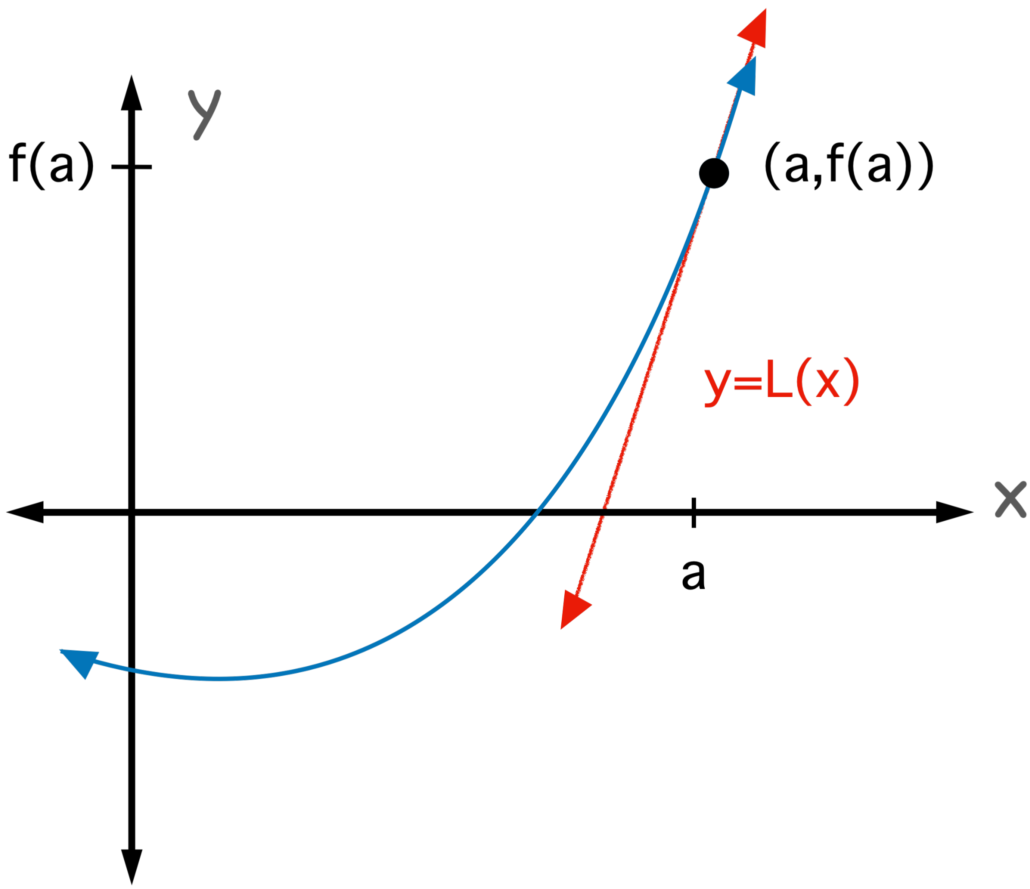
Linearization Function $L(x)$

The linearization function is a good way to approximate a function $f(x)$ at a location on the curve $P(a, f(a))$. It is based on the proverbial "determine the equation of a line at the point $P(a, f(a))$". Consider the point slope form from Beginning Algebra.



$$y - f(a) = f'(a)(x - a)$$

Which implies

$$y = f(a) + f'(a)(x - a)$$

We let $\boldsymbol{L(x) = f(a) + f'(x)(x - a)}$

which is called the **Linearization Function** for $f(x)$.

$L(x) \approx f(x)$ for x "near" a.

