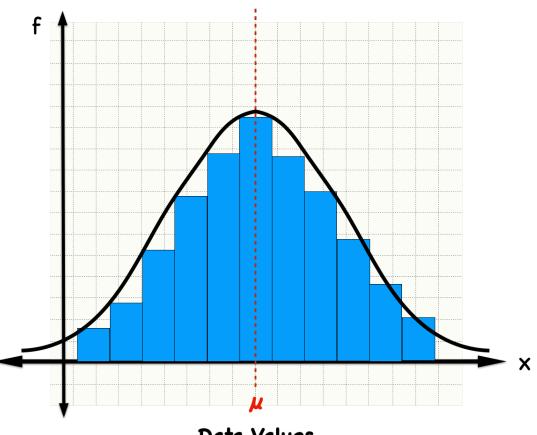
## **Normal Probability Distribution**

Aka, the "Bell Shaped Curve"

Recall the type of "pictures of data" you get when creating a Histogram for a frequency table, or relative frequency table, when you gather data. There is a special 'picture" where most of the data is centered in the middle around the mean and tappers off significantly as the data values become more extreme.



Data Values

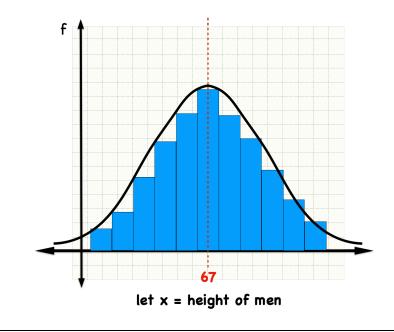
This picture represents a distribution of data values for a **continuous random variable x** and can be represented mathematically be a function.

$$P(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

This distribution is known formally as the **Normal Probability Distribution**, aka the Bell-Shaped Curve as demonstrated by its shape. Where  $\mu$  is the mean of the distribution and  $\sigma$  is the standard deviation for the distribution.

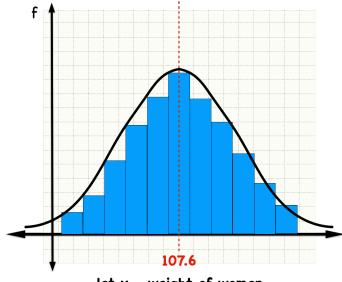
## What can be represented by the Normal Probability Distribution?

Things that grow naturally, behave naturally, reproduced and manufactured "identically" by man or nature. Some common examples are the following.



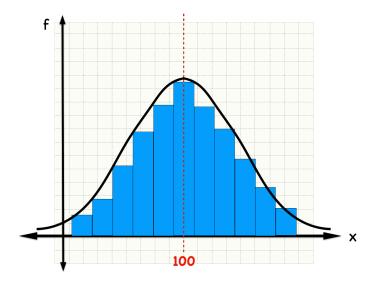
The height of men where  $\mu = 67$  *inches* (*United States*) and  $\sigma = 2.9$  *inches* 

The weight of women  $\mu = 107.6$  *pounds* (*United States*) and  $\sigma = 12.6$  *pounds* 

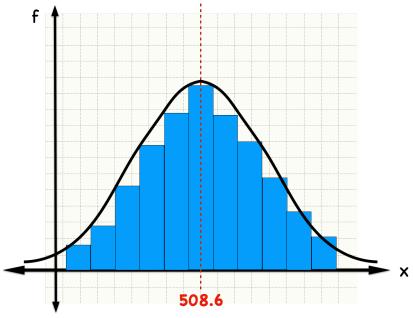


let x = weight of women

The IQ scores of people  $\mu=100$  and  $\sigma=15$ 

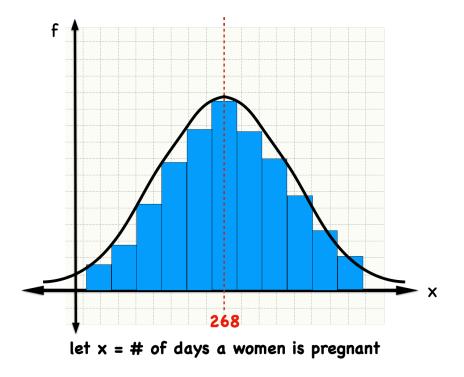


The volume of water placed in a 500-mL bottle manufactured by a particular company  $\mu=508.6~mL$  and  $\sigma=1.5~mL$ 

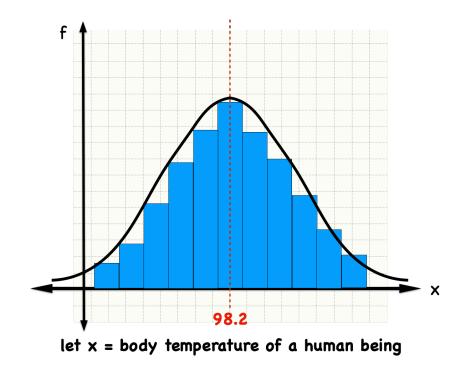


let x = volume of water filled into a 500 mL bottle

The duration of pregnancies  $\mu=268~days$  and  $\sigma=15~days$ 



Body temperature  $= 98.2^{o} F$  and  $\sigma = 0.2^{o} F$ 

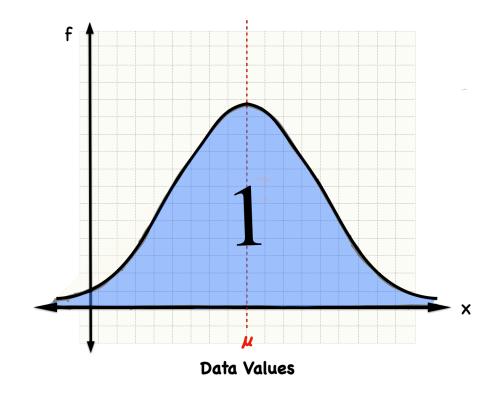


## Facts about the Normal Distribution.

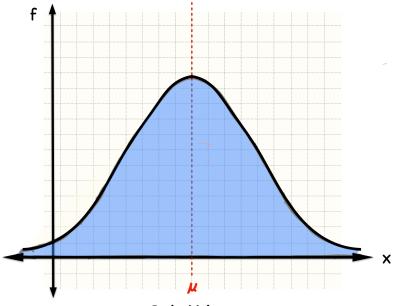
The distribution is symmetric about the mean  $\boldsymbol{\mu}$ 

The area shaded in blue under the Bell-Shaped curve is equal to 1

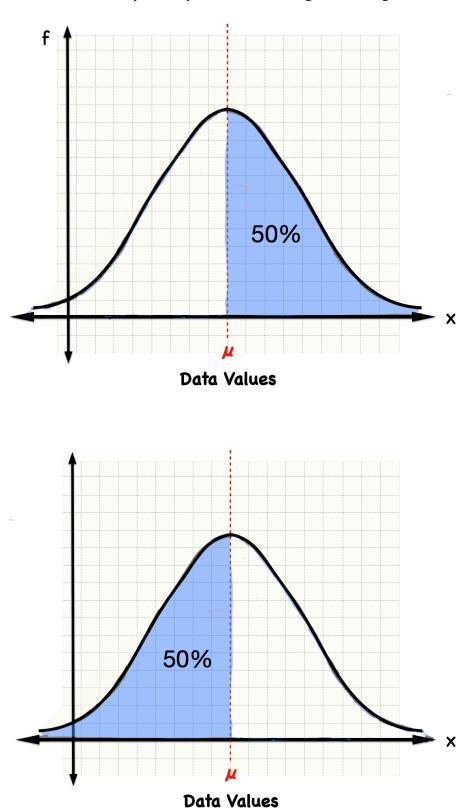
$$\int_{-\infty}^{\infty} P(x) dx = \int_{-\infty}^{\infty} \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}} dx = 1 \text{ as } \sum_{all \ x} P(x) = 1$$



Most of what you need to know in answering Normal Probability Distribution questions is based on the following exercise.

