One Way ANOVA
Analysis of Variance

When we are interested in determining if the means from at least three different populations are the same we use the **Method of Analysis of Variance.** This is done in the form of a formal **Hypothesis Testing Procedure**.

$H_0: \mu_1 = \mu_2 = \mu_3 = \cdots \mu_k$ **Claim**
H_1: at least one μ is not equal

Because this process is very complicated Mathematically, we emphasize the use of technology and the interpretation of the results.

One Way ANOVA is a method of testing the equality of three or more population means by analyzing the sample variances. One way Analysis of Variance is used with data categorized with **one factor (treatment)**. This is a single factor used to separate the sample data into distinct groups.

Requirements

- The populations have distributions that are approximately normal. This is a loose requirement.
- The populations have the same population variance σ^2 or standard deviation σ. This is a loose requirement.
- The samples are random and independent from one another.
- The different samples are from populations that are categorized in one way.

Procedure for Testing $H_0: \mu_1 = \mu_2 = \mu_3 = \cdots \mu_k$ **Claim**

- Use technology to obtain the **Test Statistic** $\boldsymbol{F}$ and $\boldsymbol{p}$ **value**.
- The ANOVA Test is a Right Tailed Test.

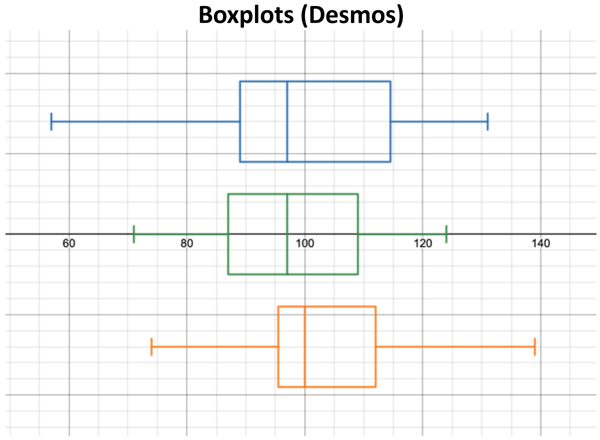
We will study examples to get a good feel for Analysis of Variance known as ANOVA.

Performance IQ Scores based on the Height of Men (Short, Medium, Tall)

Sample Statistics are in the table below along with Boxplots (Desmos) for **short men** (below 5 ft 5), **medium height men** (between 5 ft 5 inches and 5 ft 9 inches) and **tall men** (above 5 ft 10 inches).

Tall Men							
86	93	108	59	125	96	90	92
96	88	97	117	103	120	121	113
92	86	83	104	57	119	121	121
93	97	77	113	95	131	116	118
110	95	85	74	100	113	82	102
Mean	99.685						
Variance	300.709						
SD	17.341						
n	40						
Medium Height Men							
79	72	119	103	91	97		
109	109	114	78	97	113		
106	96	110	105	105	83		
93	96	71	85	87	101		
81	88	118	124	92	105		
Mean	97.563						
Variance	199.991						
SD	14.142						
n	30						
Short Men							
87	117	114	139	83			
100	129	74	105	104			
96	95	74	97	99			
100	98	101	107	118			
95	104	97	122	110			
Mean	102.631						
Variance	232.805						
SD	15.258						
n	25						

Boxplots (Desmos)



