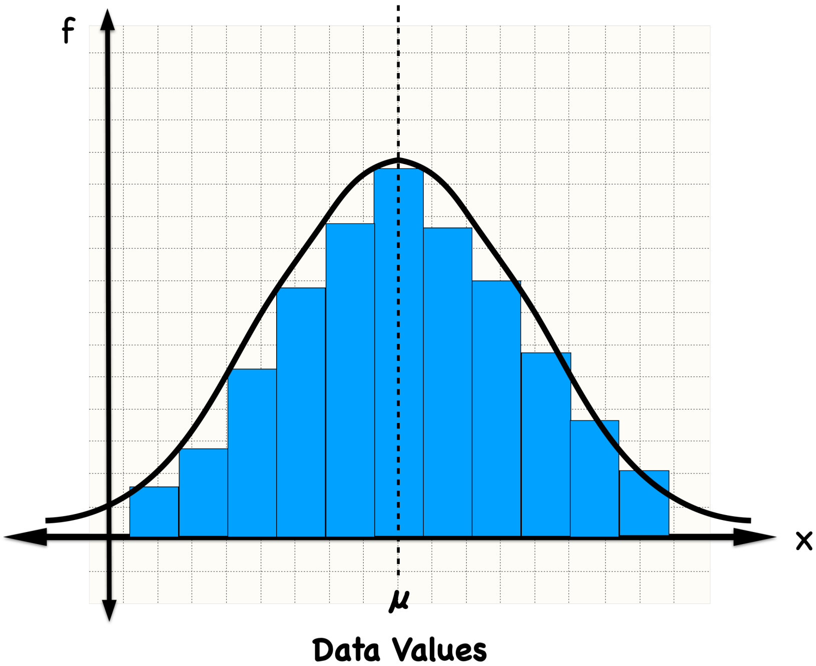
Normal Probability Distribution Notes

Aka, the “Bell Shaped Curve”

Recall the type of “pictures of data” you get when creating a Histogram for a frequency table, or relative frequency table, when you gather data. There is a special ‘picture” where most of the data is centered in the middle around the mean and tappers off significantly as the data values become more extreme.



This picture represents a distribution of data values for a **continuous random variable x** and can be represented mathematically be a function.

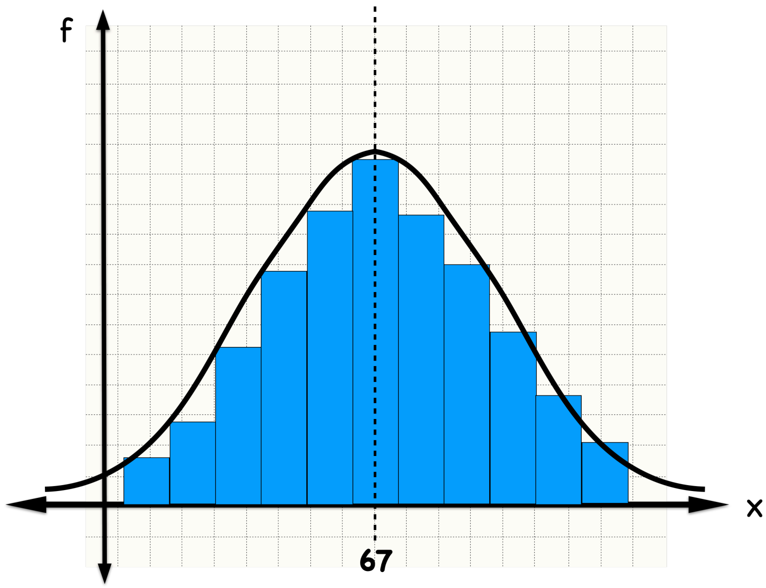
$$P(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

This distribution is known formally as the **Normal Probability Distribution**, aka the **Bell-Shaped Curve** as demonstrated by its shape. Where $\boldsymbol{\mu}$ is the **mean** of the distribution and $\boldsymbol{\sigma}$ is the **standard deviation** for the distribution.

What can be represented by the Normal Probability Distribution?

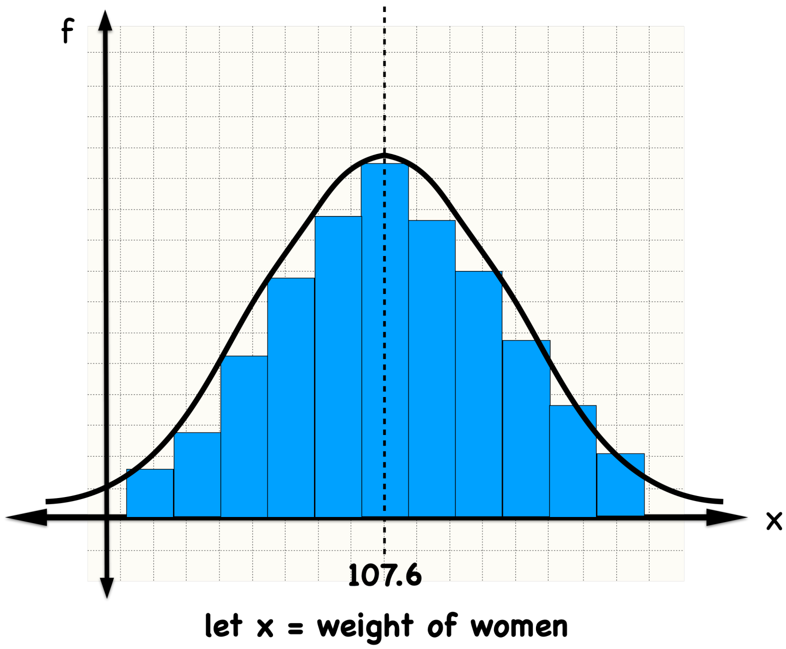
Things that grow naturally, behave naturally, reproduced, and manufactured "identically" by man or nature. Some common examples are the following.

The height of men where $\mu = 67\ inches\ (United\ States)$ and $\sigma = 2.9\ inches$

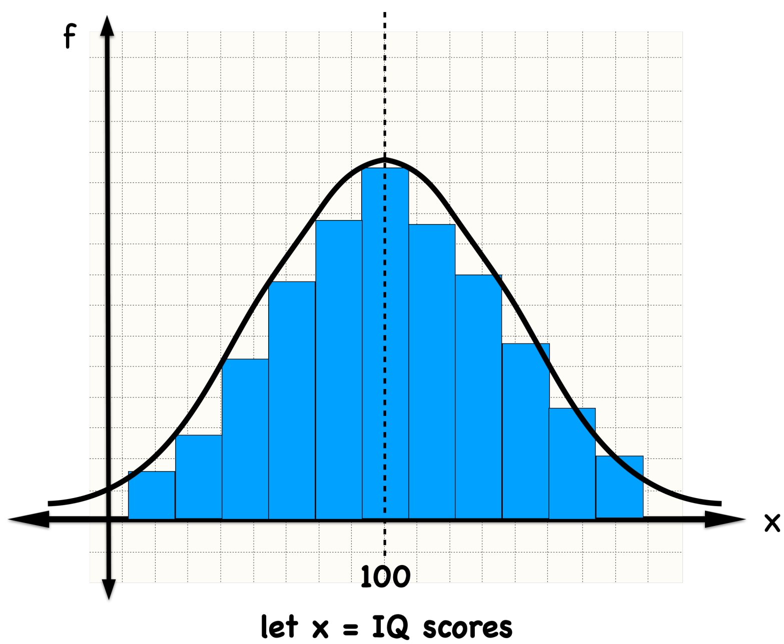


let x = height of men

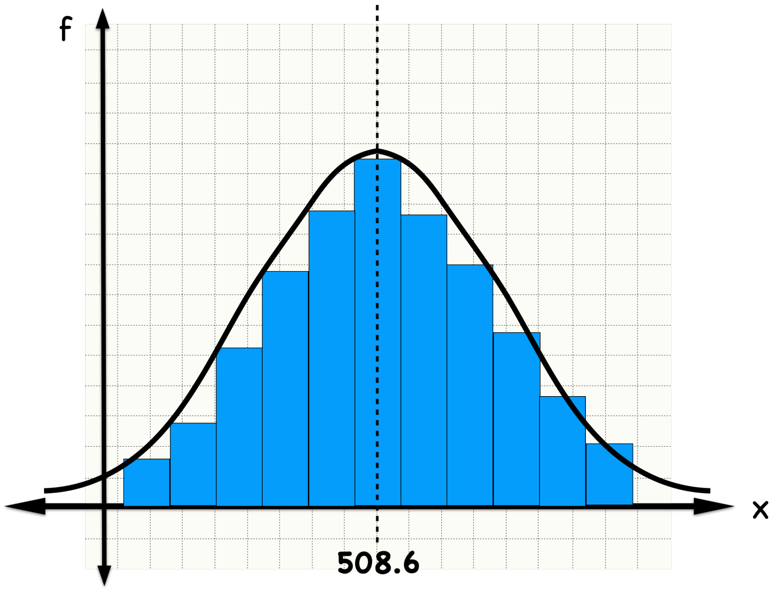
The weight of women $\mu = 107.6\ pounds\ (United\ States)$ and $\sigma = 12.6\ \ pounds$