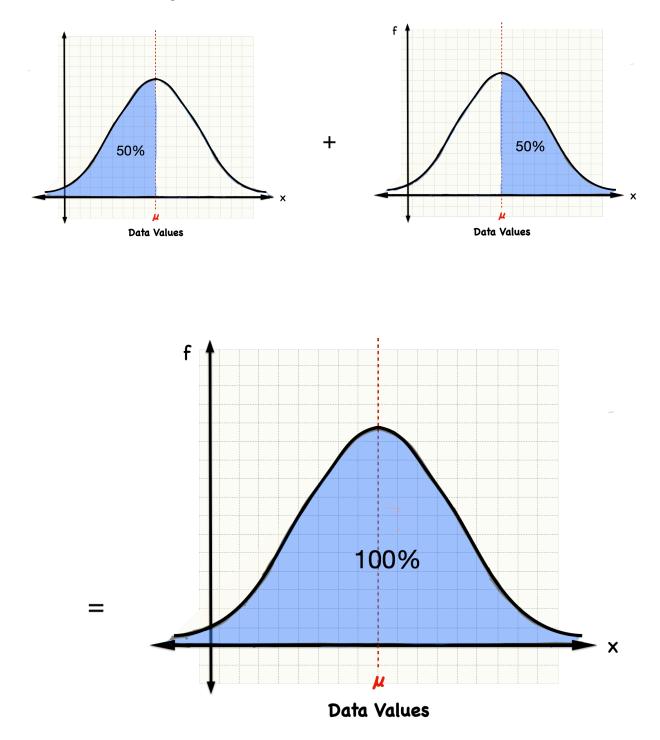


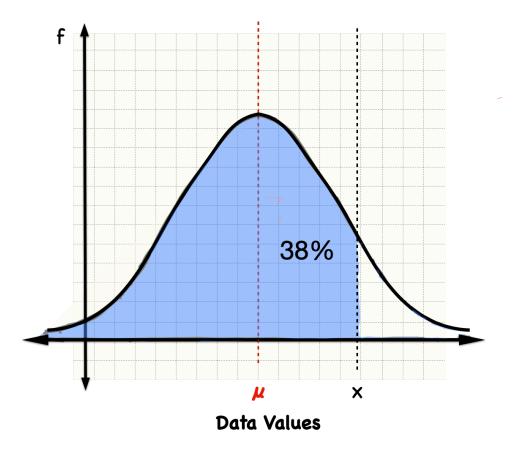
Data Values

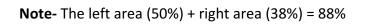


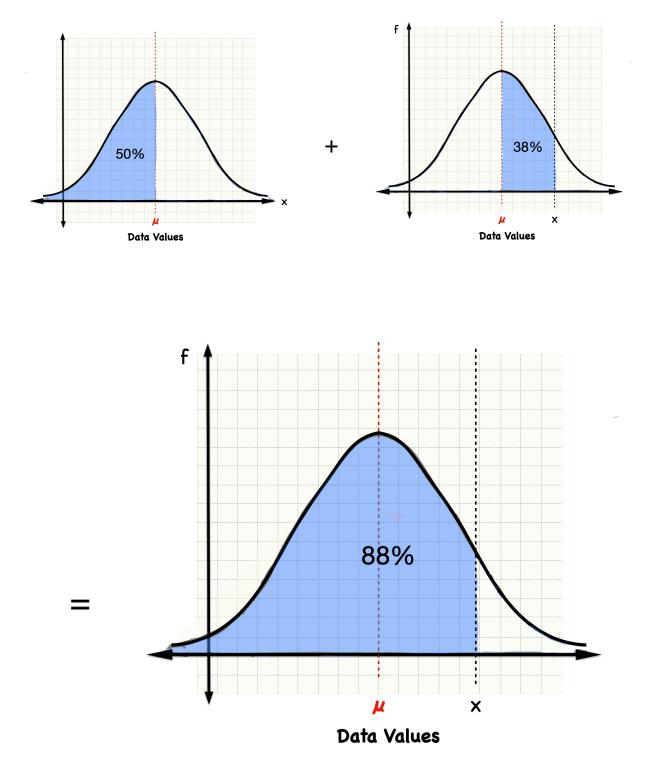
We will use symmetry and the following shaded regions.

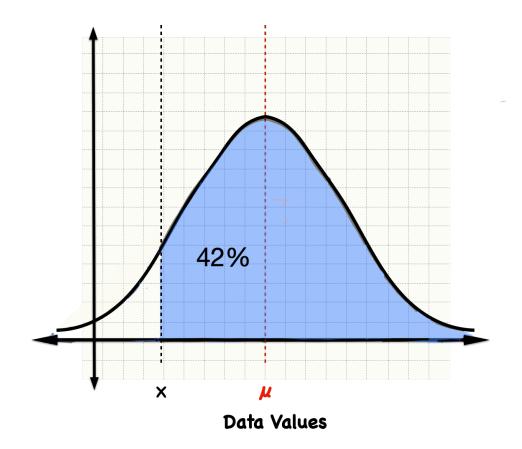
Note- The left area + right area= 100%

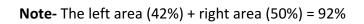


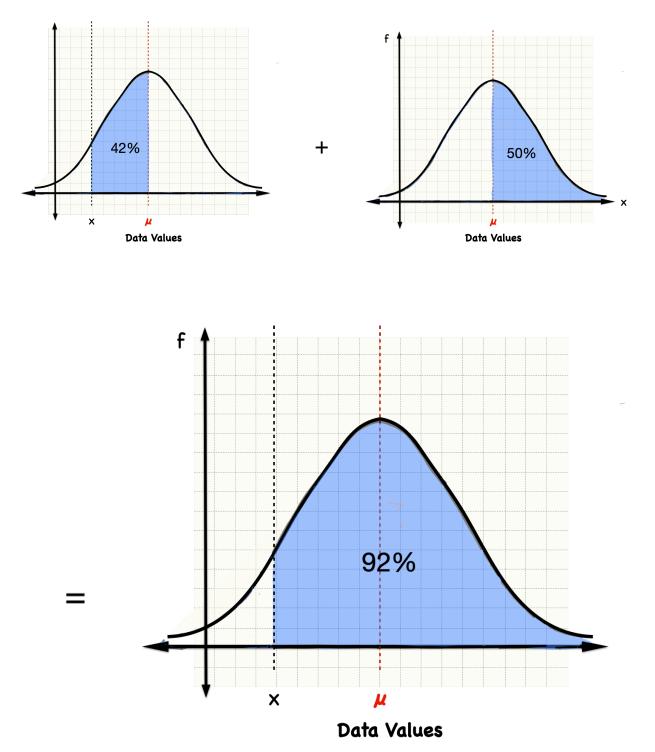




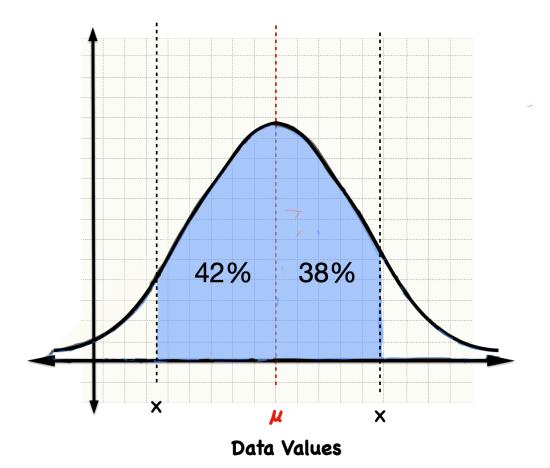


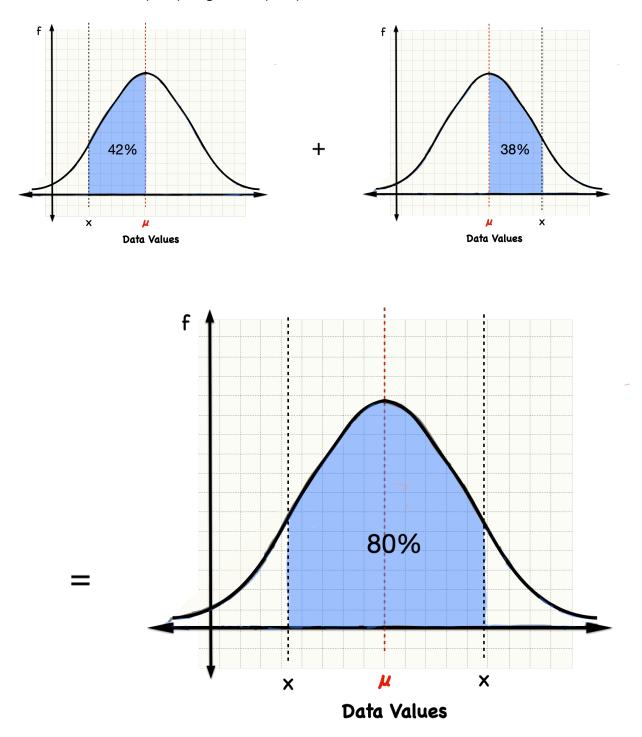






What percent of the Bell is shaded?



