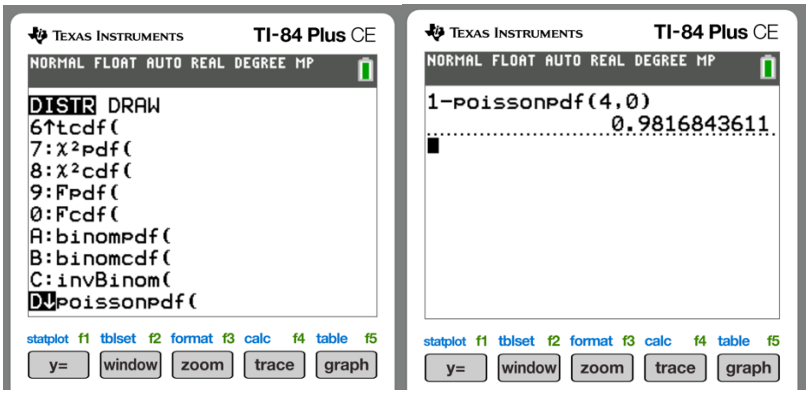


$$p(4) \approx 0.195$$

Of course, my favorite questions are:

6. At least one student visits?

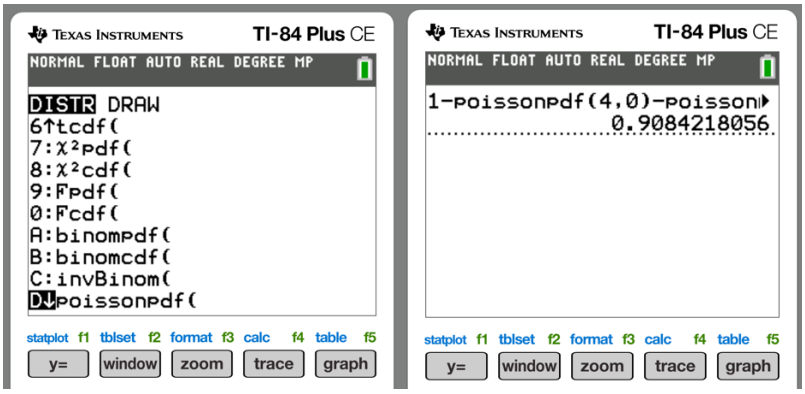
$$p(x \geq 1) = p(1) + p(2) + p(3) + \cdots = 1 - p(0)$$



$$p(x \geq 1) \approx 0.982$$

7. At least two students visit?

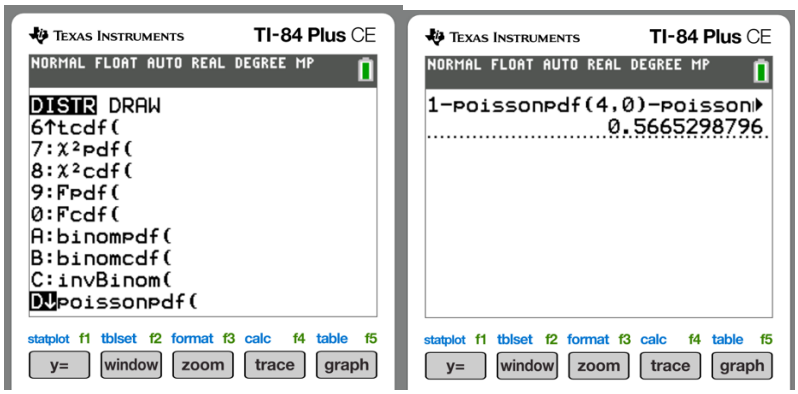
$$p(x \geq 2) = p(2) + p(3) + p(4) + \cdots = 1 - p(0) - p(1)$$



$$p(x \geq 2) \approx 0.908$$

8. More than three students visit?

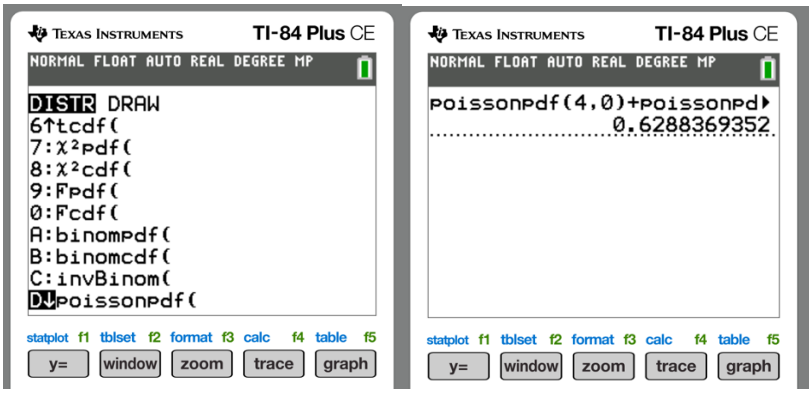
$$p(x > 3) = p(4) + p(5) + p(6) + \cdots = 1 - p(0) - p(1) - p(2) - p(3)$$