The IQ scores of people $\mu = 100$ and $\sigma = 15$

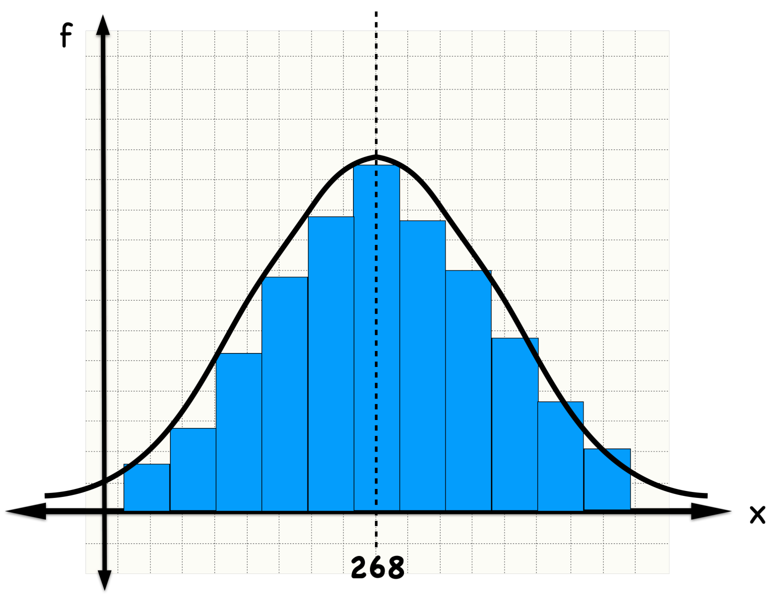


The volume of water placed in a 500-mL bottle manufactured by a particular company
$\mu = 508.6\ mL$ and $\sigma = 1.5\ mL$



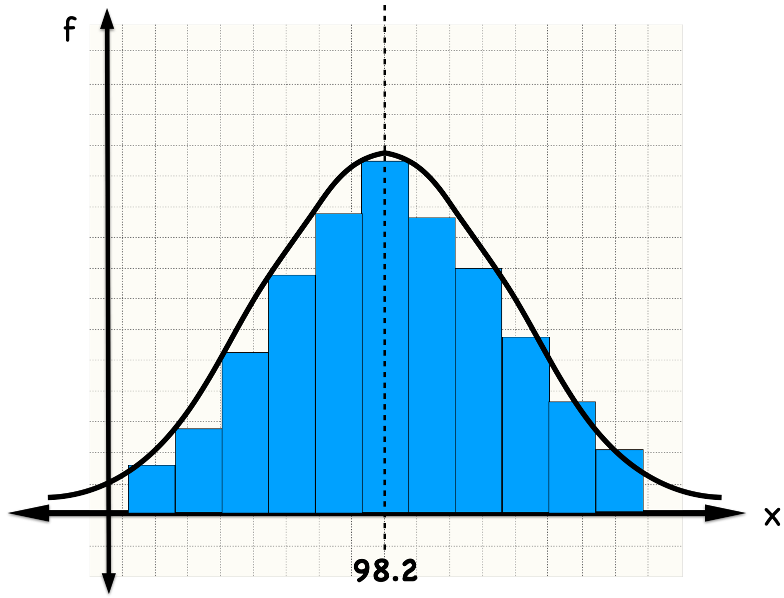
let x = volume of water filled into a 500 mL bottle

The duration of pregnancies $\mu = 268\ days$ and $\sigma = 15\ days$



let x = # of days a women is pregnant

Body temperature $\mu = 98.2^o\ F$ **and** $\sigma = \ 0.2^o\ F$



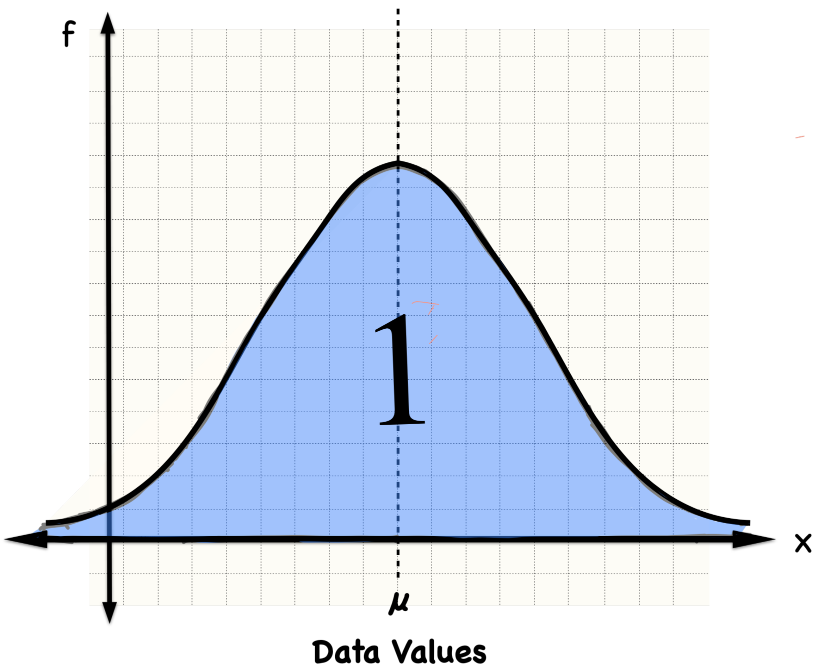
let x = body temperature of a human being

Facts about the Normal Distribution.

The distribution is symmetric about the mean μ

The area shaded in blue under the Bell-Shaped curve is equal to 1

$$\int_{-\infty}^{\infty} P(x)dx = \int_{-\infty}^{\infty} \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}} dx = 1 \text{ as } \sum_{all\ x} P(x) = 1$$

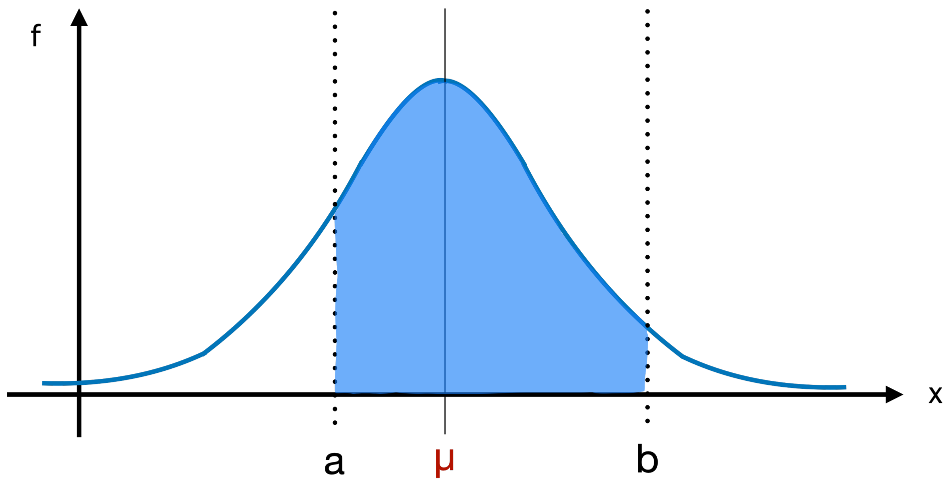


Most of what you need to know in answering Normal Probability Distribution questions is based on the following exercise.

What percent of the Bell is shaded?