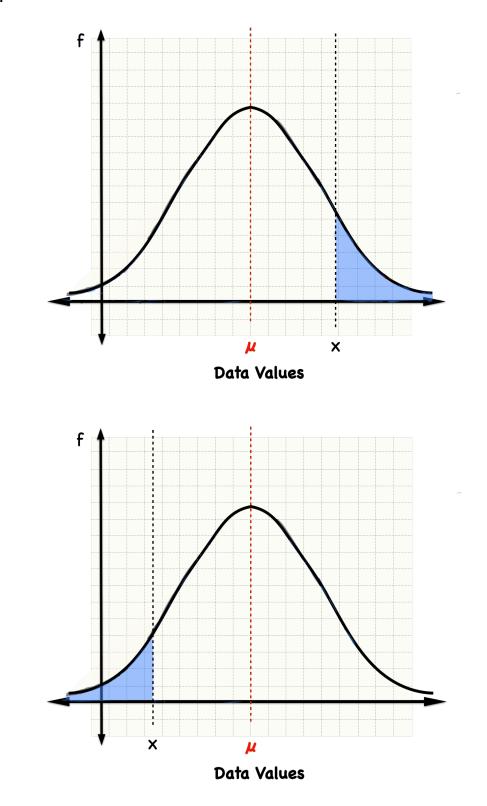


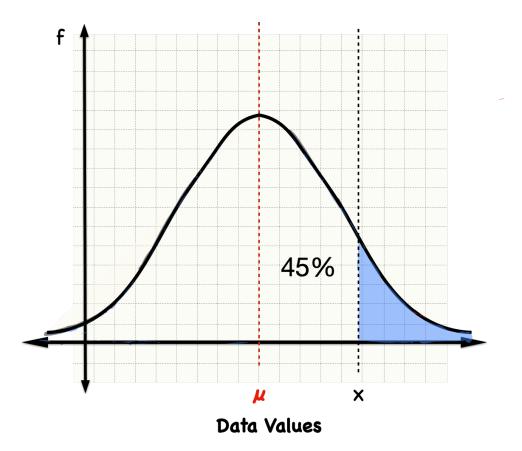
**Note-** The left area (42%) + right area (38%) = 80%

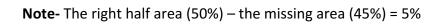
The following shaded regions are vitally important to many Normal Probability Distribution questions.