Example

Find the linearization function for $f(x) = \frac{1}{x^2}$ at $a = -1$

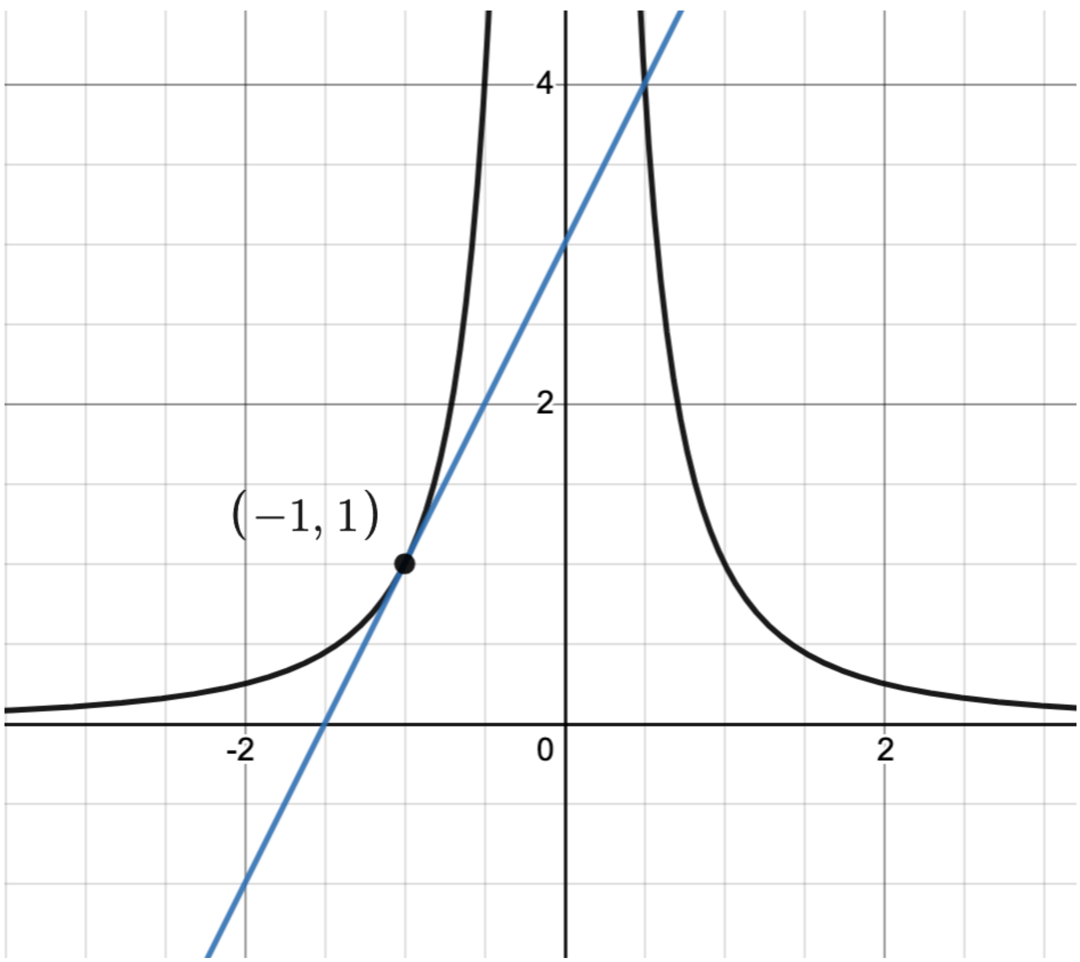
$$f'(x) = -\frac{2}{x^3}$$