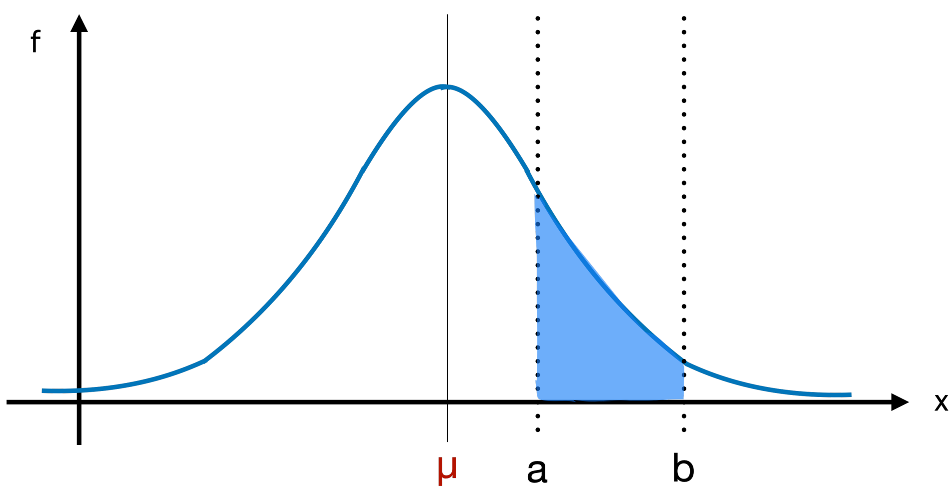
Between a and b

$$p(a \leq x \leq b)$$



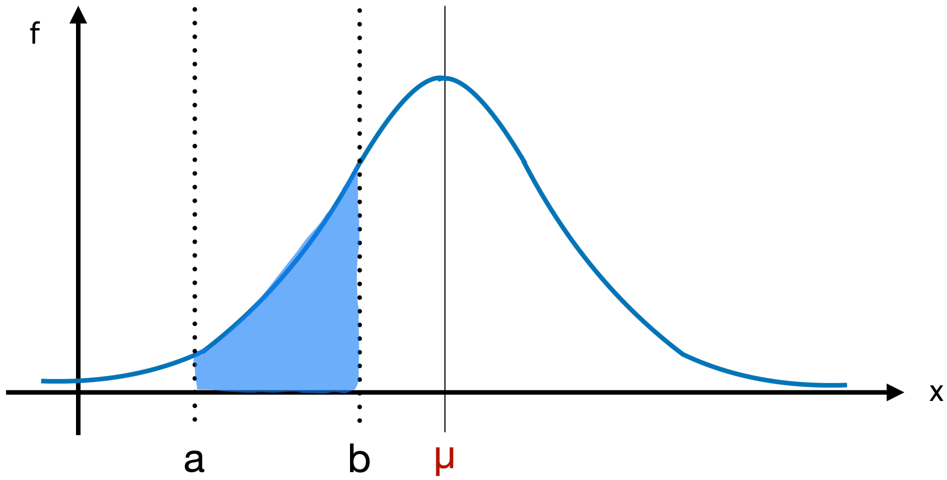
Between a and b

$$p(a \leq x \leq b)$$



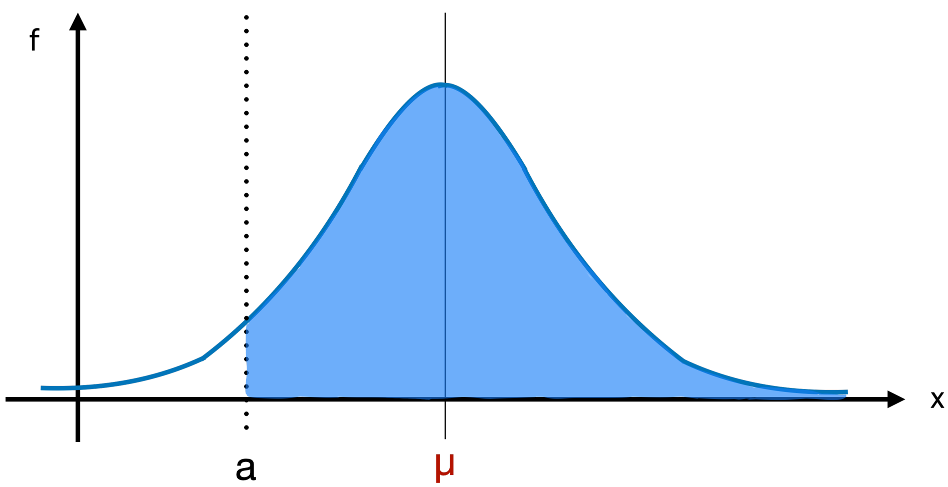
Between a and b

$$p(a \leq x \leq b)$$



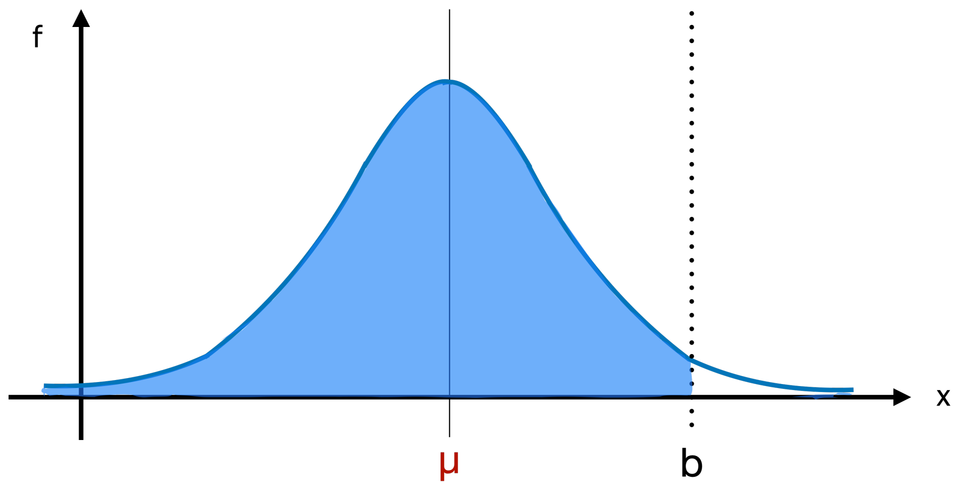
At least a or More than a

$$p(x \geq a) \text{ or } p(x > a)$$



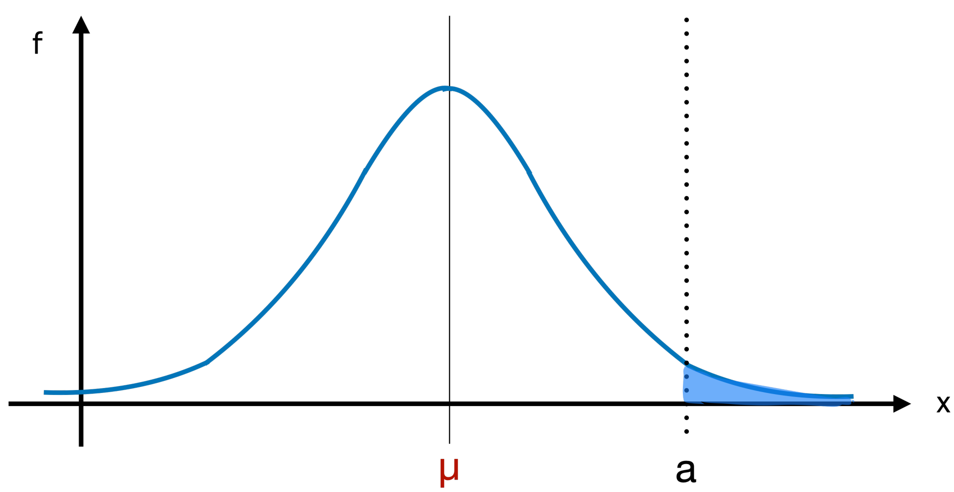
No more than b or Less than b

$$\boldsymbol{p(x \leq b)} \quad or \quad \boldsymbol{p(x < b)}$$



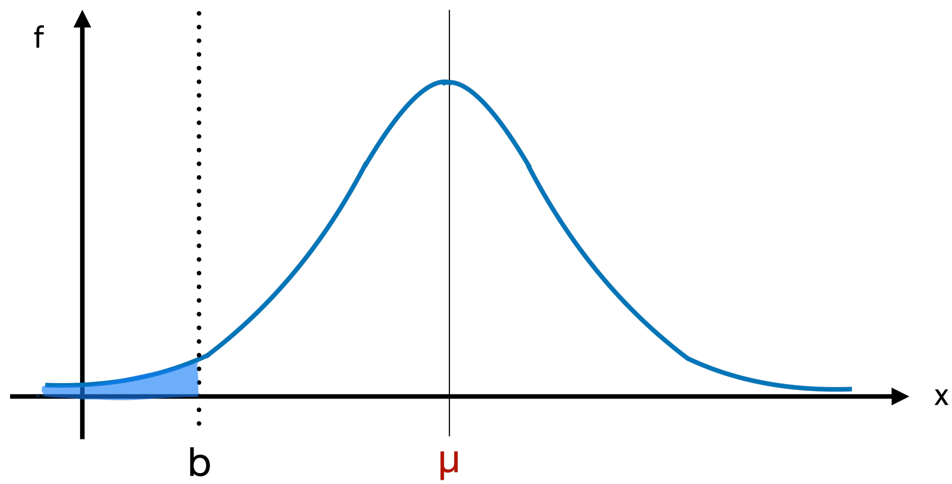
At least a or More than a

$$\boldsymbol{p(x \geq a)} \ or \ \boldsymbol{p(x > a)}$$



No more than b or Less than b

$$p(x \le b) \quad or \quad p(x < b)$$



Applications of the Normal Probability Distribution

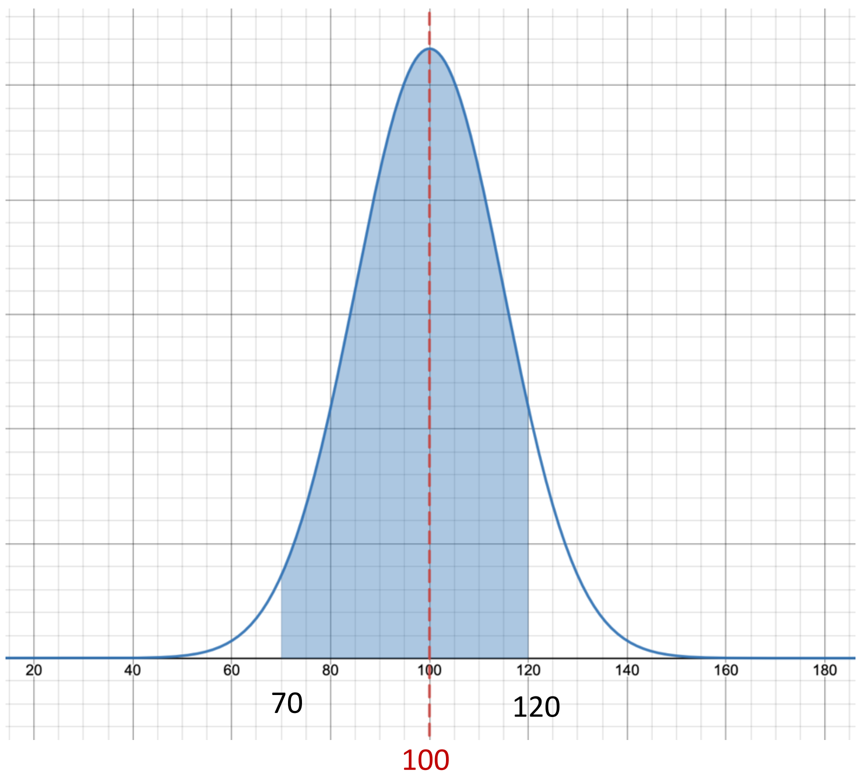
IQ Scores

Intelligence is measured by an IQ (Intelligence Quotient) which is **Normally Distributed** with a **mean** of 100 and a **standard deviation** of 15. If you select a person at random, what's the probability the person has an IQ score that is:

1. Between 70 and 120?

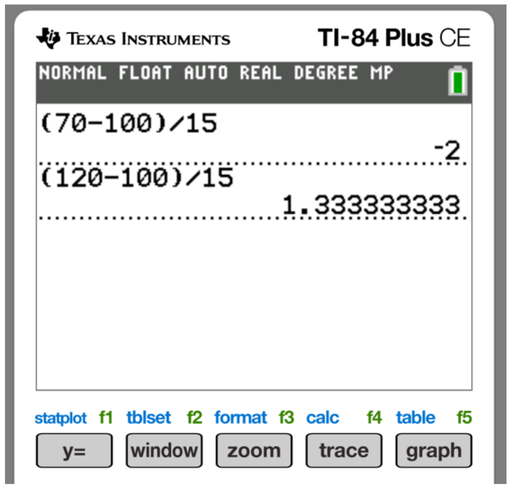
You will need to follow the procedure outlined below.

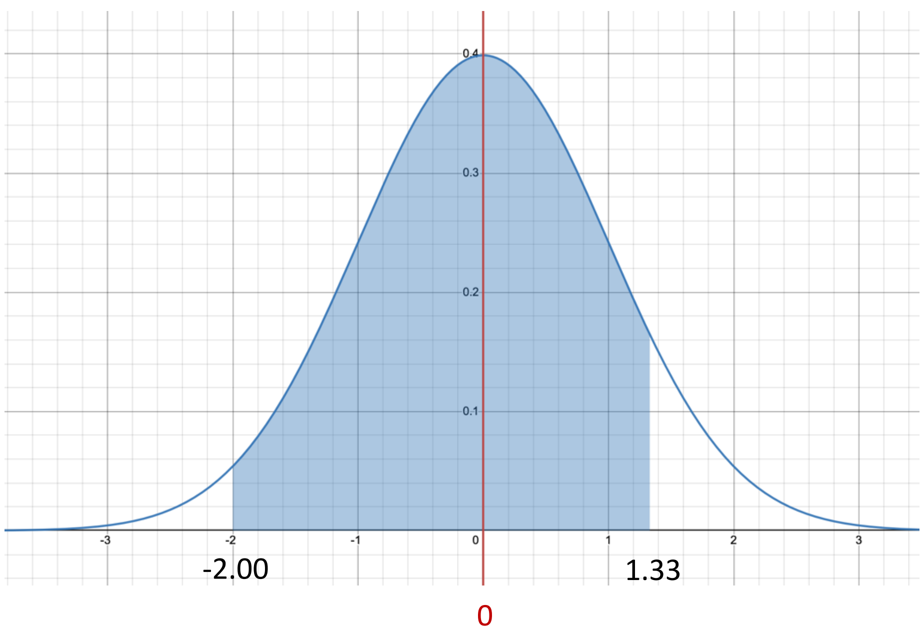
Step 1 Draw a picture of the question.



Step 2 Convert the **Normal Probability Distribution x** to the **Standard Normal Probability Distribution Z** using the **Z-Value** definition (aka, the Standard Value)

$$z = \frac{x - \mu}{\sigma}$$





Step 3 Use the Z-table and logic to deduce the percent of the Bell that is shaded. However, write your answer as a probability as the question was posed as a probability. If the question is posed as a percent, then write your answer as a percent.

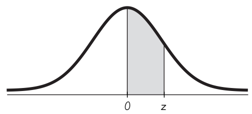


TABLE A-2 Standard Normal (z) Distribution

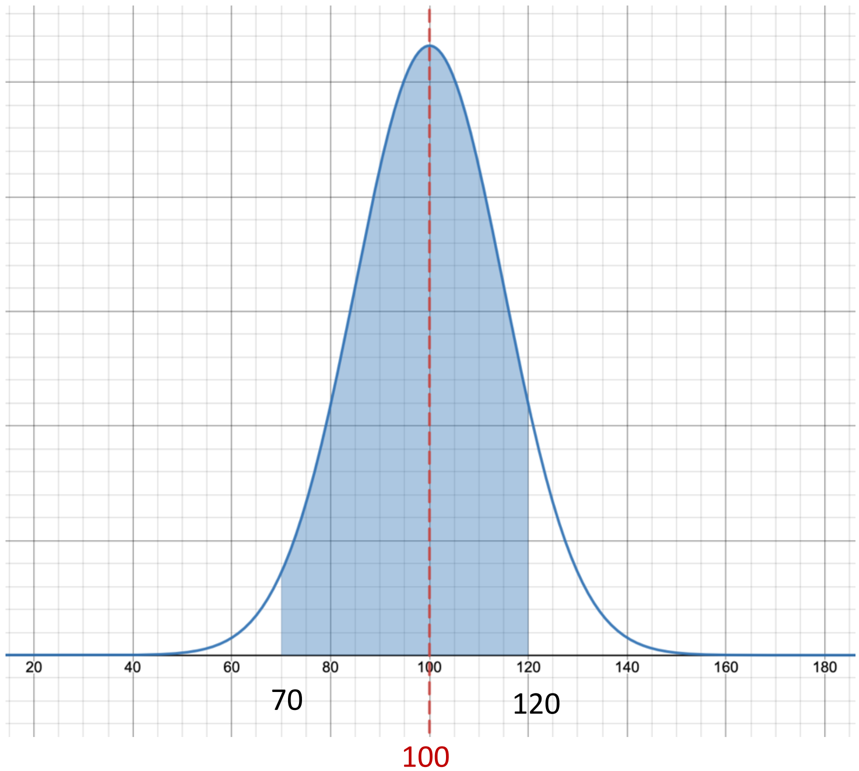
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495 *	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	$\boxed{.4750}$	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949 *	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.10 and higher	.4999									