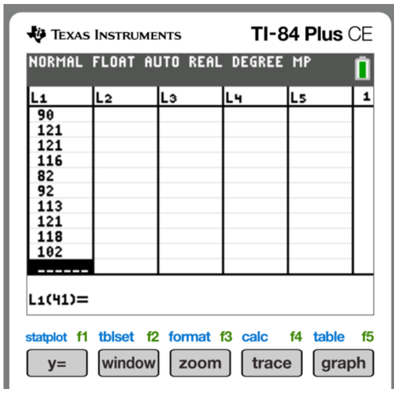
Inconclusive

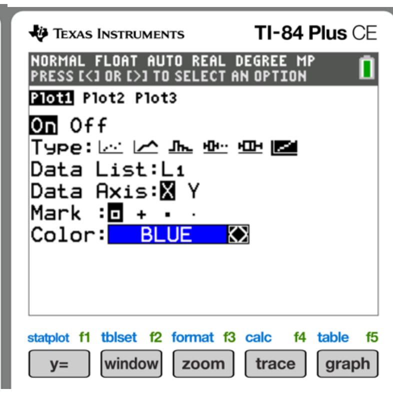
Normal

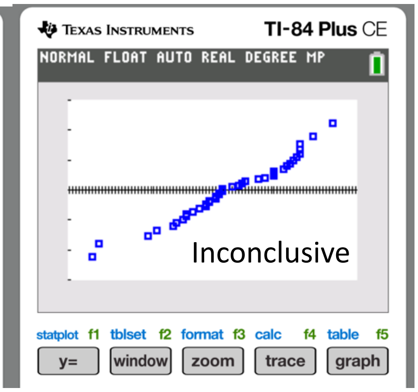
Right Skew

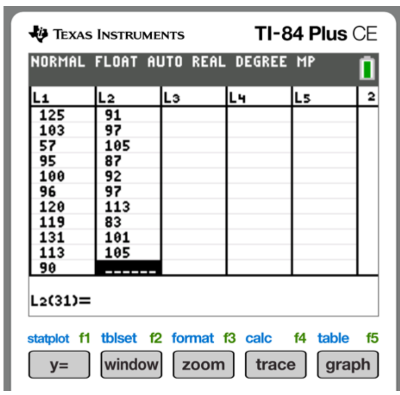
It is a good thing they need to be "loosely" Normal Distributed.

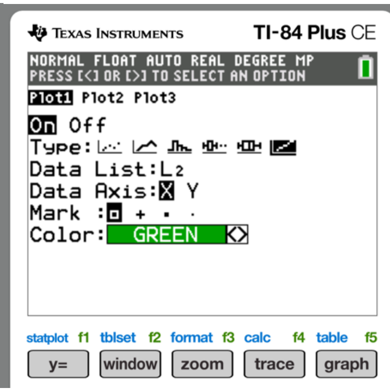
We can **Access for Normality** being creating a **Normal Quantile Plot** in the **TI-84 Calculator.** Let L_1 represent the IQ Scores for tall men, L_2 represent the IQ Scores for medium height men, L_3 represent the IQ Scores for short men.