$$L(x) = f(-1) + f'(-1)(x - (-1))$$

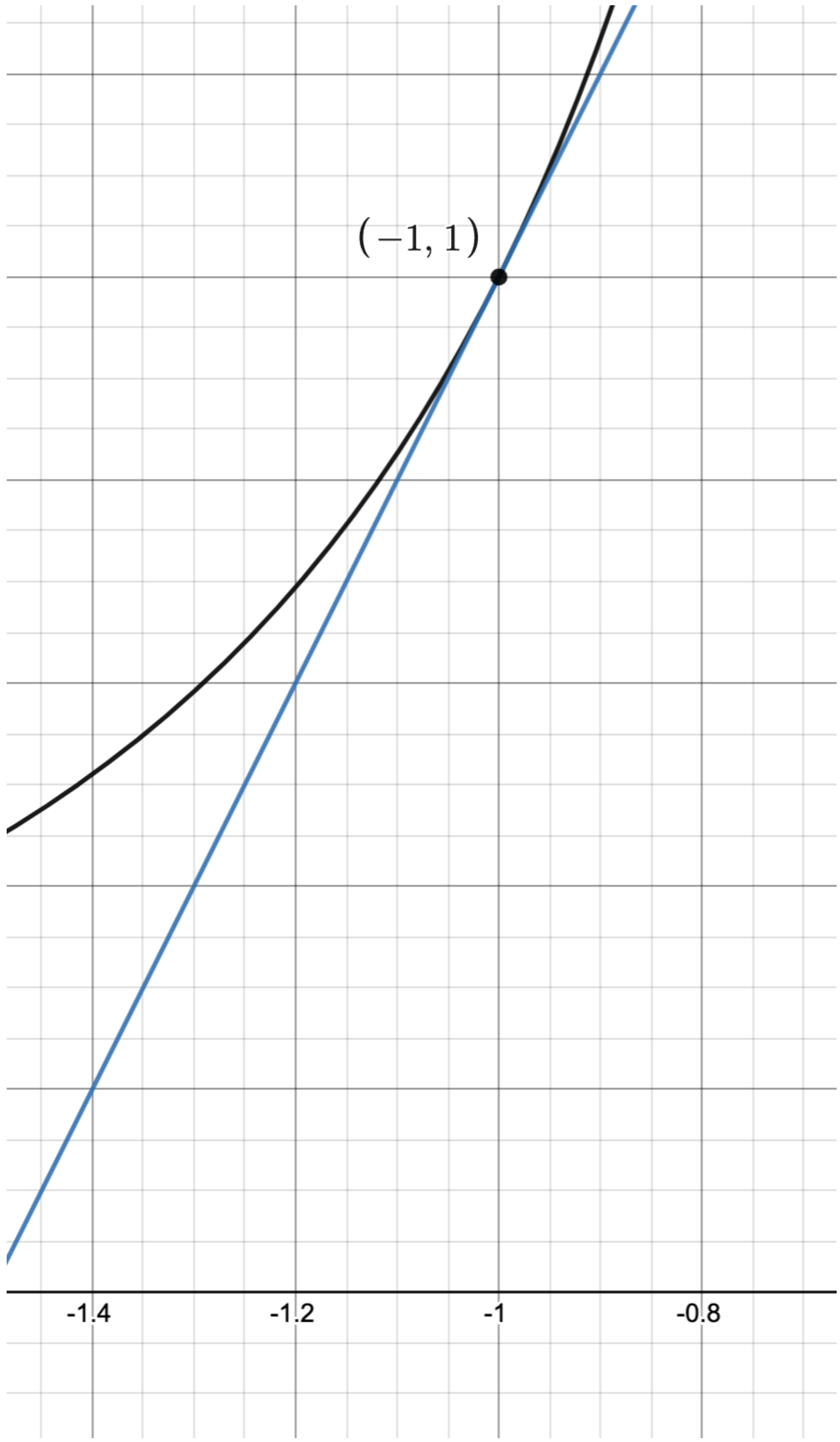
$$L(x) = \frac{1}{(-1)^2} + \left[-\frac{2}{(-1)^3}\right](x - (-1))$$