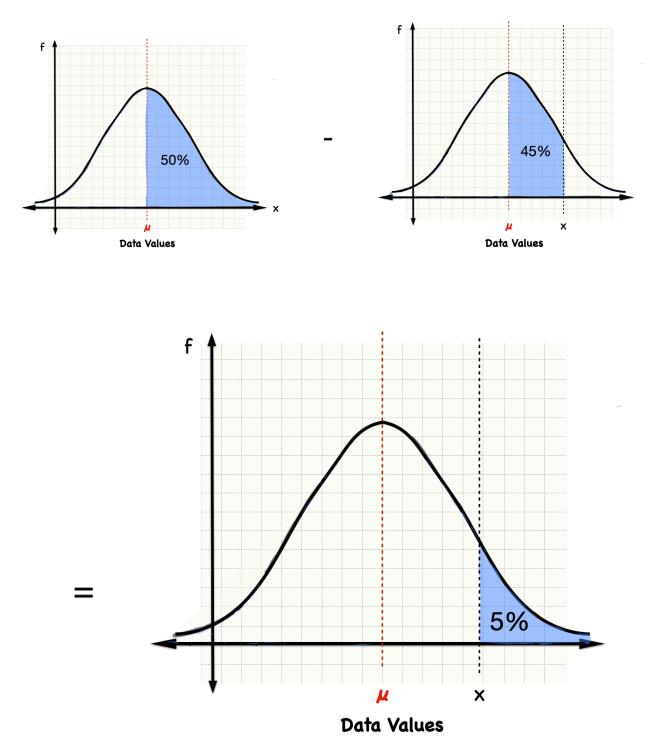
## **Right Tail**

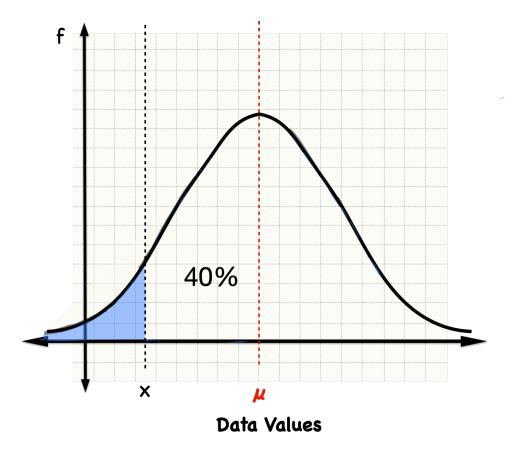
Left Tail

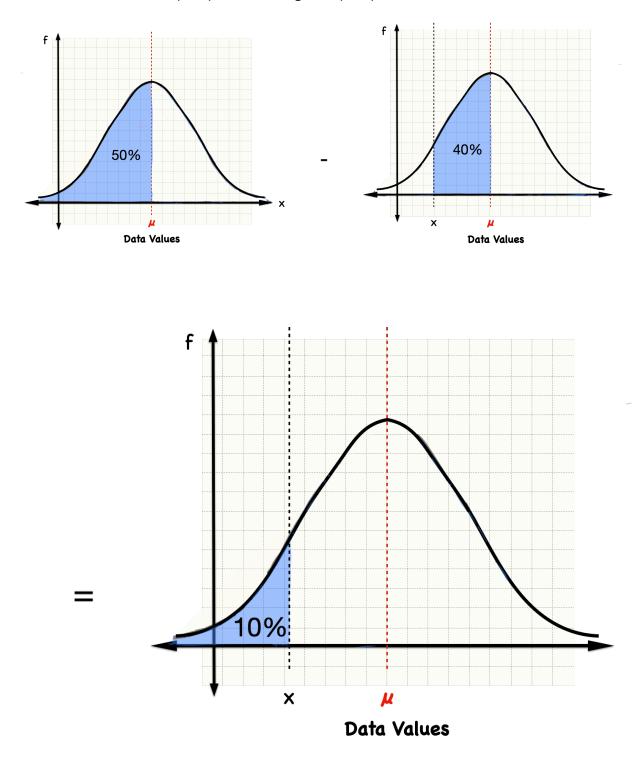




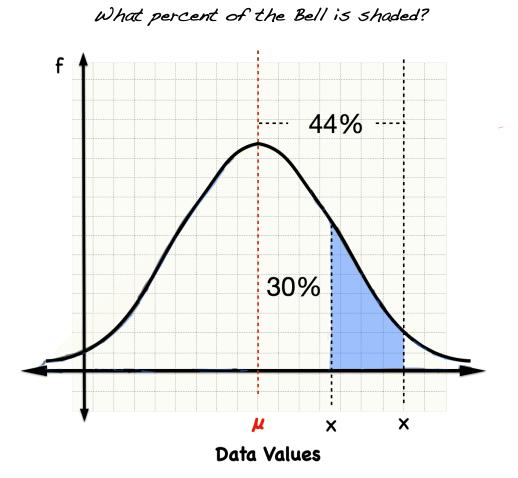


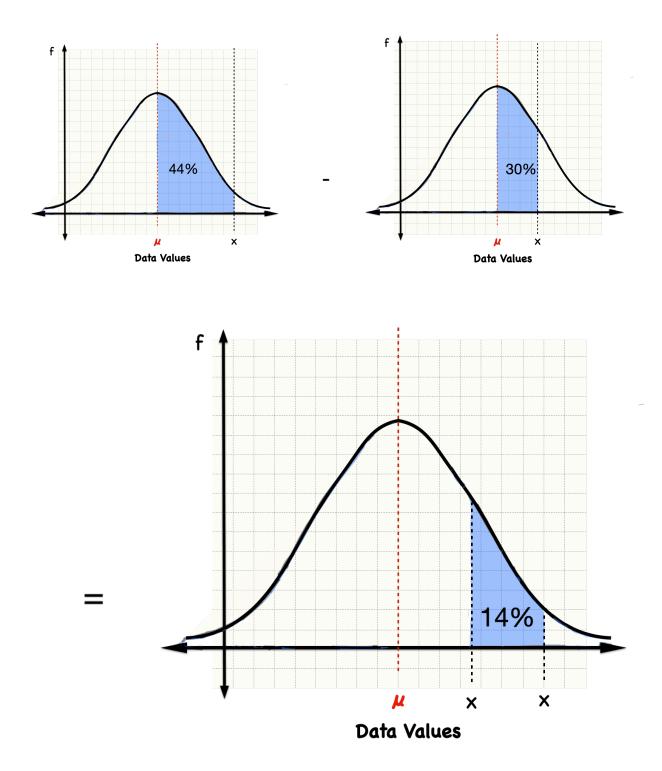




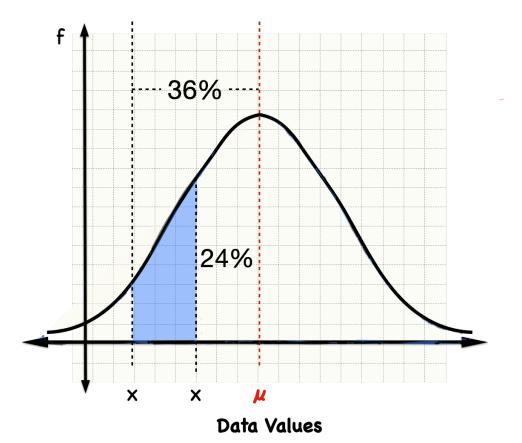


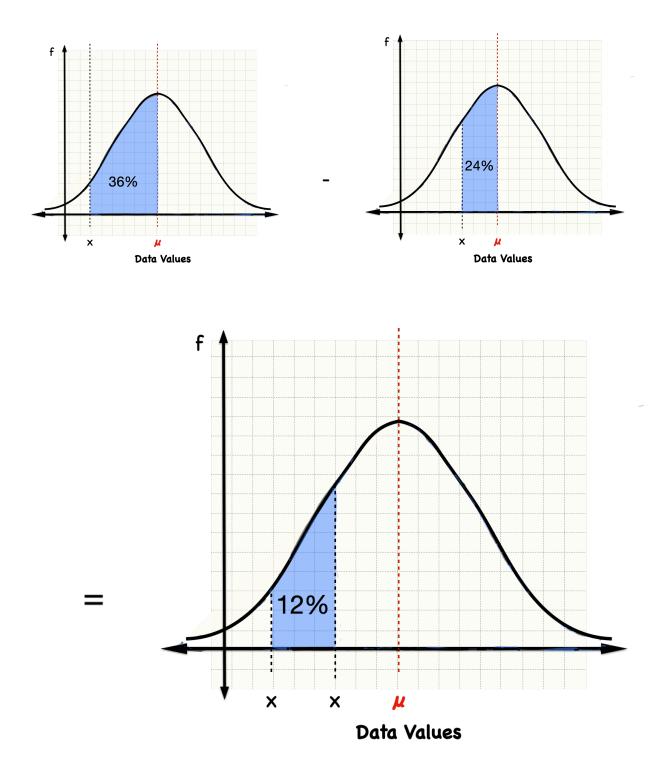
Note- The left half area (50%) – the missing area (42%) = 8%





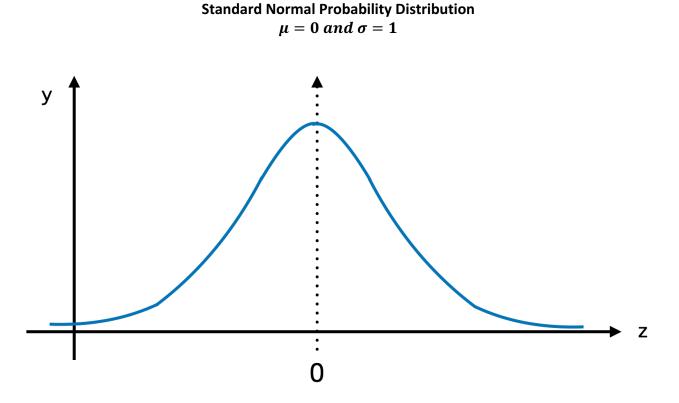
Note- The larger right side area (44%) – the missing right side area (30%) = 14%



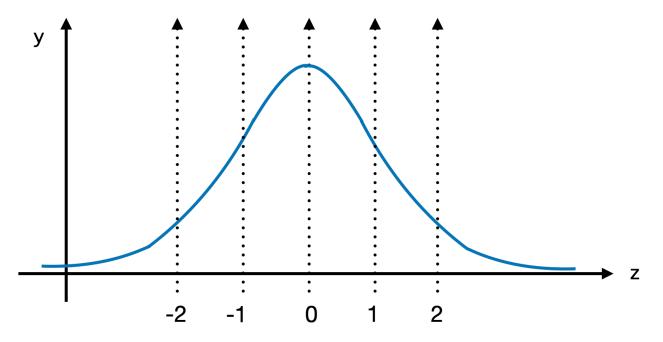


Note- The larger left side area (36%) – the missing left side area (24%) = 12%

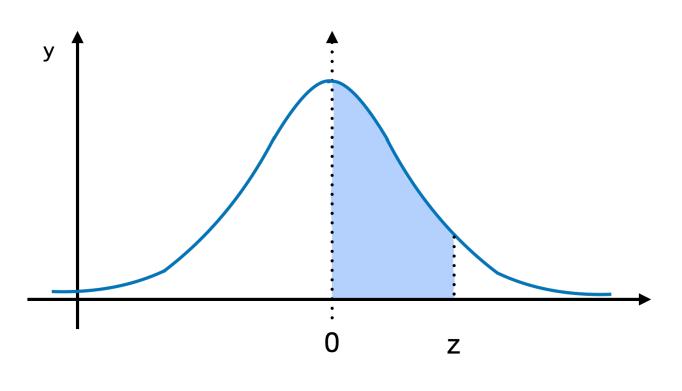
The key to answering Normal Probability Distribution questions is to work with the **Standard Normal Probability** distribution. This is a special Normal Probability Distribution with a mean of 0 and a standard deviation of 1 and is represented by the letter z.



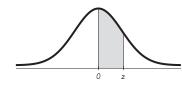
Including the standard deviations, we have a more comprehensive description.



The key to working with the Standard Normal Probability Distribution is to use a table with the following shaded portion of the Bell-Shaped curve as determined by the **Z-table**.



A copy is found on my website with your formula sheet <u>http://www.ddjudge.com/assets/formulacard.pdf</u>



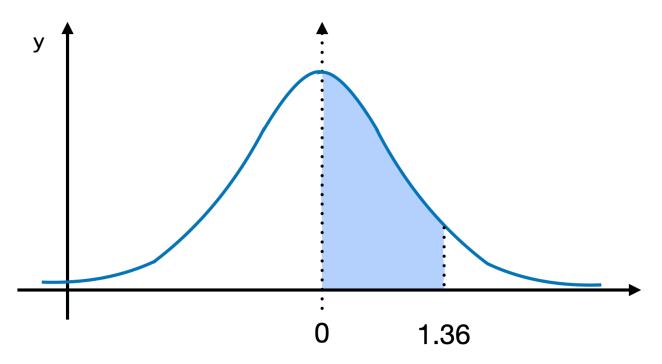
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.0
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.03
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.07
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.11
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.15
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.18
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.22
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.25
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.28
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.31
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.33
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.36
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.38
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.40
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.41
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.43
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.44
1.6	.4452	.4463	.4474	.4484		∗ .4505	.4515	.4525	.4535	.45
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.46
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.47
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.47
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.48
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.48
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.48
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.49
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.49
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949 *	11201	.49
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.49
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.49
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.49
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.49
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.49
3.10										
and	.4999									
nigher										