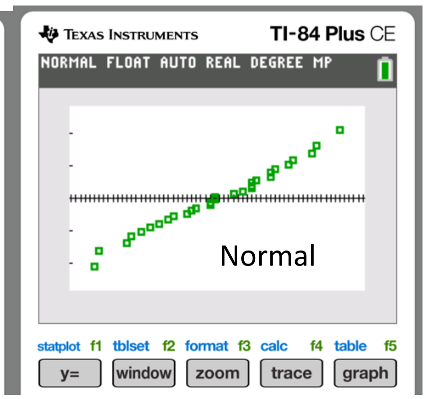


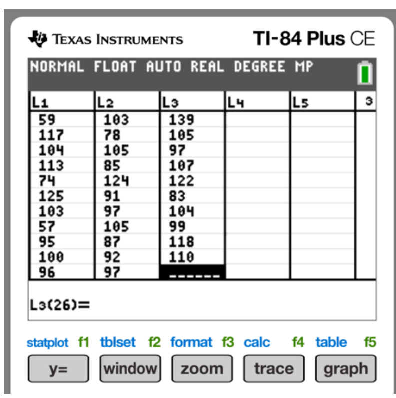


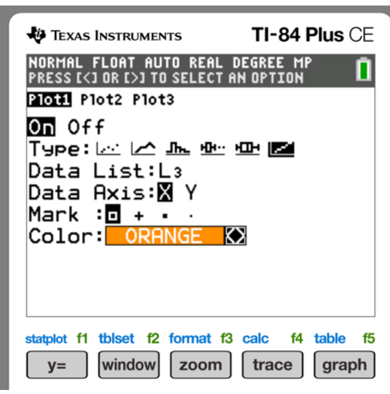


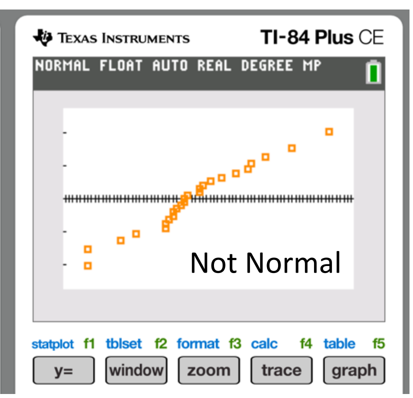




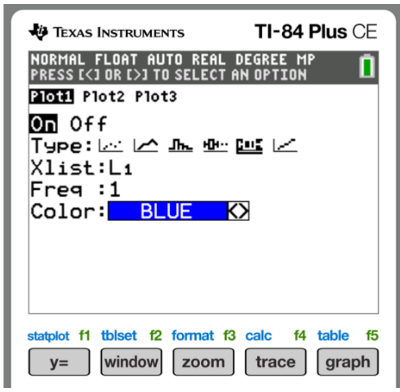


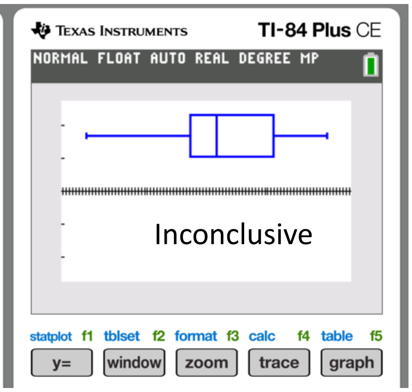


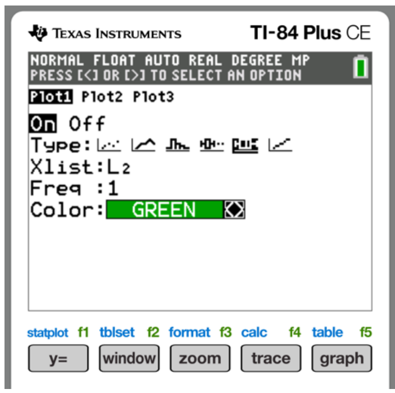


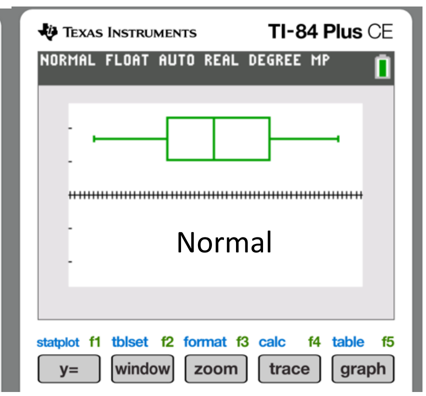


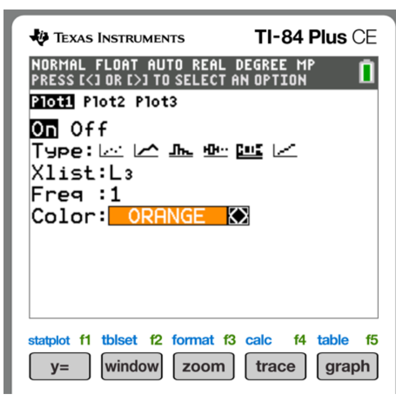
We can create **Boxplots** with the **TI-84 Plus CE calculator** as well.

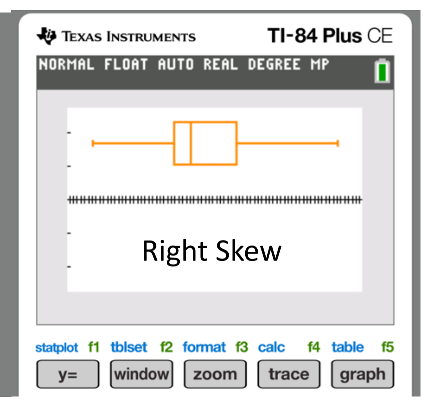










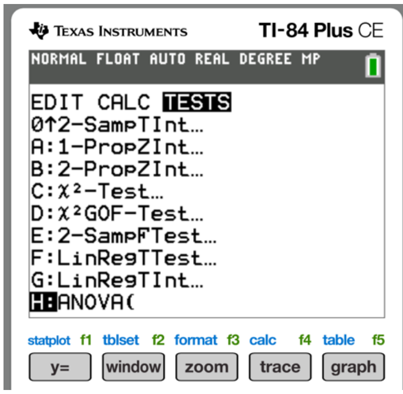


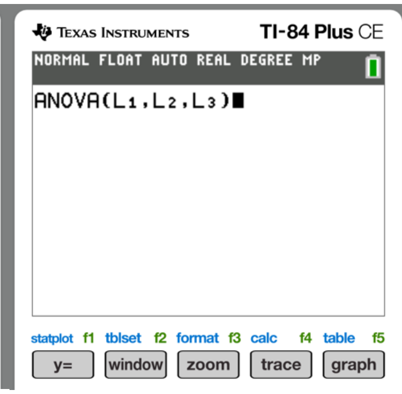
Modern (P Value Method) Hypothesis Test for Analysis of Variance with $\alpha = 5\%$

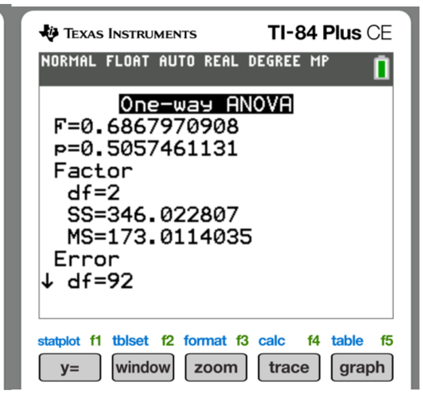
$H_0: \mu_1 = \mu_2 = \mu_3$ **Claim**

H_1: at least one μ is not equal

Using the $\boldsymbol{ANOVA(L_1, L_2, L_3)}$ in the **TI-84 Plus CE Calculator**