NOTE: For values of z above 3.09, use 0.4999 for the area.
*Use these common values that result from interpolation:

z score	Area
1.645	0.4500
2.575	0.4950

From Frederick C. Mosteller and Robert E. K. Rourke, *Sturdy Statistics*, 1973, Addison-Wesley Publishing Co., Reading, MA. Reprinted with permission of Frederick Mosteller.

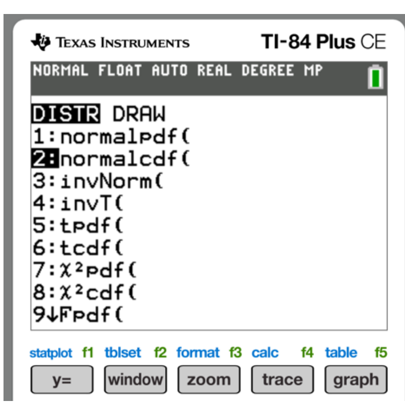
However, we can use the **TI-83 or TI-84 Calculator** from the beginning.

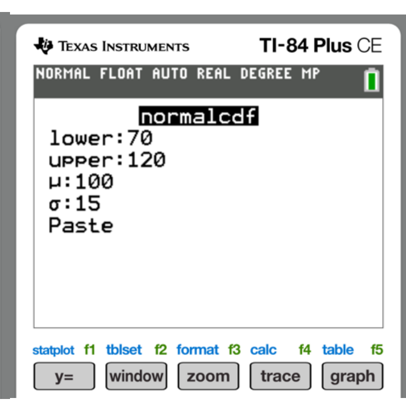


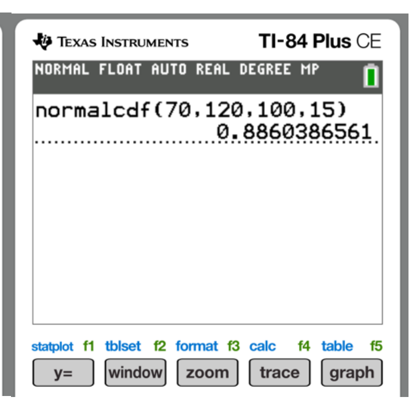
TI-83 or TI-84 Plus Finding Area Between Two x Values

1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **normalcdf** and click **enter**.
3. Enter the desired lower Z value and the upper Z value.
   Enter 100 for μ and 15 for σ **normalcdf(lower x, upper x, μ, σ)** and press **enter.**

normalcdf(70,120,100,15)

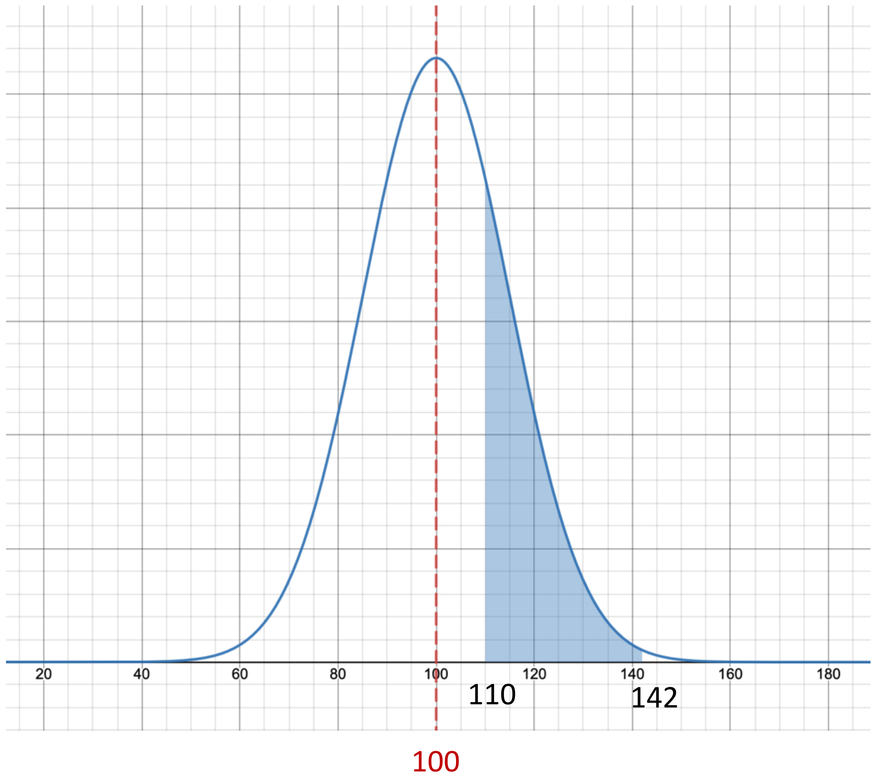






$$\boldsymbol{p(70 \le x \le 120) \approx 0.886}$$

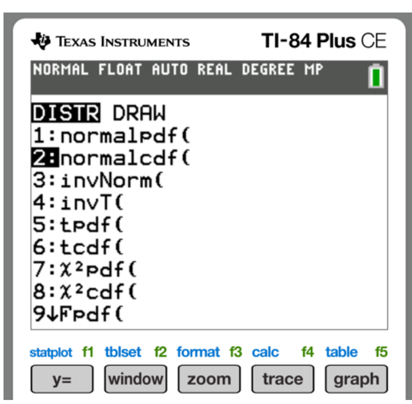
2. Between 110 and 142?

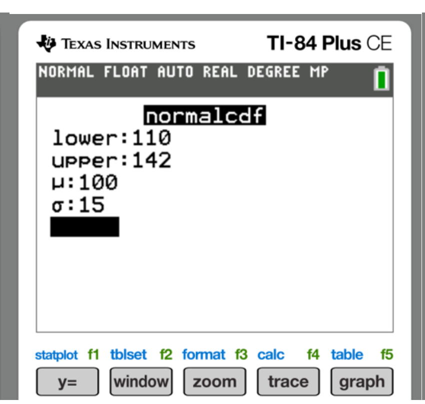


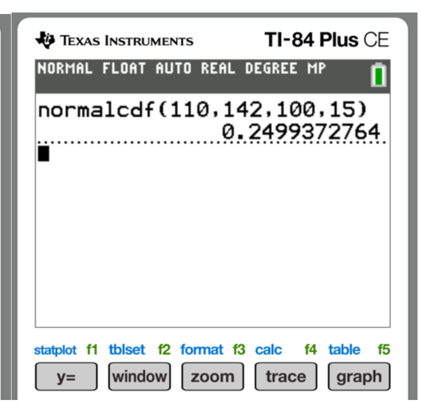
TI-83 or TI-84 Plus Finding Area Between Two x Values

1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **normalcdf** and click **enter**.
3. Enter the desired lower Z value and the upper Z value.
   Enter 100 for μ and 15 for σ **normalcdf(lower x, upper x, $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$)** and press **enter.**

Normalcdf(110,142,100,15)