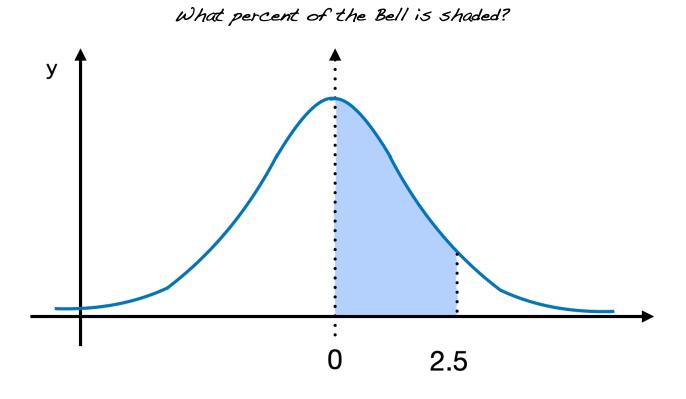
z score Area

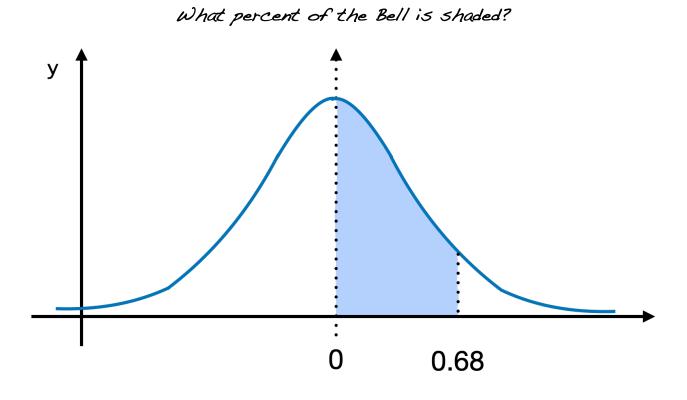
1.645 <u>0.4500</u>

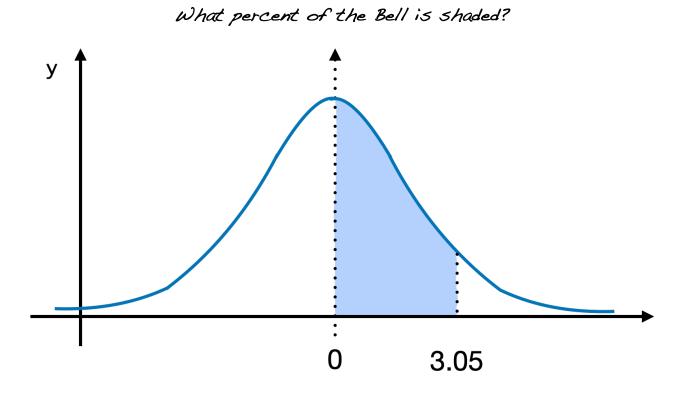
2.575 0.4950

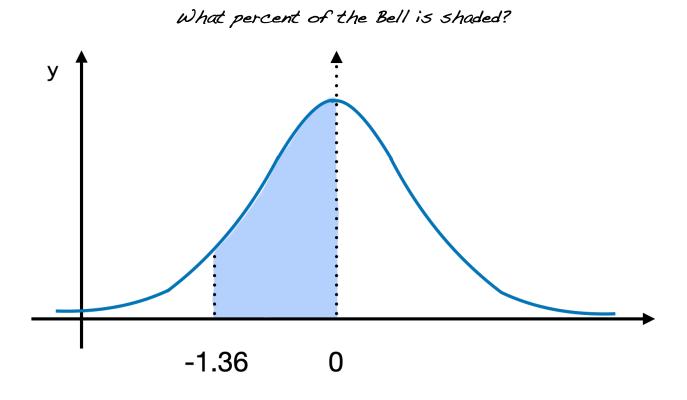
From Frederick C. Mosteller and Robert E. K. Rourke, *Sturdy Statistics*, 1973, Addison-Wesley Publishing Co., Reading, MA. Reprinted with permission of Frederick Mosteller.

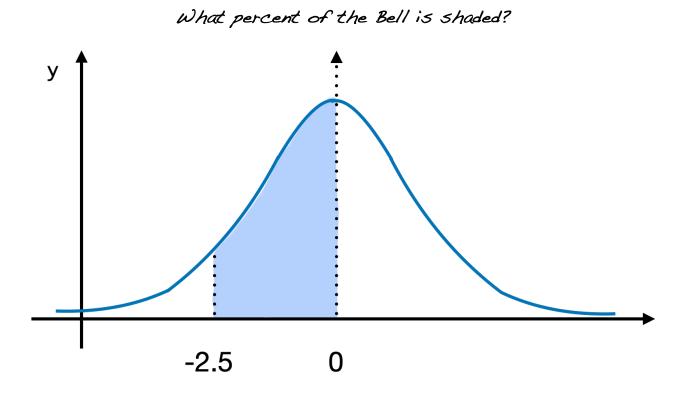


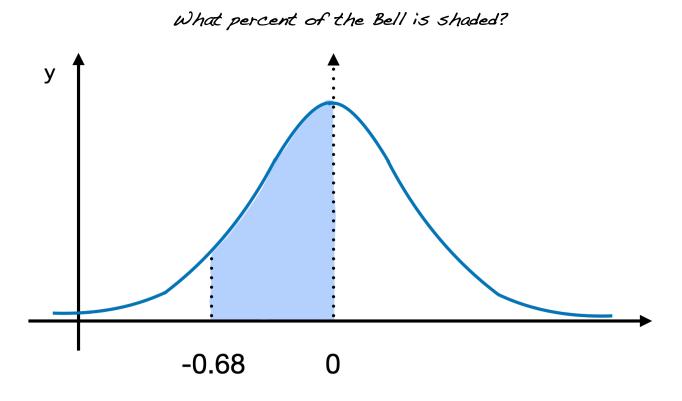


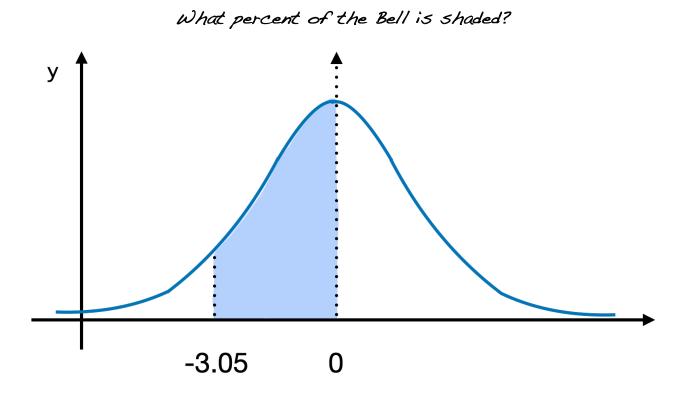


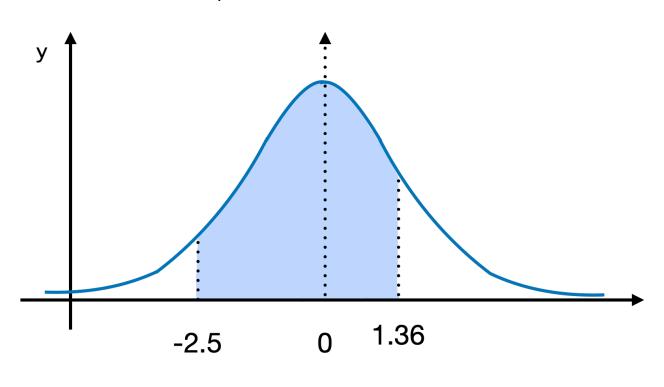








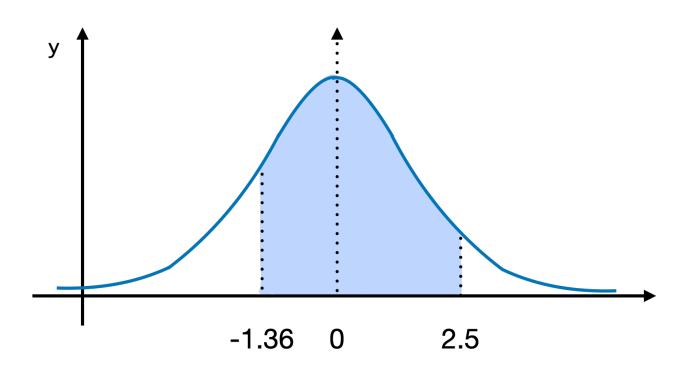




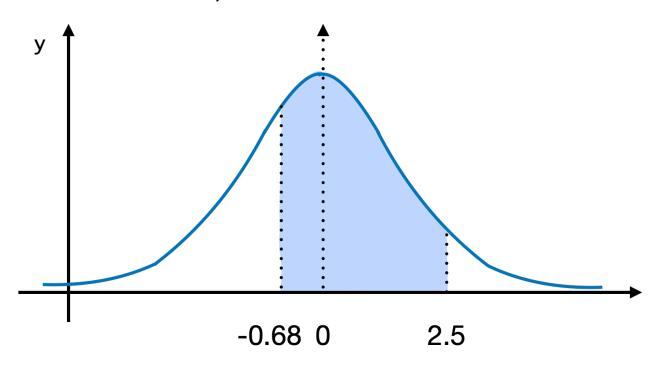
TI-83 or TI-84 Plus Finding Area Between Two Z-Scores

- 1. Press **2**<sup>nd</sup> then **vars** to access DISTR (distributions) menu.
- 2. Select **normalcdf** and click **enter**.
- 3. Enter the desired lower Z value and the upper Z value. Enter 0 for  $\mu$  and 1 for  $\sigma$  **normalcdf(lower Z, upper Z, \mu, \sigma)** and press **enter.**

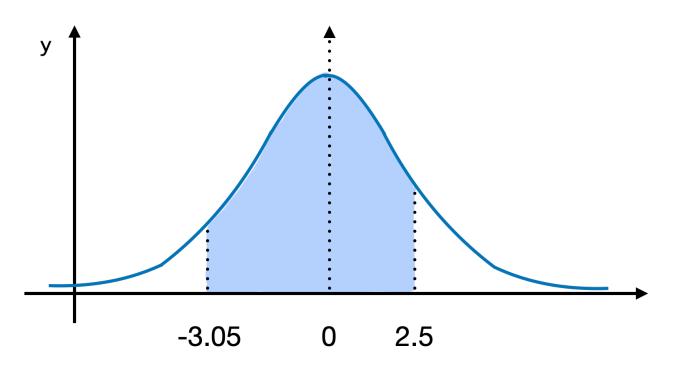
normalcdf(-2.5,1.36,0,1)



TI-83 or TI-84 Plus Finding Area Between Two Z-Scores
1. Press 2<sup>nd</sup> then vars to access DISTR (distributions) menu.
2. Select normalcdf and click enter.
3. Enter the desired lower Z value and the upper Z value. Enter 0 for μ and 1 for σ normalcdf(lower Z, upper Z, μ, σ) and press enter.