$$L(x) = 1 + 2(x + 1)$$

$$L(x) = 2x + 3$$



$$\frac{1}{x^2} \approx 2x + 3 \text{ for } x \text{ “near” } -1$$



Example

Find the linearization function for $f(x) = \frac{1}{x^2}$ at $a = -2$

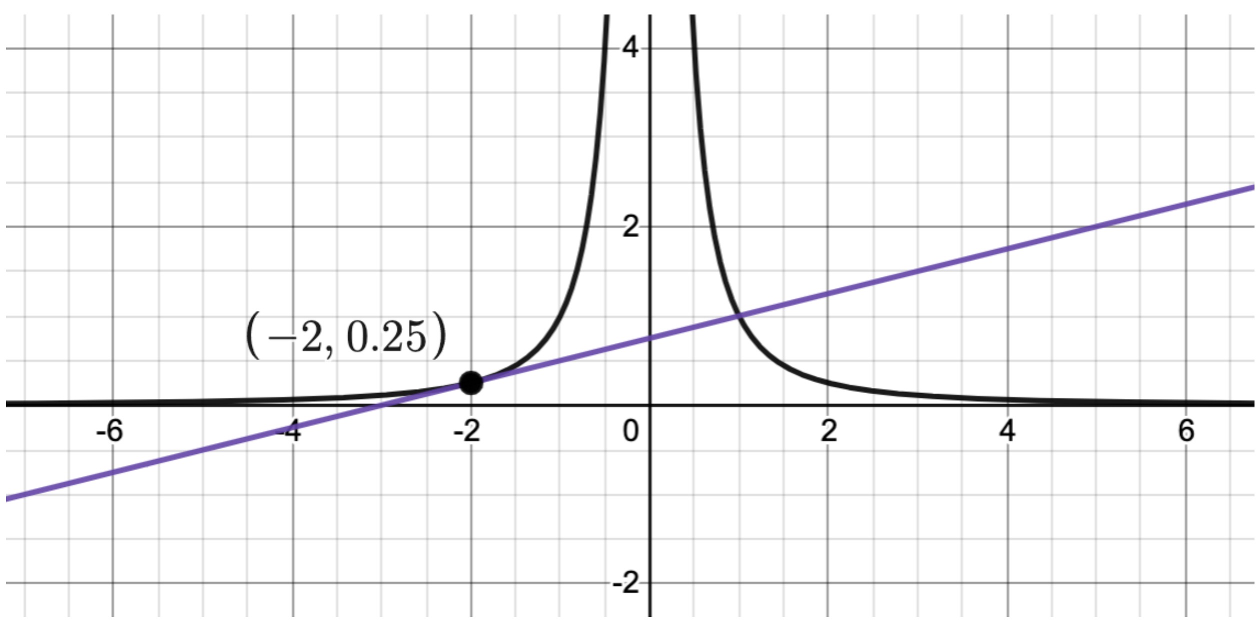
$$f'(x) = -\frac{2}{x^3}$$

$$L(x) = f(-2) + f'(-2)(x - (-2))$$

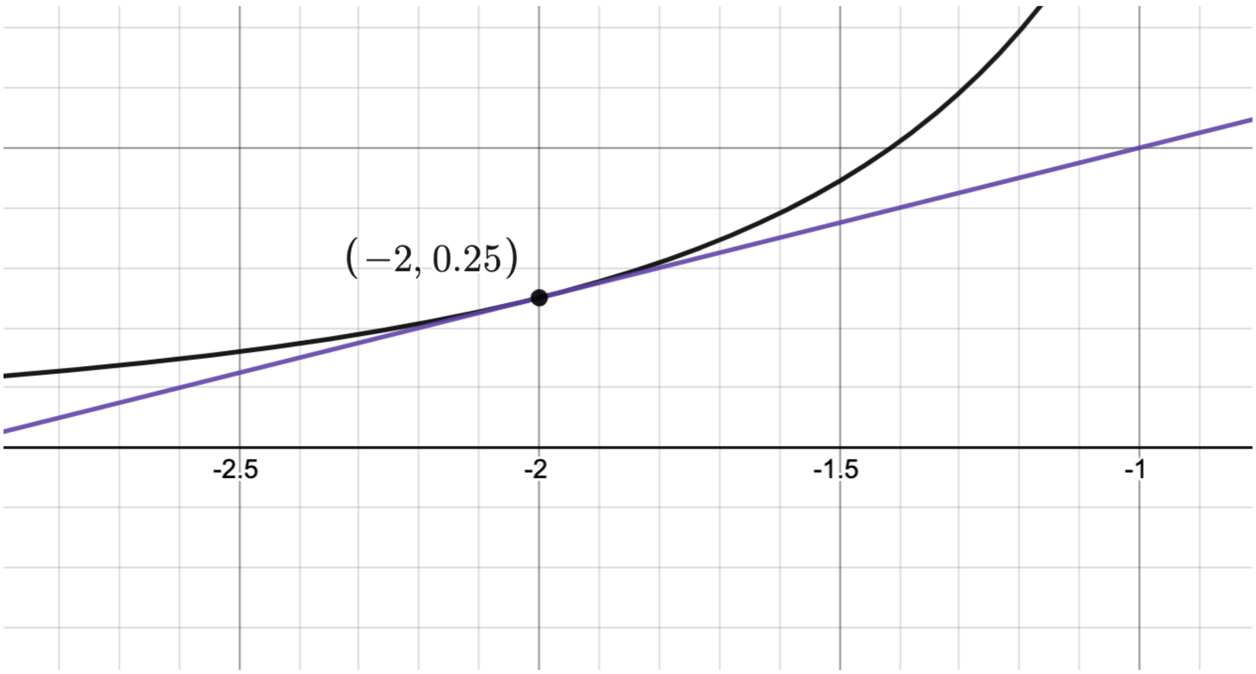
$$L(x) = \frac{1}{(-2)^2} + \left[-\frac{2}{(-2)^3}\right](x - (-2))$$