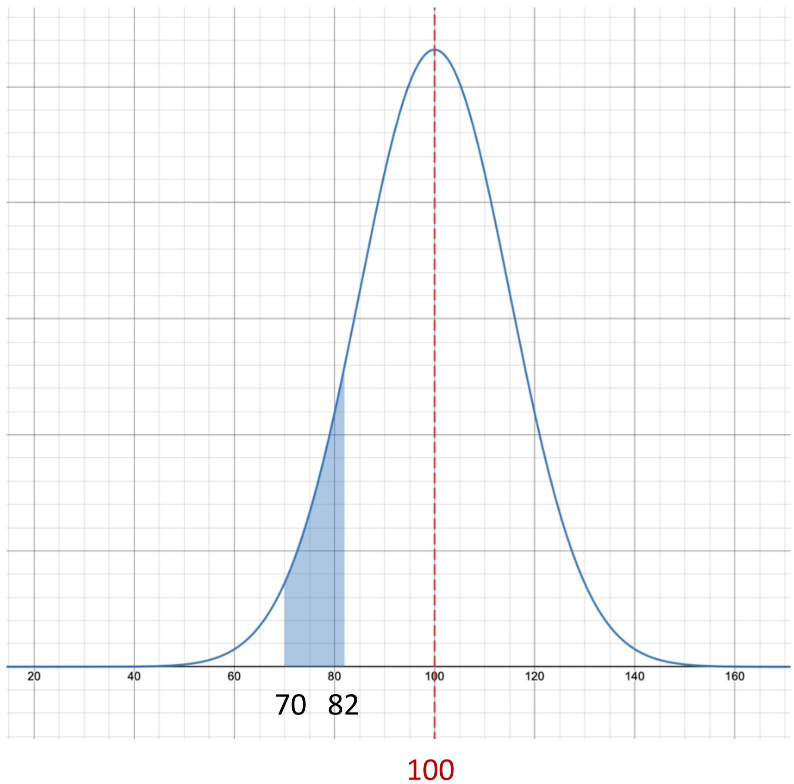






$$\boldsymbol{p(110 \leq x \leq 142) \approx 0.250}$$

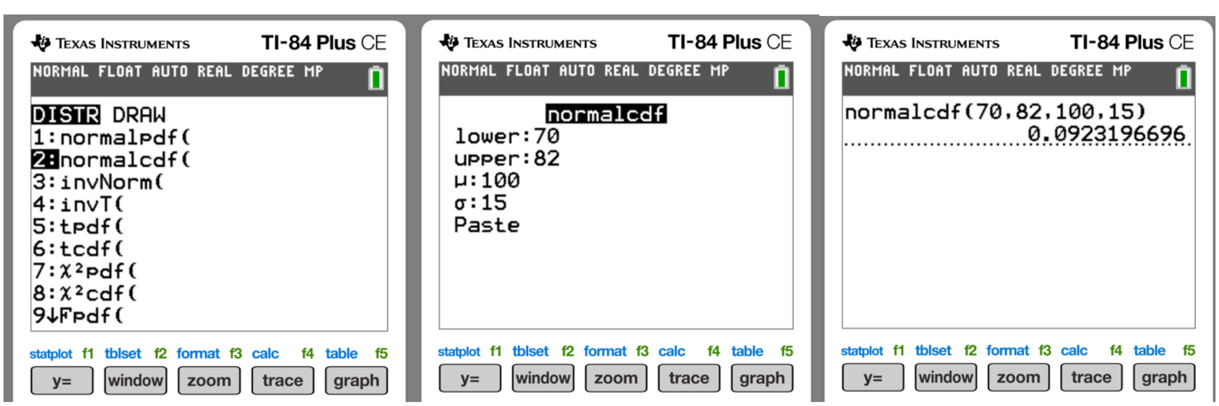
3. Between 70 and 82?



TI-83 or TI-84 Plus Finding Area Between Two x Values

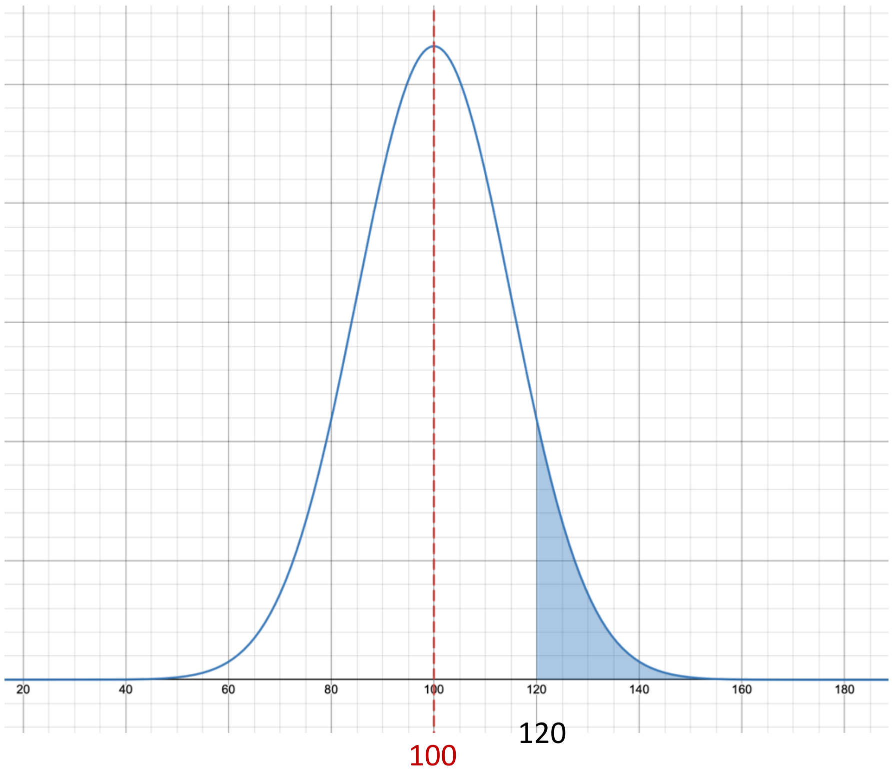
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **normalcdf** and click **enter**.
3. Enter the desired lower Z value and the upper Z value.
   Enter 100 for μ and 15 for σ **normalcdf(lower x, upper x, μ, σ)** and press **enter.**

normalcdf(70,82,100,15)



$$p(70 \le x \le 82) \approx 0.092$$

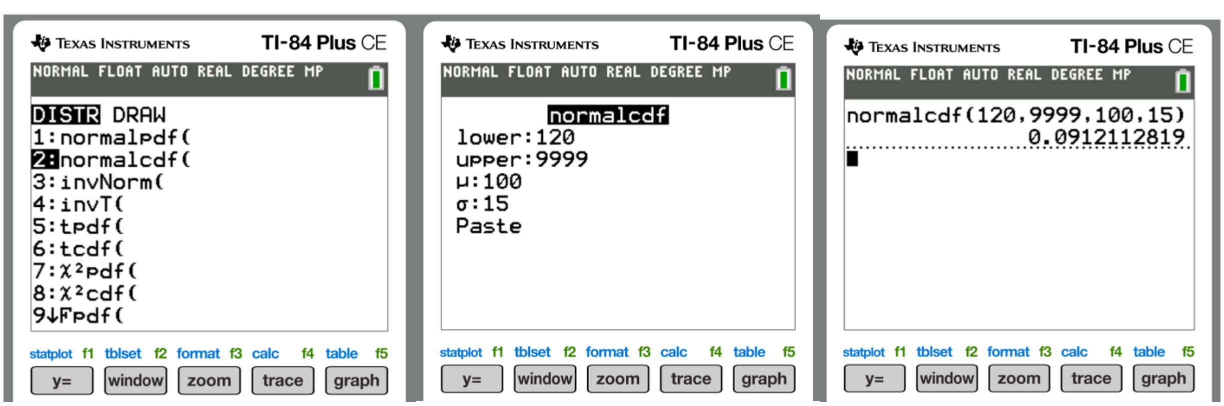
4. More than 120?