$$p(x > 3) \approx 0.567$$

9. No more than four students visit?

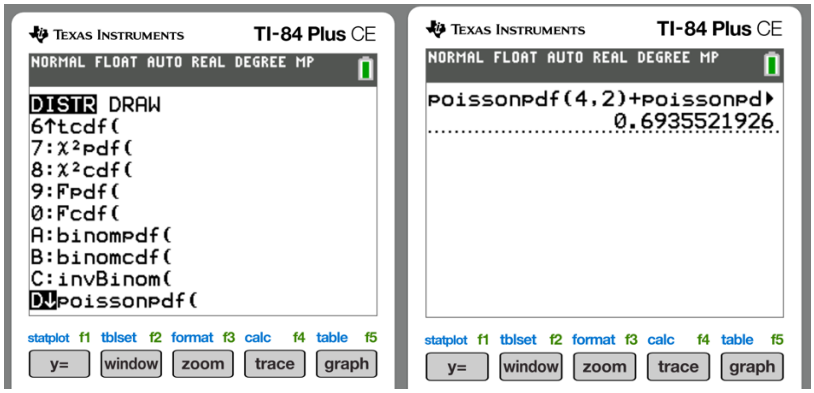
$$p(x \leq 4) = p(0) + p(1) + p(2) + p(3) + p(4)$$



$$p(x \leq 4) \approx 0.629$$

10. Between two and five students visit?

$$p(2 \leq x \leq 5) = p(2) + p(3) + p(4) + p(5)$$

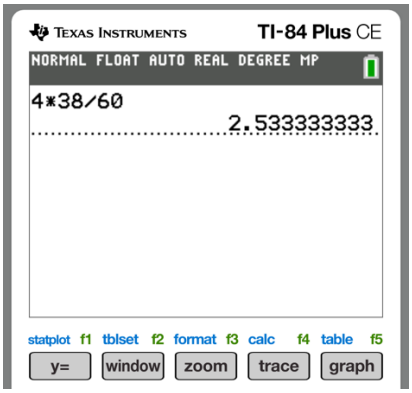


$$p(2 \leq x \leq 5) \approx 0.694$$

Interval

In the next **38 minutes** of an office hour, what's the probability that:

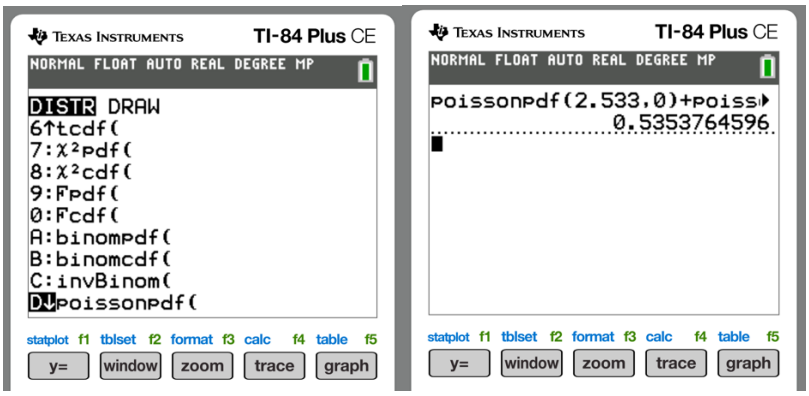
$$\frac{4\ visitors}{60\ min} = \frac{\mu\ visitors}{38\ min}$$