$$L(x) = \frac{1}{4} + \frac{1}{4}(x + 2)$$

$$L(x) = \frac{1}{4}x + \frac{3}{4}$$



$$\frac{1}{x^2} \approx \frac{1}{4}x + \frac{3}{4} \text{ for } x \text{ "near" } -2$$



Example

Find the linearization function for $f(x) = \cos(x)$ at $a = \frac{\pi}{2}$

$$f'(x) = -\sin(x)$$

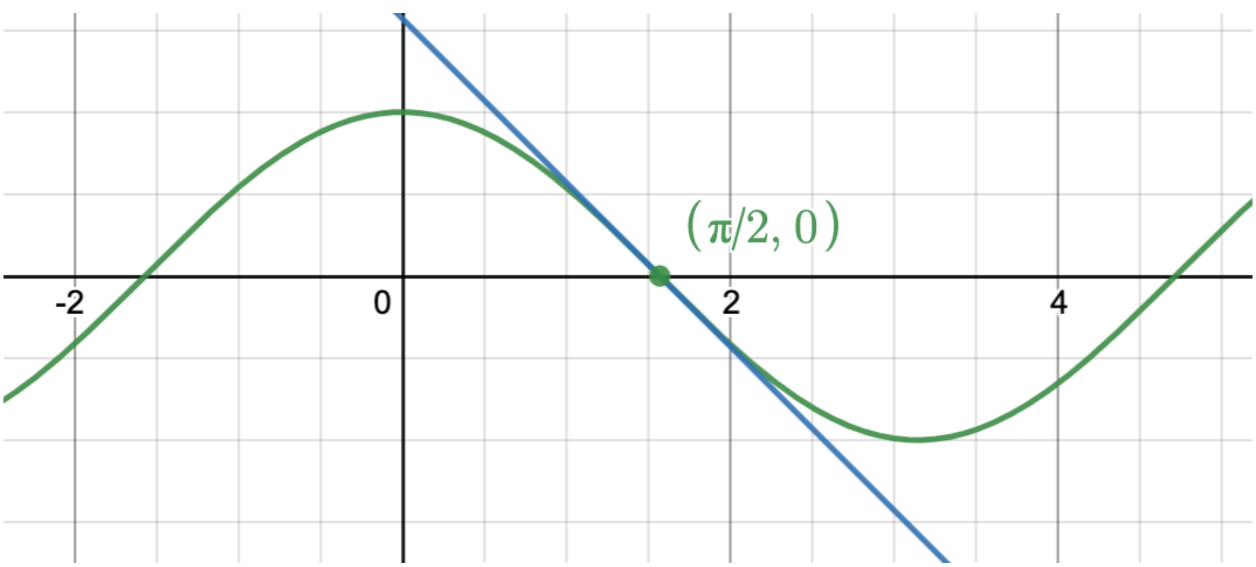
$$L(x) = f\left(\frac{\pi}{2}\right) + f'\left(\frac{\pi}{2}\right)\left(x - \frac{\pi}{2}\right)$$

$$L(x) = cos\left(\frac{\pi}{2}\right) + \left(-sin\left(\frac{\pi}{2}\right)\right)\left(x - \frac{\pi}{2}\right)$$