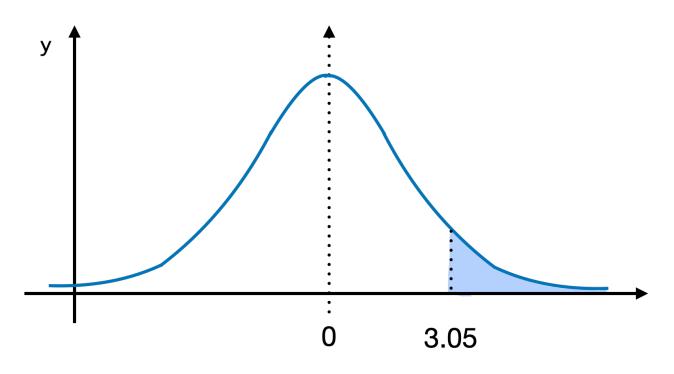
TI-83 or TI-84 Plus Finding Area Between Two Z-Scores
1. Press 2<sup>nd</sup> then vars to access DISTR (distributions) menu.
2. Select normalcdf and click enter.
3. Enter the desired lower Z value and the upper Z value. Enter 0 for μ and 1 for σ normalcdf(lower Z, upper Z, μ, σ) and press enter.



TI-83 or TI-84 Plus Finding Area Between Two Z-Scores

- 1. Press **2<sup>nd</sup>** then **vars** to access DISTR (distributions) menu.
- 2. Select **normalcdf** and click **enter**.
- 3. Enter the desired lower Z value and the upper Z value. Enter 0 for  $\mu$  and 1 for  $\sigma$  **normalcdf(lower Z, upper Z, \mu, \sigma)** and press **enter.**

normalcdf(-3.05,2.5,0,1)

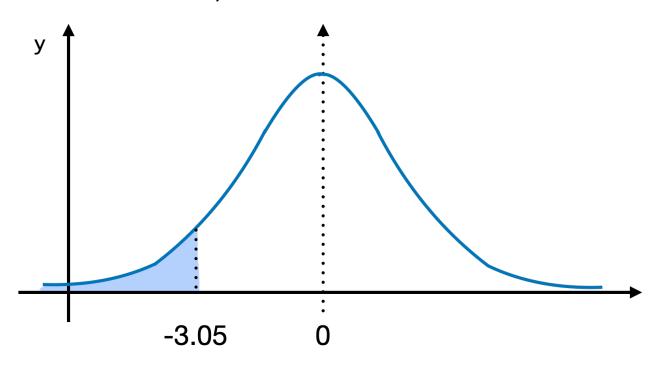


TI-83 or TI-84 Plus Finding Area Between Two Z-Scores

- 1. Press **2<sup>nd</sup>** then **vars** to access DISTR (distributions) menu.
- 2. Select normalcdf and click enter.
- 3. Enter the desired lower Z value and the upper Z value. Enter 0 for  $\mu$  and 1 for  $\sigma$  **normalcdf(lower Z, upper Z, \mu, \sigma)** and press **enter.**

# normalcdf(3.05,100,0,1)

When there is **no upper Z value**, you can use 100 when working with the Standard Normal Probability Formula.

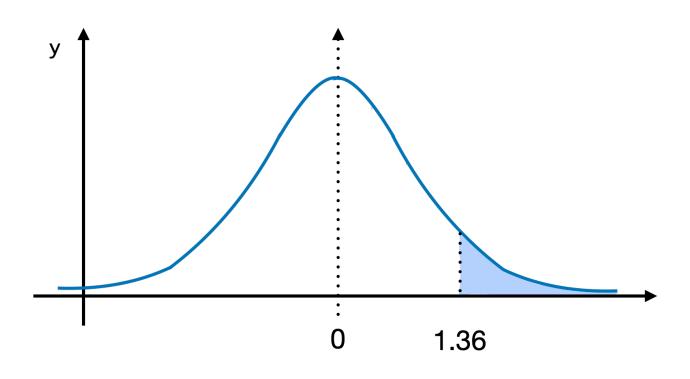


TI-83 or TI-84 Plus Finding Area Between Two Z-Scores

- 1. Press **2<sup>nd</sup>** then **vars** to access DISTR (distributions) menu.
- 2. Select **normalcdf** and click **enter**.
- 3. Enter the desired lower Z value and the upper Z value. Enter 0 for  $\mu$  and 1 for  $\sigma$  normalcdf(lower Z, upper Z,  $\mu$ ,  $\sigma$ ) and press enter.

normalcdf(-100,-3.05,0,1)

When there is **no lower Z value**, you can use **-100** when working with the Standard Normal Probability Formula.

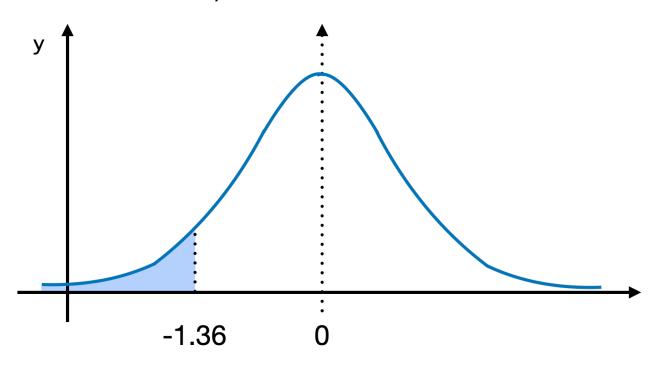


TI-83 or TI-84 Plus Finding Area Between Two Z-Scores

- 1. Press **2<sup>nd</sup>** then **vars** to access DISTR (distributions) menu.
- 2. Select normalcdf and click enter.
- 3. Enter the desired lower Z value and the upper Z value. Enter 0 for  $\mu$  and 1 for  $\sigma$  **normalcdf(lower Z, upper Z, \mu, \sigma)** and press **enter.**

normalcdf(1.36,100,0,1)

When there is **no upper Z value**, you can use 100 when working with the Standard Normal Probability Formula.

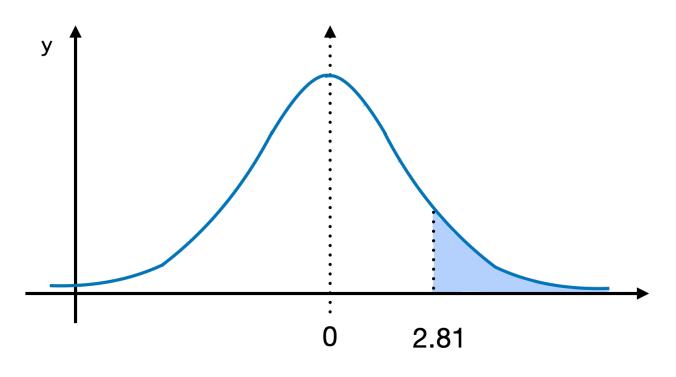


TI-83 or TI-84 Plus Finding Area Between Two Z-Scores

- 1. Press **2<sup>nd</sup>** then **vars** to access DISTR (distributions) menu.
- 2. Select normalcdf and click enter.
- 3. Enter the desired lower Z value and the upper Z value. Enter 0 for  $\mu$  and 1 for  $\sigma$  **normalcdf(lower Z, upper Z, \mu, \sigma)** and press **enter.**

normalcdf(-100,-1.36,0,1)

When there is **no lower Z value**, you can use **-100** when working with the Standard Normal Probability Formula.