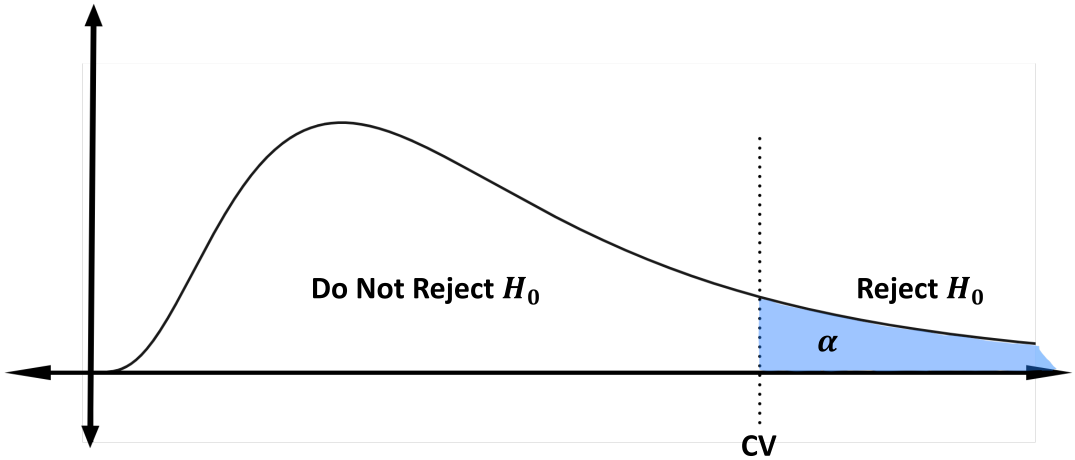
$\boldsymbol{p \approx 0.506; p \nless \alpha}$**; Accept** $\boldsymbol{H_0}$

The Sample Supports the Claim

The data comes from populations having the same means

However, if you would like to use the **Traditional Hypothesis Testing Method** for ANOVA that is based on the F Distribution.

F Distribution



- The F distribution is not symmetric and is right skewed.
- The values of the F distribution are non-negative.
- The slope of the F distribution is dependent on two different degrees of freedom.

Test Statistic $\boldsymbol{F = \frac{n s_{\bar{x}}{}^2}{s_p{}^2}}$

The test Statistic is the ratio of two estimates for σ^2

1. The **variation between samples** (based on variation among the sample means).

$$\boldsymbol{n s_{\bar{x}}{}^2}$$

2. **Variation within samples** (based on the sample variances).

$$\boldsymbol{s_p{}^2}$$

$$F = \frac{\textbf{variation between samples}}{\textbf{variation within samples}}$$

Performance IQ Scores based on the Height of Men (Short, Medium, Tall)

$H_0: \mu_1 = \mu_2 = \mu_3$ **Claim**
H_1: at least one μ is not equal

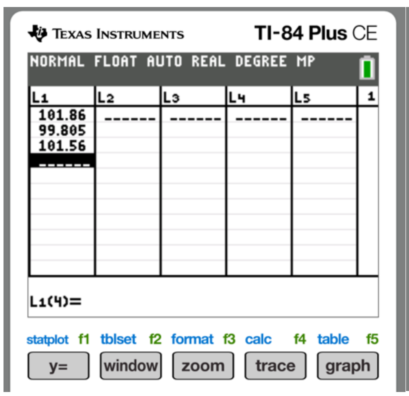
Equal Sample Sizes are needed for increased accuracy!
$n = 25$

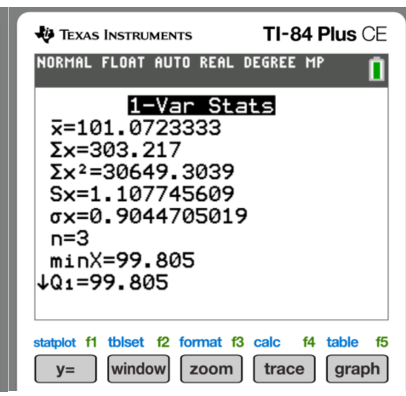
Tall Men	
Mean	101.856
Variance	288.672
SD	16.990
n	25

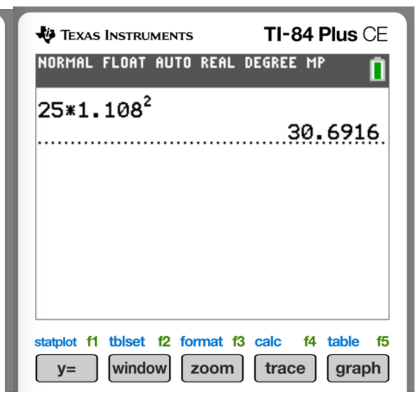
Medium Men	
Mean	99.805
Variance	210.641
SD	14.513
n	25