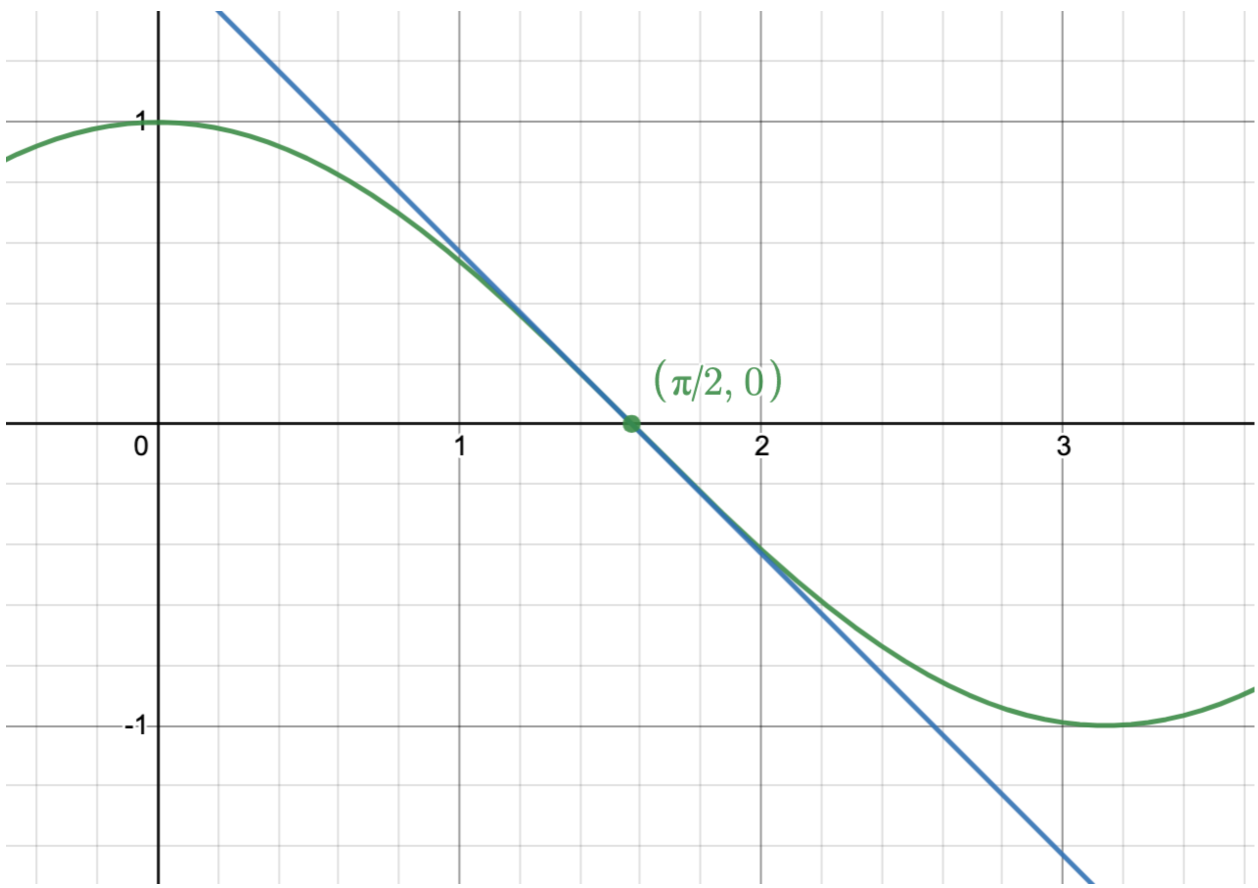
$$L(x) = cos\left(\frac{\pi}{2}\right) + \left(-sin\left(\frac{\pi}{2}\right)\right)\left(x - \frac{\pi}{2}\right)$$

$$L(x) = 0 + (-1)\left(x - \frac{\pi}{2}\right)$$

$$L(x) = -x + \frac{\pi}{2}$$



$$\cos(x) \approx -x + \frac{\pi}{2} \text{ for } x \text{ "near" } \frac{\pi}{2}$$



Use a linear approximation to approximate the following numbers.

1. $\sqrt[3]{1.98}$

2. $\frac{1}{1002}$

3. $sin(44^{O})$

4. $sec(0.08)$

5. 5.02^4