TI-83 or TI-84 Plus Finding Area Between Two x Values

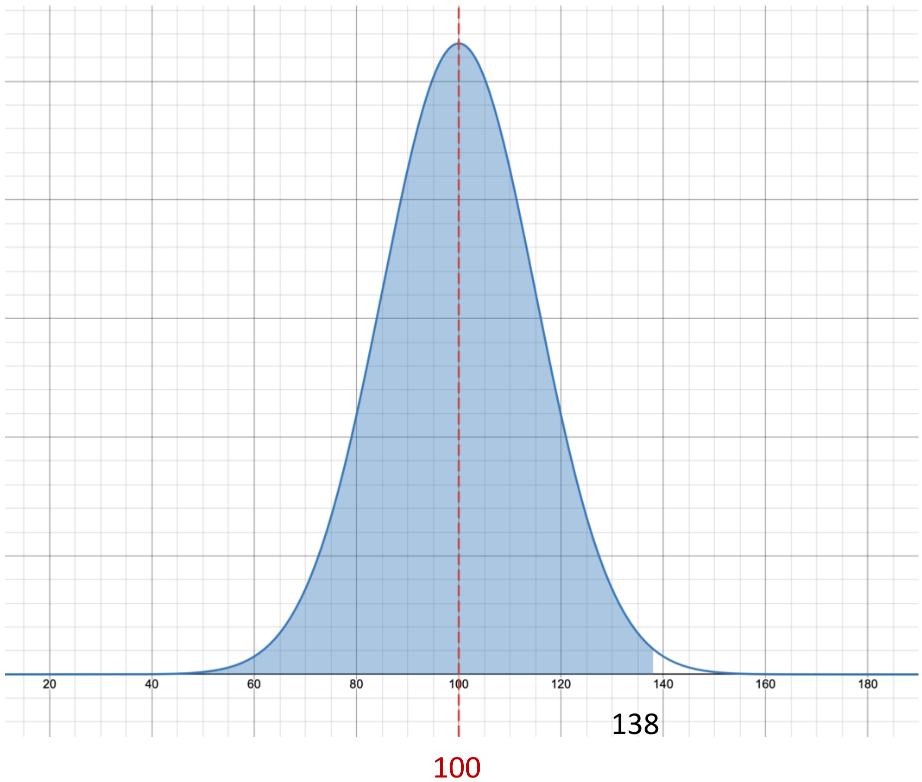
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **normalcdf** and click **enter**.
3. Enter the desired lower Z value and the upper Z value.
   Enter 100 for μ and 15 for σ **normalcdf(lower x, upper x, μ, σ)** and press **enter.**

normalcdf(120,9999,100,15)



$$p(x > 120) \approx 0.091$$

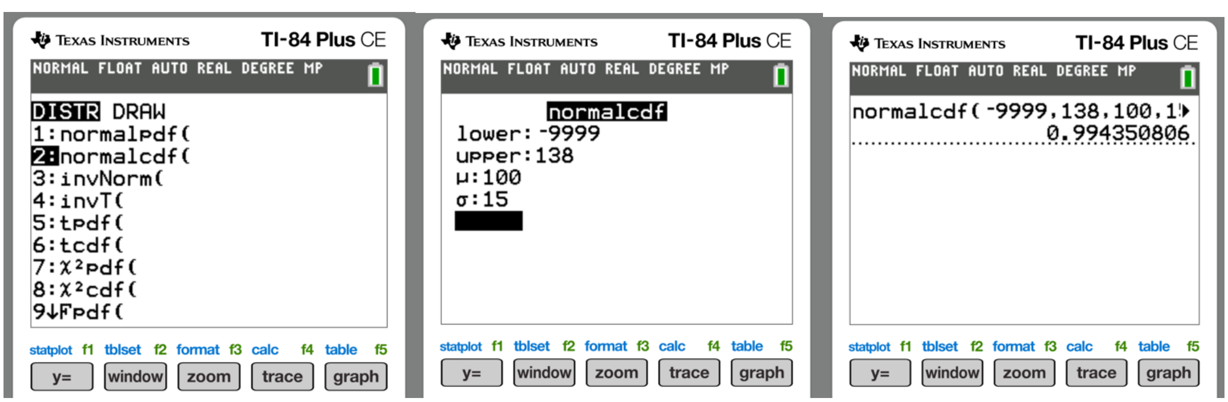
5. No more than 138?



TI-83 or TI-84 Plus Finding Area Between Two x Values

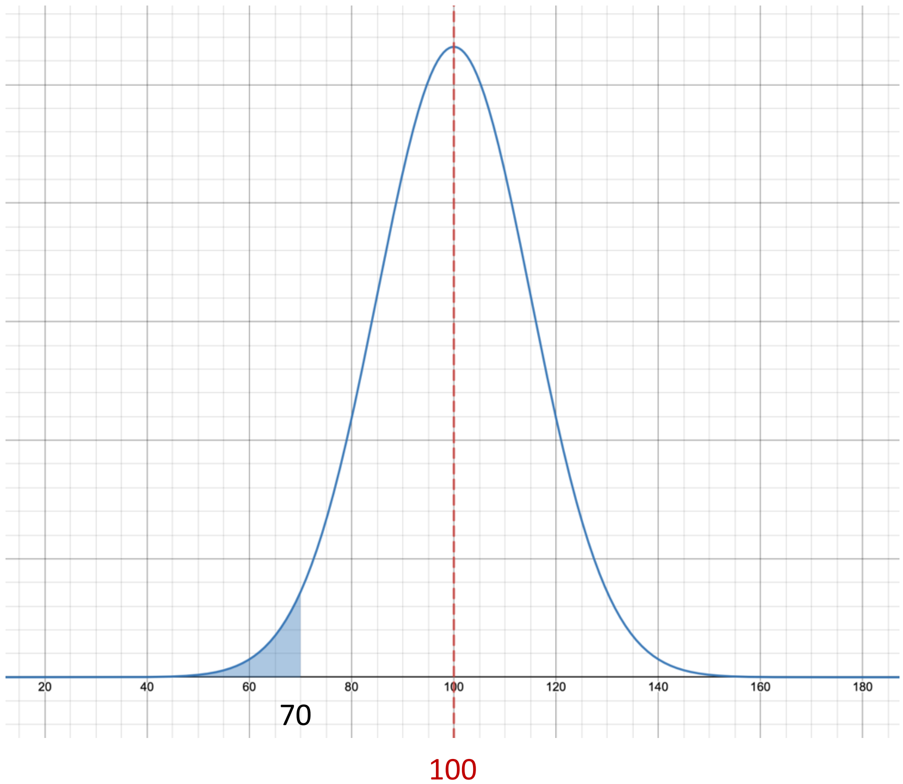
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **normalcdf** and click **enter**.
3. Enter the desired lower Z value and the upper Z value.
   Enter 100 for μ and 15 for σ **normalcdf(lower x, upper x, μ, σ)** and press **enter.**

normalcdf(-9999,138,100,15)



$$p(x \leq 138) \approx 0.994$$

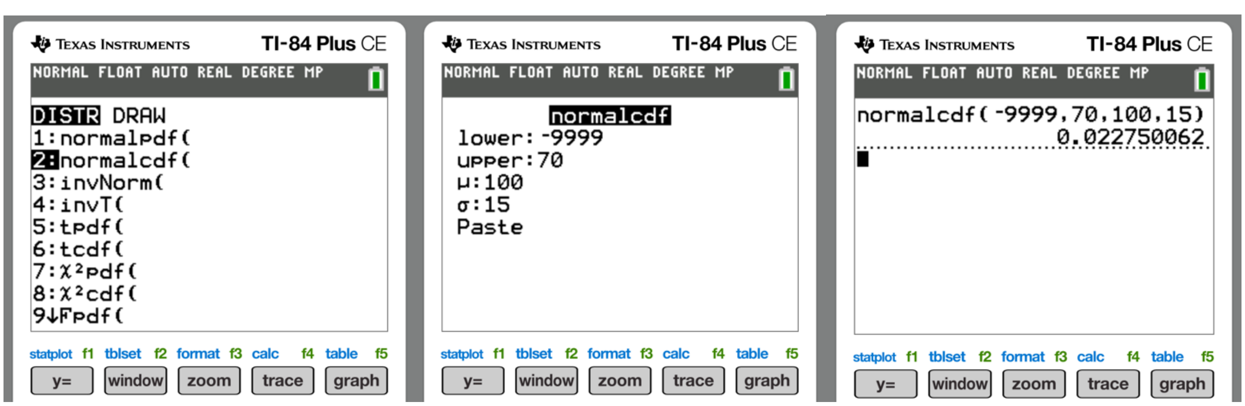
6. Less than 70?