$$\mu = \frac{4*38}{60} \approx 2.533 \text{ visitors per 38 minutes}$$

11. No more than two students visit?

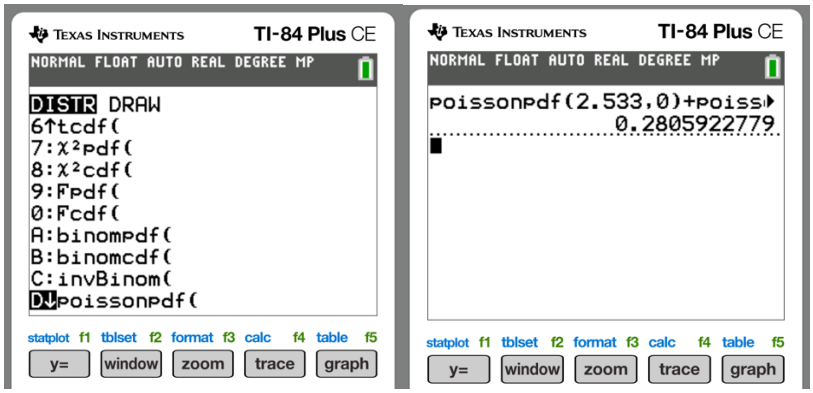
$$p(x \leq 2) = p(0) + p(1) + p(2)$$



$$p(x \leq 2) \approx 0.535$$

12. Less than two students visit?

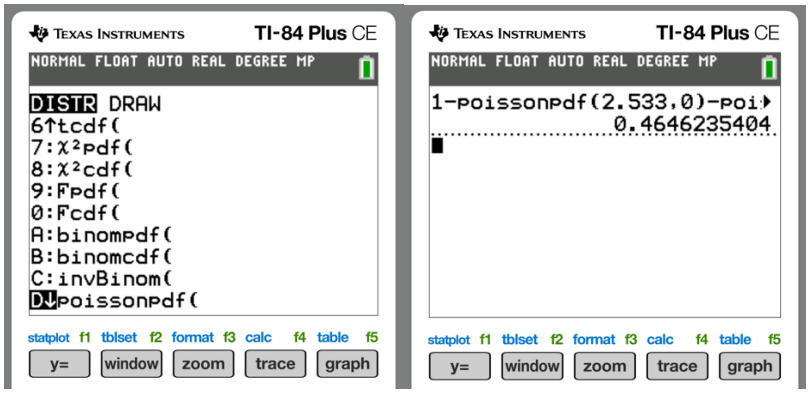
$$p(x < 2) = p(0) + p(1)$$



$$p(x < 2) \approx 0.281$$

13. At least three students visit?

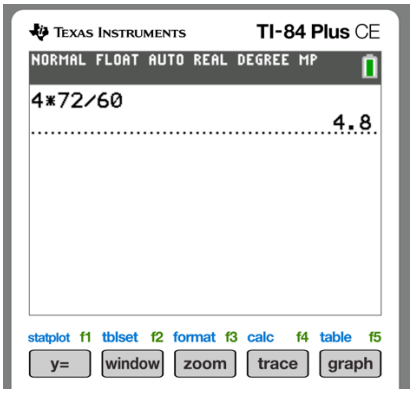
$$p(x \geq 3) = p(3) + p(4) + p(5) + \cdots = 1 - p(0) - p(1) - p(2)$$



$$p(x \geq 3) \approx 0.465$$

In the next **72 minutes** of an office hour, what's the probability that:

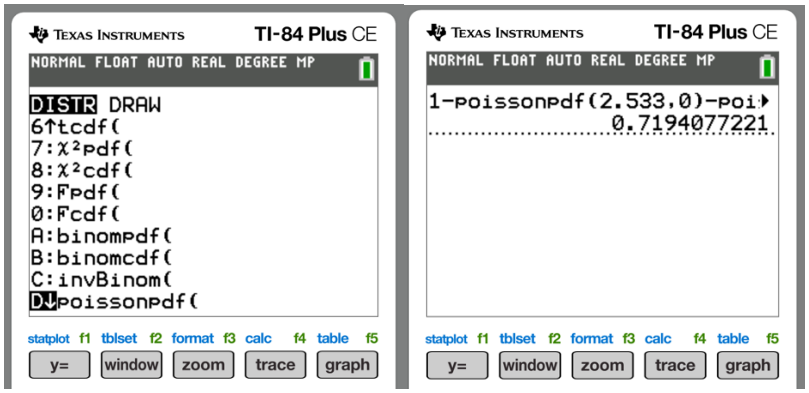
$$\frac{\mathbf{4\ visitors}}{\mathbf{60\ min}} = \frac{\boldsymbol{\mu}\ \mathbf{visitors}}{\mathbf{72\ min}}$$



$$\boldsymbol{\mu} = \frac{4*72}{60} = \mathbf{4.8}\ \text{visitors per 72 minutes}$$

14. More than one student visit?

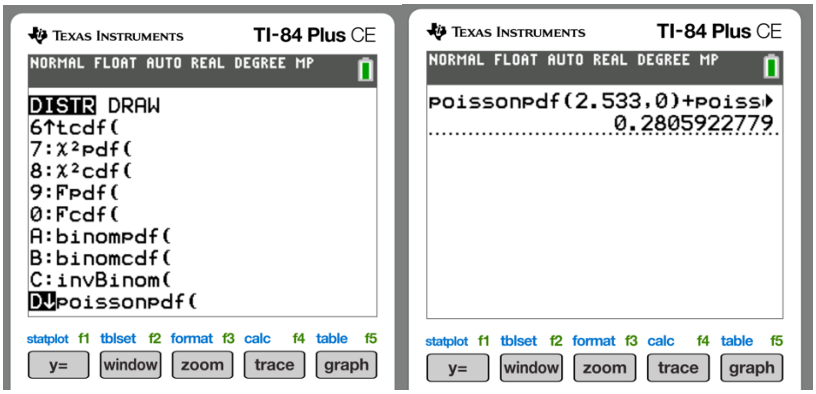
$$\boldsymbol{p(x > 1) = p(2) + p(3) + p(5) + \cdots = 1 - p(0) - p(1)}$$



$$\boldsymbol{p(x > 1) \approx 0.719}$$

15. No more than one student visits?

$$p(x \leq 1) = p(0) + p(1)$$



$$p(x \leq 1) \approx 0.281$$

This Poisson Porbability Distribution also has a way to compute its variance and standard deviation. In fact, these are the short cut formulas.

Mean μ

Variance $\sigma^2 = \mu$

Standard Deviation $\sigma = \sqrt{\mu}$