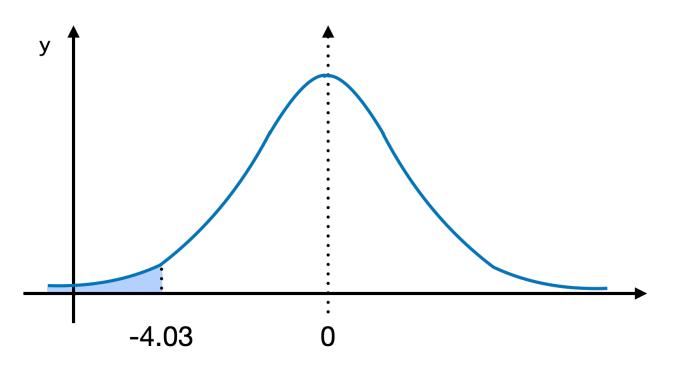
TI-83 or TI-84 Plus Finding Area Between Two Z-Scores

- 1. Press **2<sup>nd</sup>** then **vars** to access DISTR (distributions) menu.
- 2. Select normalcdf and click enter.
- 3. Enter the desired lower Z value and the upper Z value.

Enter 0 for  $\mu$  and 1 for  $\sigma$  normalcdf(lower Z, upper Z,  $\mu$ ,  $\sigma$ ) and press enter.

normalcdf(2.81,100,0,1)

When there is **no upper Z value**, you can use 100 when working with the Standard Normal Probability Formula.

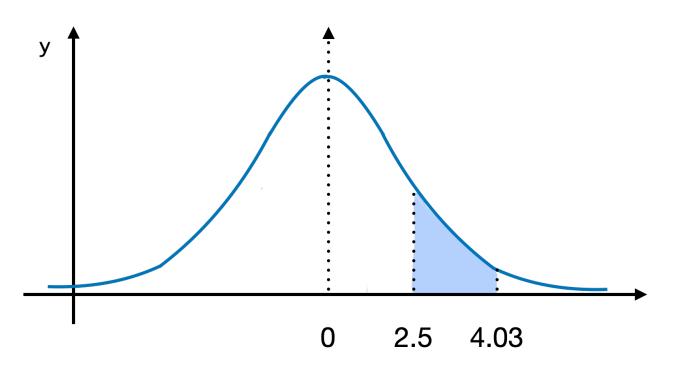


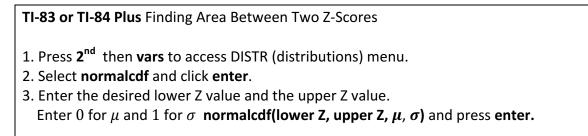
TI-83 or TI-84 Plus Finding Area Between Two Z-Scores

- 1. Press **2<sup>nd</sup>** then **vars** to access DISTR (distributions) menu.
- 2. Select normalcdf and click enter.
- 3. Enter the desired lower Z value and the upper Z value. Enter 0 for  $\mu$  and 1 for  $\sigma$  **normalcdf(lower Z, upper Z, \mu, \sigma)** and press **enter.**

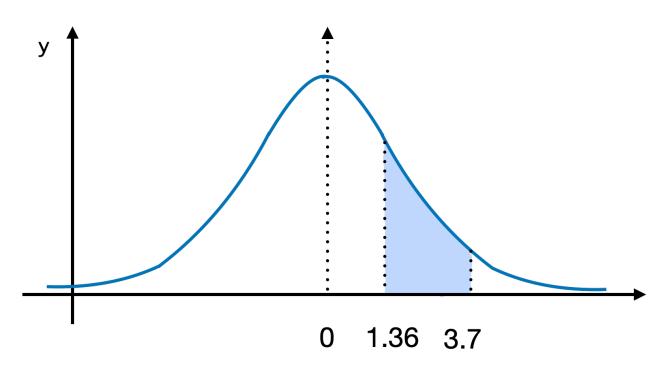
normalcdf(-100,-4.03,0,1)

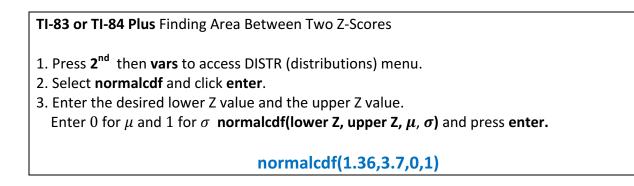
When there is **no lower Z value**, you can use **-100** when working with the Standard Normal Probability Formula.

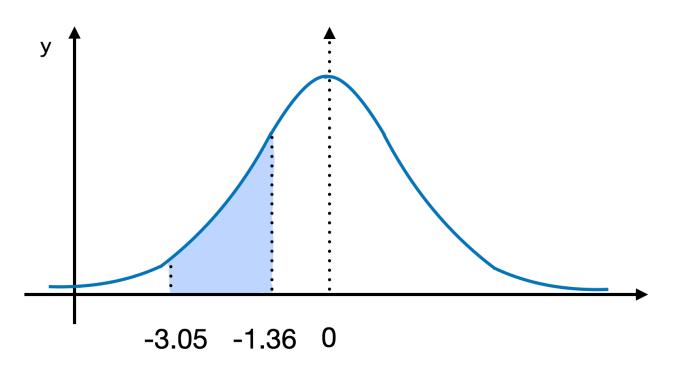




normalcdf(2.5,4.03,0,1)



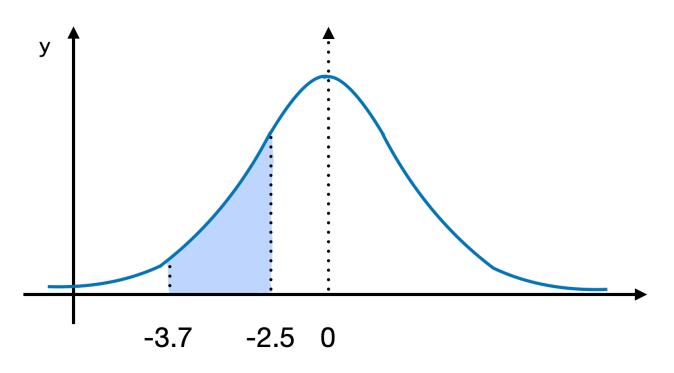




TI-83 or TI-84 Plus Finding Area Between Two Z-Scores

- 1. Press **2**<sup>nd</sup> then **vars** to access DISTR (distributions) menu.
- 2. Select normalcdf and click enter.
- 3. Enter the desired lower Z value and the upper Z value. Enter 0 for  $\mu$  and 1 for  $\sigma$  normalcdf(lower Z, upper Z,  $\mu$ ,  $\sigma$ ) and press enter.

normalcdf(-3.05,-1.36,0,1)



TI-83 or TI-84 Plus Finding Area Between Two Z-Scores

- 1. Press **2<sup>nd</sup>** then **vars** to access DISTR (distributions) menu.
- 2. Select normalcdf and click enter.
- 3. Enter the desired lower Z value and the upper Z value.

Enter 0 for  $\mu$  and 1 for  $\sigma$  normalcdf(lower Z, upper Z,  $\mu$ ,  $\sigma$ ) and press enter.

normalcdf(-3.7,-2.5,0,1)

### **Applications of the Normal Probability Distribution**

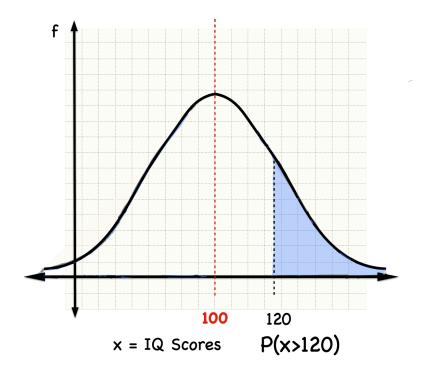
### **IQ Scores**

Intelligence is measured by an IQ (Intelligence Quotient) which is **Normally Distributed** with a **mean** of 100 and a **standard deviation** of 15. If you select a person at random, what's the probability the person has an IQ score that is:

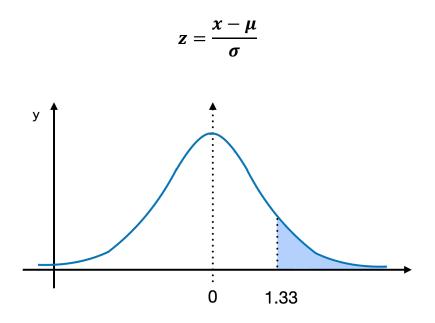
1. At least 120?

You will need to follow the procedure outlined below.