Small Men	
Mean	101.556
Variance	222.581
SD	14.919
n	25

Variation Between Samples

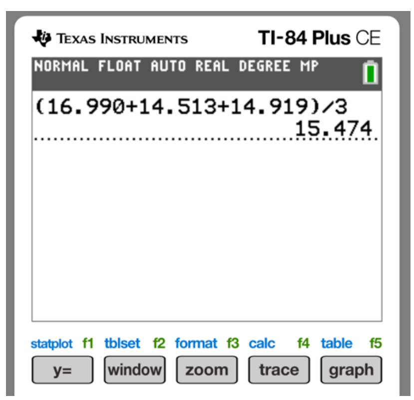




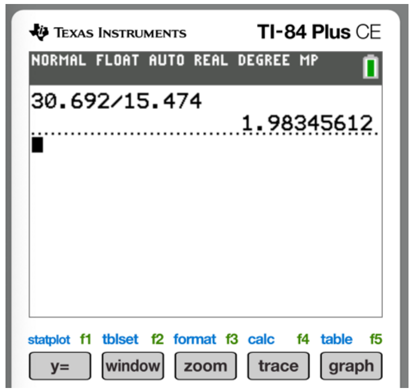


$$n s_{\bar{x}}^2 = 25 \cdot 1.108^2 \approx 30.692$$

Variation Within Samples



$$s_p^2 = \frac{16.990+14.513+14.919}{3} \approx 15.474$$ **Mean of the Sample Variances**



Test Statistic $F = \frac{30.692}{15.474} \approx 1.983$

Determining Critical Values

Degree of Freedom

k =**number of samples;** $k = 3$

n=**sample size;** $n = 25$

Numerator degree of freedom $= k - 1$

numerator df$= 3 - 1 = 2$

Denominator degree of freedom$= k(n - 1)$

denominator df$= 3(25 - 1) = 92$

F Table

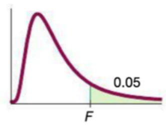
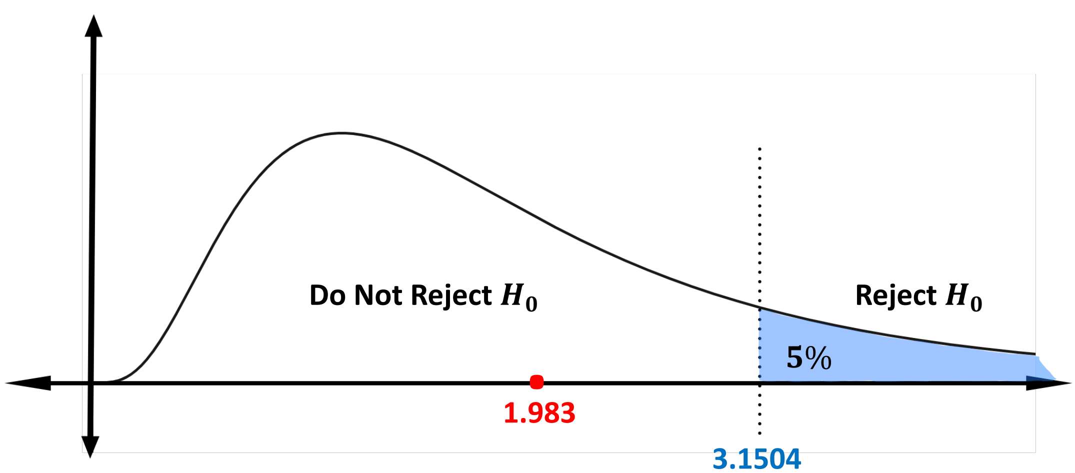


TABLE A-5 *(continued) F* Distribution ($\alpha = 0.05$ in the right tail)

Denominator degrees of freedom (df_2)	Numerator degrees of freedom (df_1): 1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385
3	10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.0410	6.9988
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.0990
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767
8	5.3177	4.4590	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789
10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876
16	4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377
17	4.4513	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563
19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227
20	4.3512	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3660
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419
23	4.2793	3.4221	3.0280	2.7955	2.6400	2.5277	2.4422	2.3748	2.3201
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002
25	4.2417	3.3852	2.9912	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821
26	4.2252	3.3690	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655
27	4.2100	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501
28	4.1960	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.2360
29	4.1830	3.3277	2.9340	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107
40	4.0847	3.2317	2.8387	2.6060	2.4495	2.3359	2.2490	2.1802	2.1240
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.0970	2.0401
120	3.9201	3.0718	2.6802	2.4472	2.2899	2.1750	2.0868	2.0164	1.9588
∞	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799



The Sample Supports the Claim

The data comes from populations having the same means

Lifespan of Americans (African, Hispanic, Asian, White)

Sample Statistics in the table below represent the lifespans of various American ethnic groups. Use the **5% level of significance** to test the claim that they come from populations with the same mean.