TI-83 or TI-84 Plus Finding Area Between Two x Values

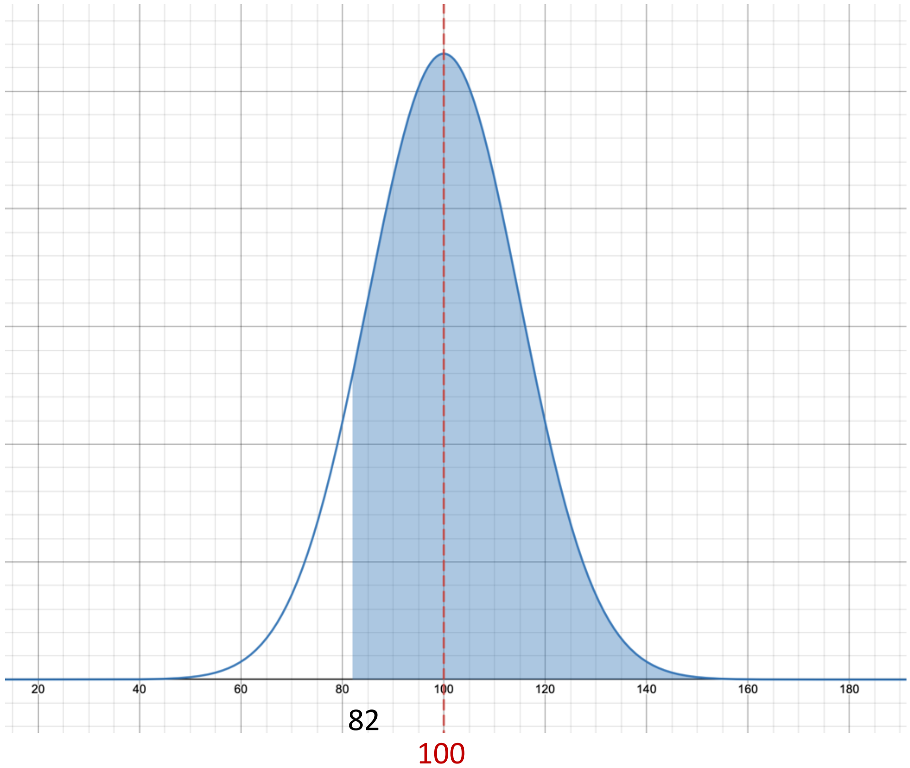
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **normalcdf** and click **enter**.
3. Enter the desired lower Z value and the upper Z value.
   Enter 100 for μ and 15 for σ **normalcdf(lower x, upper x, μ, σ)** and press **enter.**

normalcdf(-9999,70,100,15)



$$p(x < 70) \approx 0.023$$

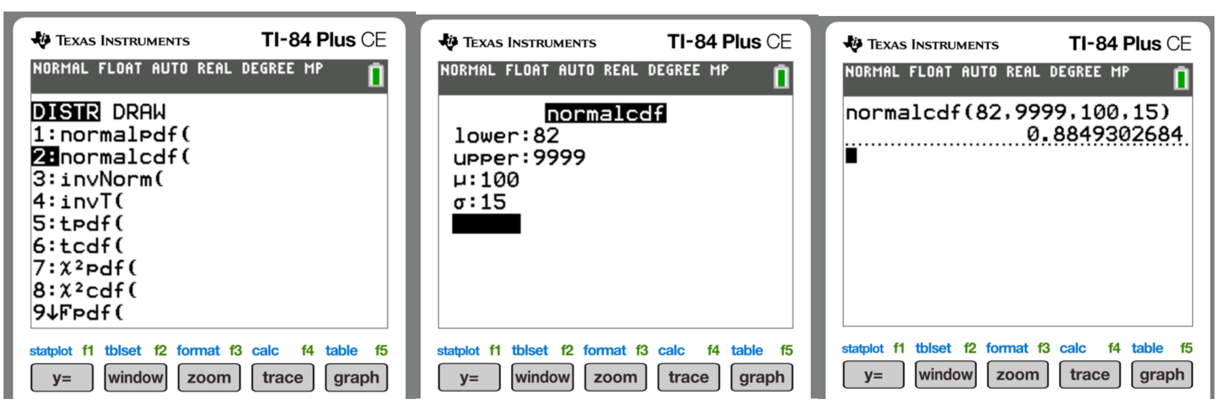
7. More than 82?



TI-83 or TI-84 Plus Finding Area Between Two x Values

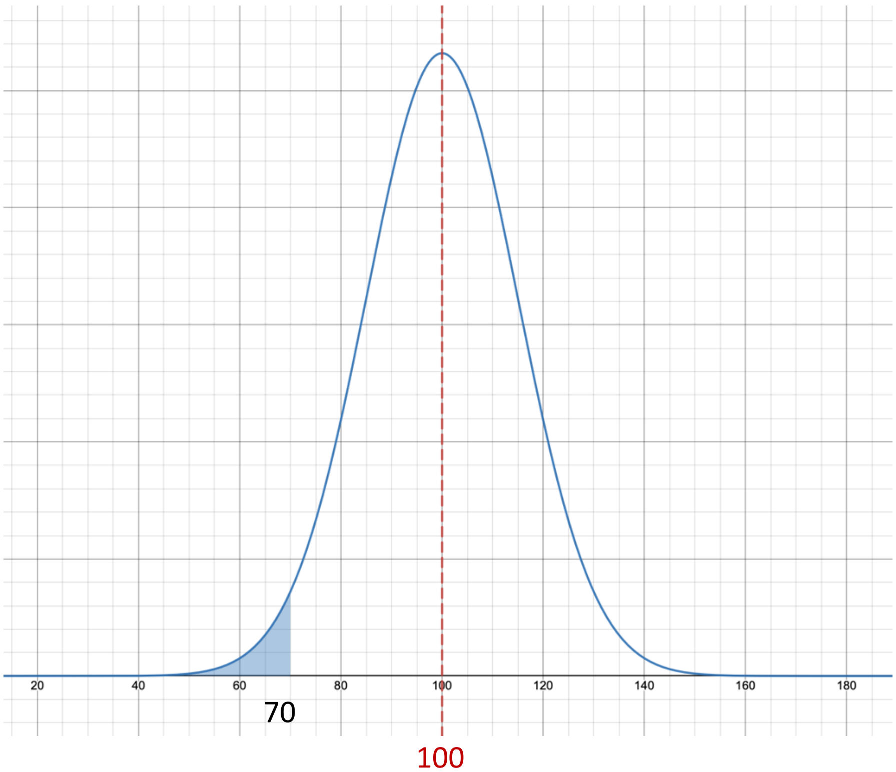
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **normalcdf** and click **enter**.
3. Enter the desired lower Z value and the upper Z value.
   Enter 100 for μ and 15 for σ **normalcdf(lower x, upper x, μ, σ)** and press **enter.**

normalcdf(82,9999,100,15)



$$p(x > 82) \approx 0.885$$

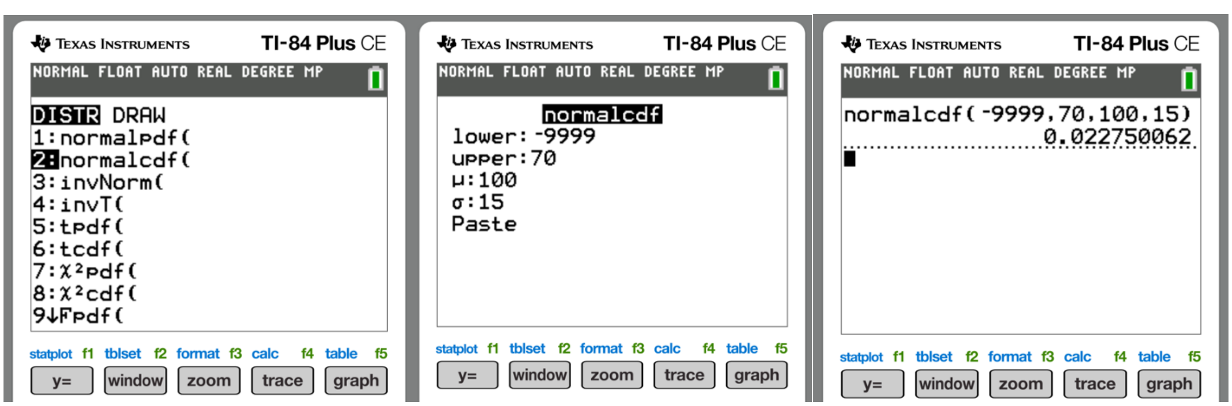
8. No more than 70?



TI-83 or TI-84 Plus Finding Area Between Two x Values

1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **normalcdf** and click **enter**.
3. Enter the desired lower Z value and the upper Z value.
   Enter 100 for μ and 15 for σ **normalcdf(lower x, upper x, μ, σ)** and press **enter.**

normalcdf(-9999,70,100,15)



$$p(x \leq 70) \approx 0.023$$