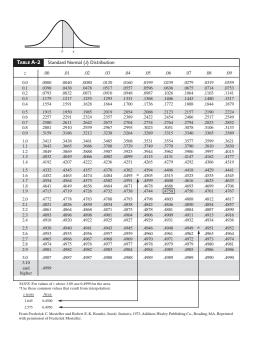
Step 1 Draw a picture of the question



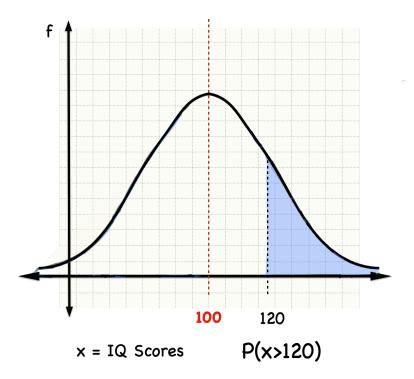
**Step 2** Convert the **Normal Probability Distribution x** to the **Standard Normal Probability Distribution Z** using the **Z-Value** definition (aka, the Standard Value)



**Step 3** Use the Z-table and logic to deduce the percent of the Bell that is shaded. However, write your answer as a probability as the question was posed as a probability. If the question is posed as a percent, then write your answer as a percent.



However, we can use the **TI-83 or TI-84 Calculator** from the beginning.



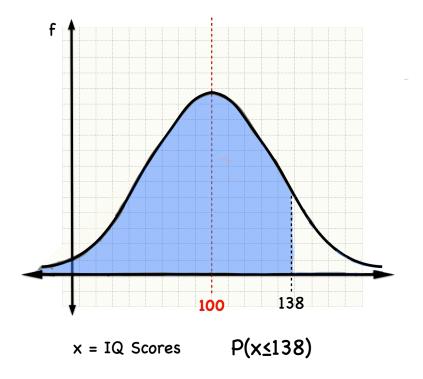
TI-83 or TI-84 Plus Finding Area Between Two x Values

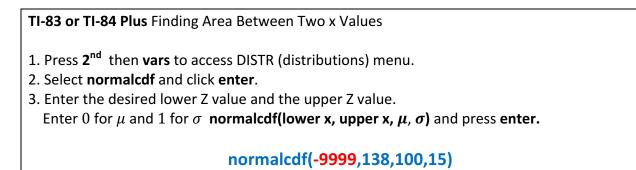
- 1. Press **2<sup>nd</sup>** then **vars** to access DISTR (distributions) menu.
- 2. Select normalcdf and click enter.
- 3. Enter the desired lower Z value and the upper Z value.

Enter 0 for  $\mu$  and 1 for  $\sigma$  normalcdf(lower x, upper x,  $\mu$ ,  $\sigma$ ) and press enter.

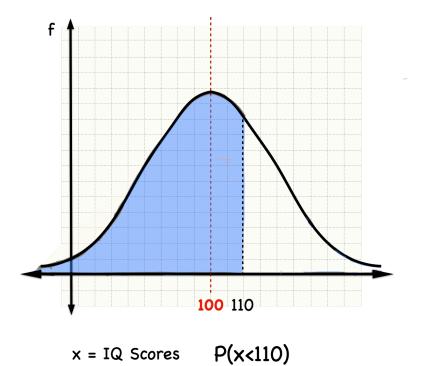
normalcdf(120,9999,100,15)

### 2. No more than 138?





# 3. Less than 110



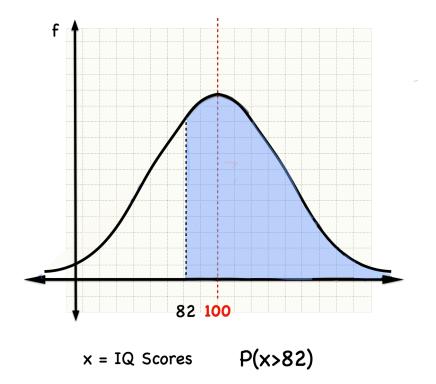
TI-83 or TI-84 Plus Finding Area Between Two x Values

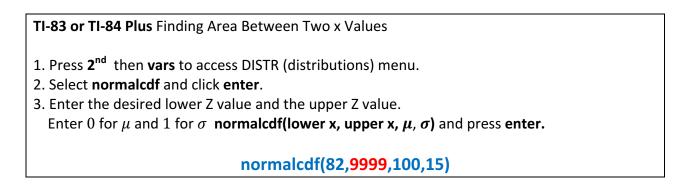
- 1. Press **2<sup>nd</sup>** then **vars** to access DISTR (distributions) menu.
- 2. Select normalcdf and click enter.
- 3. Enter the desired lower Z value and the upper Z value.

Enter 0 for  $\mu$  and 1 for  $\sigma$  normalcdf(lower x, upper x,  $\mu$ ,  $\sigma$ ) and press enter.

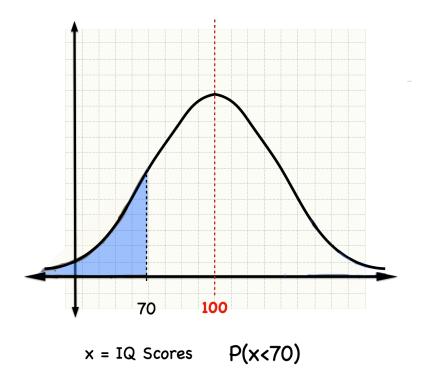
normalcdf(-9999,110,100,15)

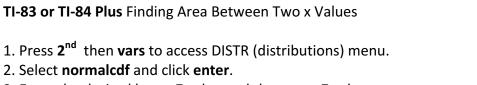
# 4. More than 82?





5. Less than 70?

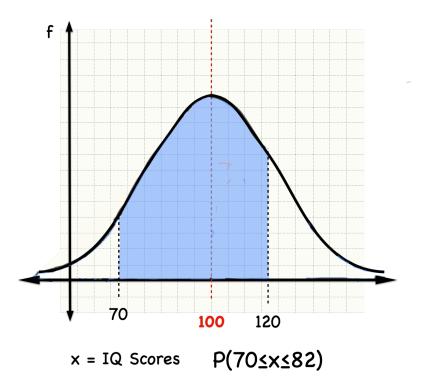




3. Enter the desired lower Z value and the upper Z value. Enter 0 for  $\mu$  and 1 for  $\sigma$  normalcdf(lower x, upper x,  $\mu$ ,  $\sigma$ ) and press enter.

normalcdf(-9999,70,100,15)

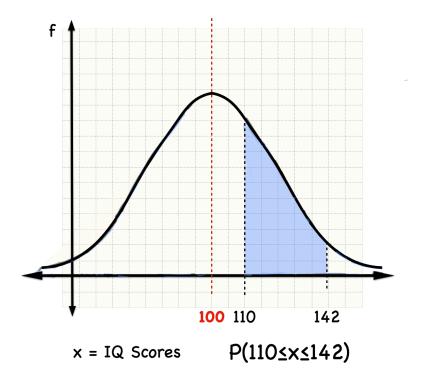
6. Between 70 and 120?

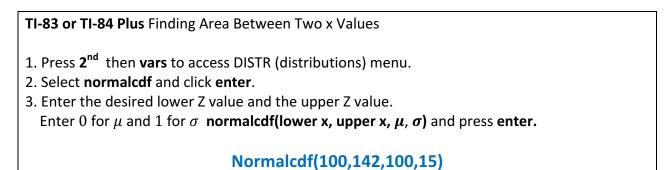


 TI-83 or TI-84 Plus Finding Area Between Two x Values
 Press 2<sup>nd</sup> then vars to access DISTR (distributions) menu.
 Select normalcdf and click enter.
 Enter the desired lower Z value and the upper Z value. Enter 0 for μ and 1 for σ normalcdf(lower x, upper x, μ, σ) and press enter.

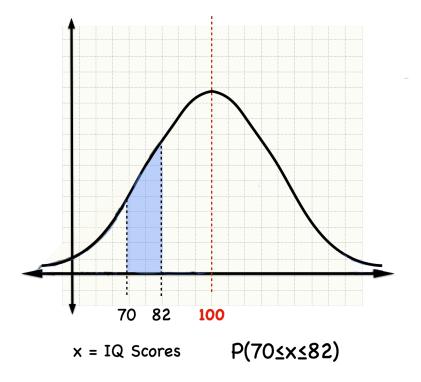
normalcdf(70,120,100,15)

# 7. Between 110 and 142?





# 8. Between 70 and 82?



TI-83 or TI-84 Plus Finding Area Between Two x Values

- 1. Press **2<sup>nd</sup>** then **vars** to access DISTR (distributions) menu.
- 2. Select normalcdf and click enter.
- 3. Enter the desired lower Z value and the upper Z value.

Enter 0 for  $\mu$  and 1 for  $\sigma$  normalcdf(lower x, upper x,  $\mu$ ,  $\sigma$ ) and press enter.

normalcdf(70,82,100,15)