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$ **Claim**
H_1: at least one μ is not equal

L_1

African American				
57	73	54	57	78
73	74	73	74	83
70	85	78	77	90
73	63	83	67	77
80	77	62	78	68
Mean	73.061			
Variance	81.672			
SD	9.037			
n	25			

L_3

Asian American				
95	99	86	100	92
79	66	71	95	77
66	84	104	85	69
97	78	94	94	88
84	78	82	68	89
Mean	84.804			
Variance	125.327			
SD	11.195			
n	25			

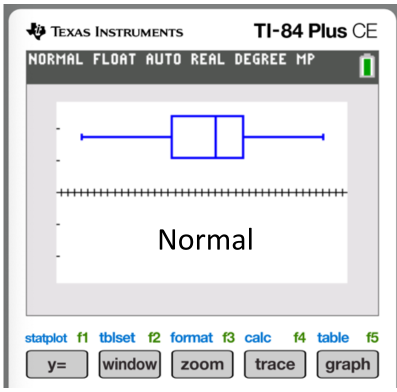
L_2

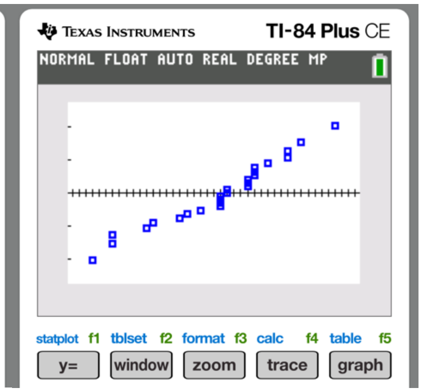
Hispanic American				
99	72	77	60	89
78	97	80	72	90
78	75	74	79	61
65	62	74	75	88
89	62	58	71	95
Mean	76.783			
Variance	141.977			
SD	11.915			
n	25			

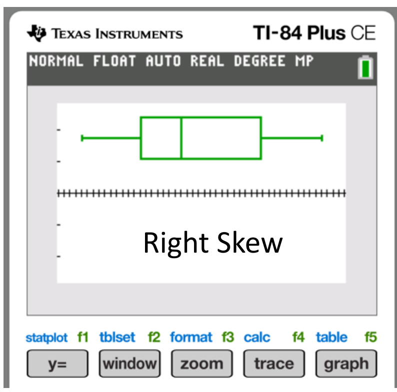
L_4

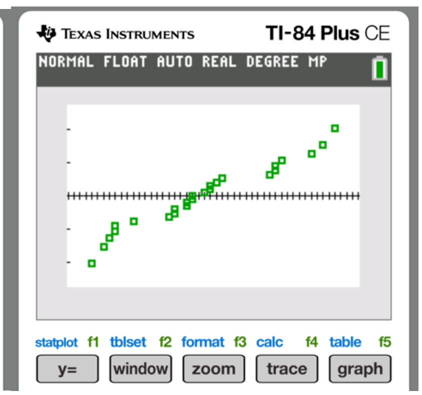
White Americans				
81	71	75	91	82
60	71	86	77	66
77	67	88	72	64
72	72	76	73	67
92	82	88	67	90
Mean	76.299			
Variance	82.628			
SD	9.090			
n	25			

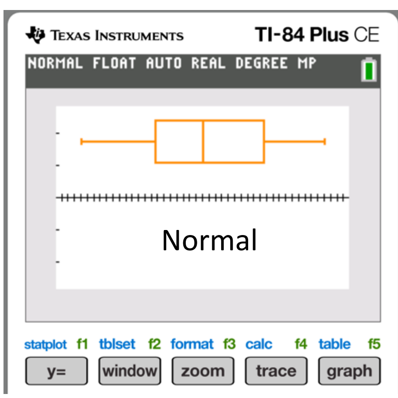
Boxplots and Normal Quantile Plots

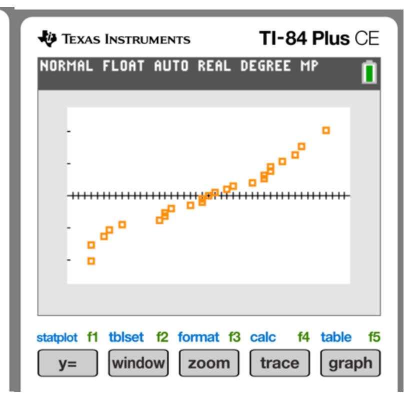


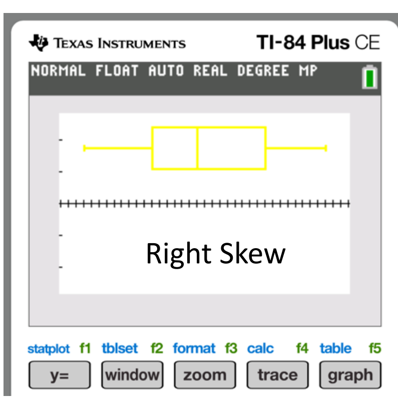


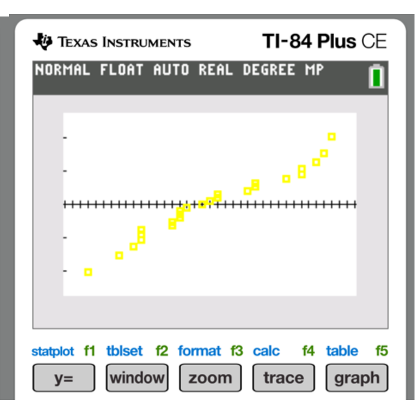




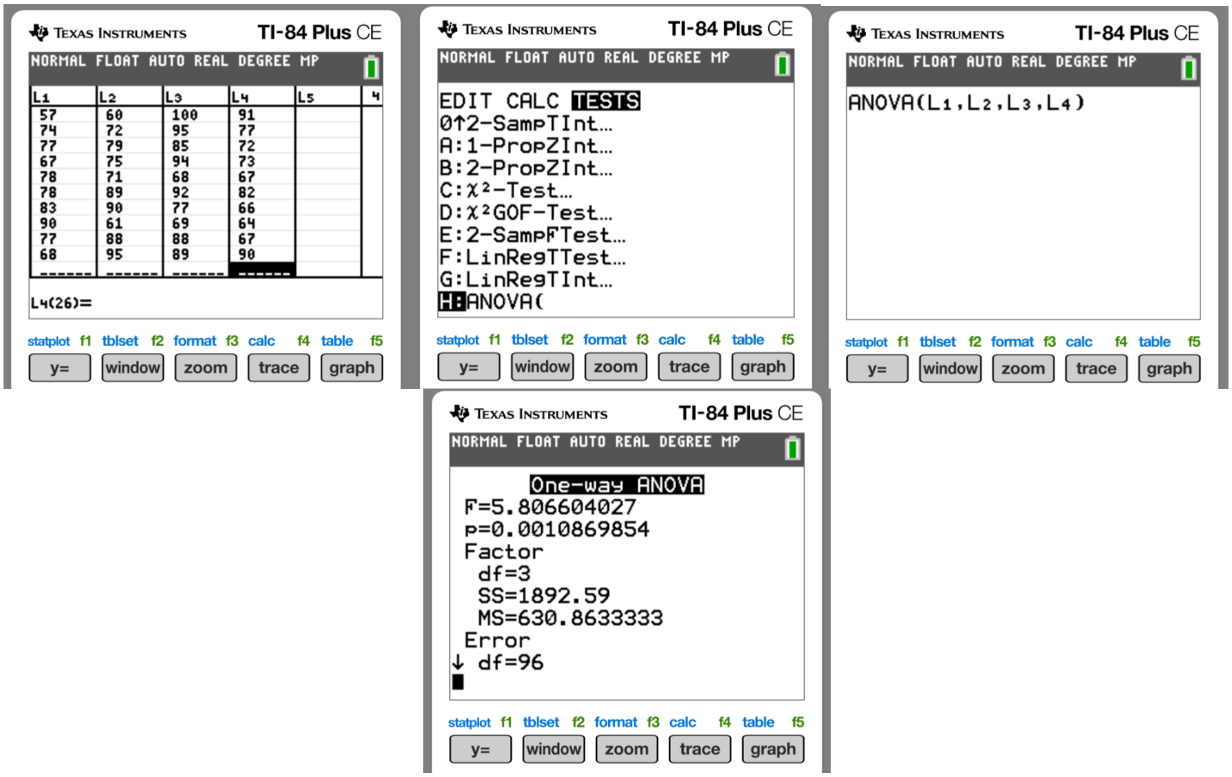








ANOVA

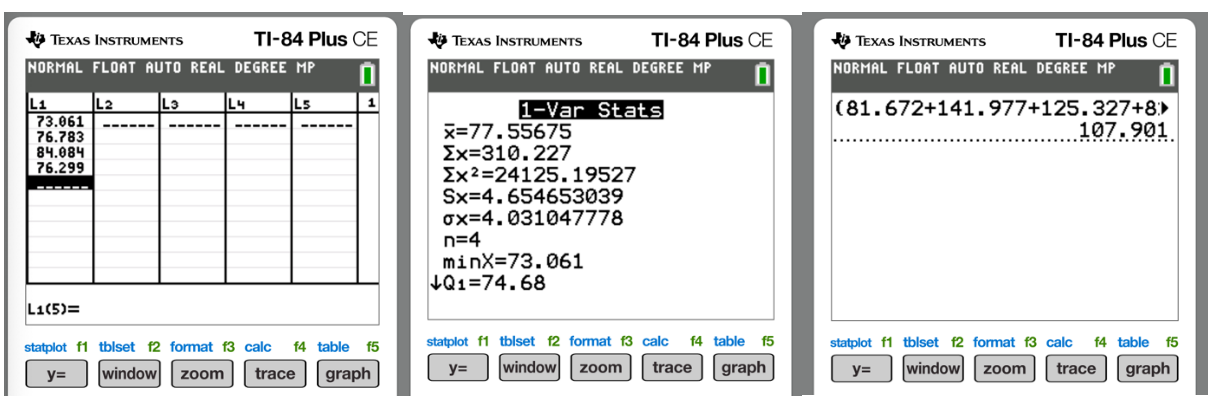


$p \approx 0.001$; $p < \alpha$; H_0 **is too low,** H_0 **has to go!**

The Sample Does Not Support the Claim

The data comes from populations with unequal means

Traditional Method



Test Statistic $F = \frac{n s_{\bar{x}}^2}{s_p{}^2} = \frac{25 \cdot 4.655^2}{107.901} \approx 5.021$

Determining Critical Values

Numerator degree of freedom $= k - 1$

numerator df $= \mathbf{4 - 1 = 3}$

Denominator degree of freedom $= k(n-1)$

denominator df $= \mathbf{4(25 - 1) = 96}$

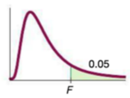
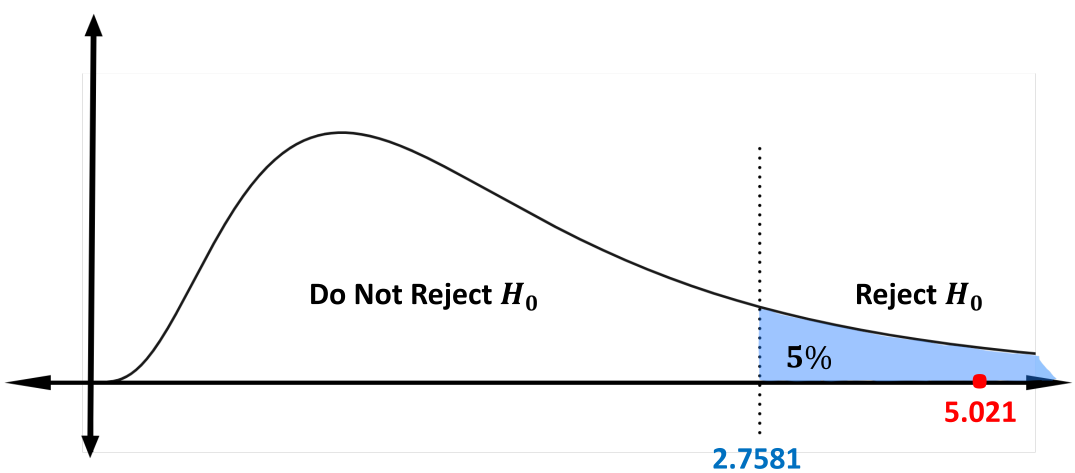


TABLE A-5 *(continued)* *F* Distribution ($\alpha = 0.05$ in the right tail)

Denominator degrees of freedom (df_2)	Numerator degrees of freedom (df_1): 1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385
3	10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.0410	6.9988
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.0990
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767
8	5.3177	4.4590	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789
10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876
16	4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377
17	4.4513	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563
19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227
20	4.3512	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3660
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419
23	4.2793	3.4221	3.0280	2.7955	2.6400	2.5277	2.4422	2.3748	2.3201
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002
25	4.2417	3.3852	2.9912	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821
26	4.2252	3.3690	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655
27	4.2100	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501
28	4.1960	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.2360
29	4.1830	3.3277	2.9340	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107
40	4.0847	3.2317	2.8387	2.6060	2.4495	2.3359	2.2490	2.1802	2.1240
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.0970	2.0401
120	3.9201	3.0718	2.6802	2.4472	2.2899	2.1750	2.0868	2.0164	1.9588
∞	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799



The Sample Does Not Support the Claim

The data comes from populations having unequal means