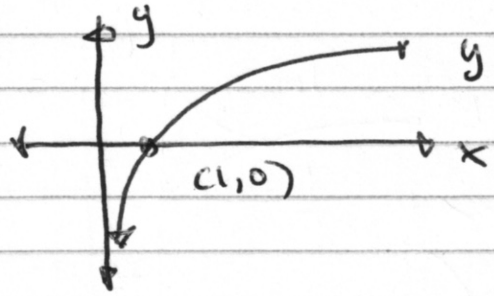
limits for logarithmic Functions

(1) $\lim\limits_{x \to \infty} \log(x)$



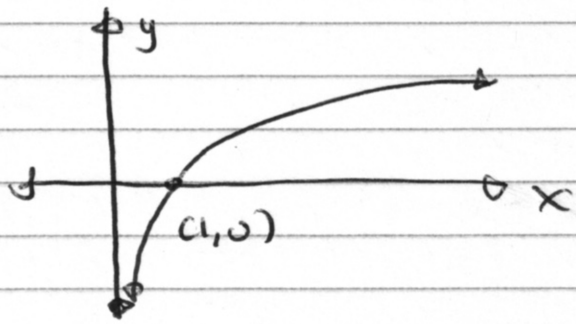
$y = \log_{10}(x)$

as $x \to \infty$; $\log(x) \to \infty$

(3) $\lim\limits_{x \to \infty} \log_5(x)$

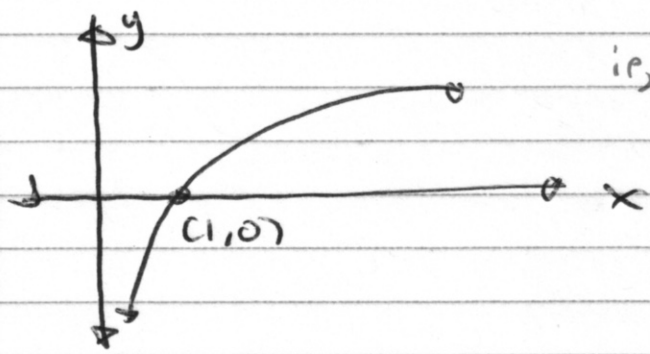
as $x \to \infty$

$\log_5(x) \to \infty$



(5) $\lim\limits_{x \to 2} \log_5(x)$

$y = \log_5(x)$ is continuous at $x = 2$

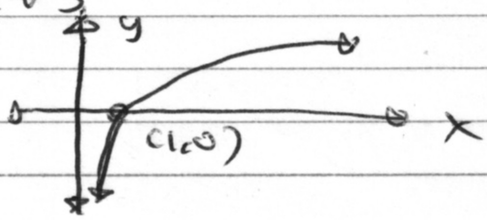


ie, $\lim\limits_{x \to 2} \log_5(x)$

$= \log_5(2)$

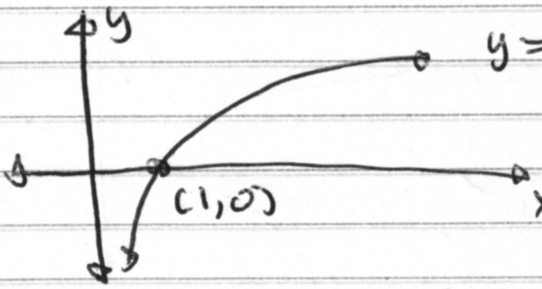
(7) $\lim\limits_{x \to 5} \log_2(x)$

$y = \log_2(x)$ is continuous at $x = 5$



ie, $\lim\limits_{x \to 5} \log_2(x) = \log_2(5)$

(9) $\lim_{x \to 4} \log(x)$



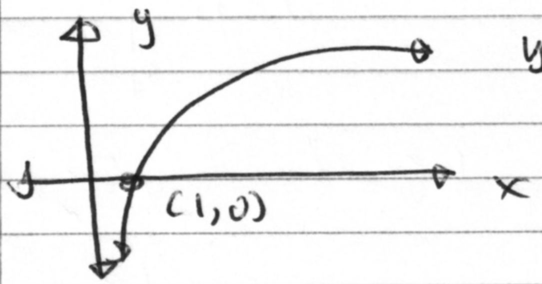
$y = \log_{10}(x)$

$y = \log_{10}(x)$ is continuous at $x = 4$

ie, $\lim_{x \to 4} \log(x)$

$= \log(4)$

(11) $\lim_{x \to 10} \log(x)$



$y = \log_{10}(x)$

$y = \log_{10}(x)$ is continuous at $x = 10$

$\lim_{x \to 10} \log(x)$

$= \log(10) = 1$

<u>note</u> $t = \log(10)$

or $t = \log_{10}(10)$

ie, $10^t = 10$

or $t = 1$

(13) $\lim_{x \to 1000} \log(x)$

$y = \log(x)$ is continuous at $x = 1000$

ie, $\lim_{x \to 1000} \log(x) = \log(1000)$

note $t = \log_{10}(1000)$

$10^y = 1000$; $10^y = 10^3$

ie, $t = 3$

so, as $x \to 1000$; $\log(x) \to 3$

(15) $\lim_{x \to \frac{1}{10}} \log(x)$

$y = \log(x)$ is continuous at $x = 1/10$

ie, $\lim_{x \to \frac{1}{10}} \log(x) = \log\left(\frac{1}{10}\right)$

note $\log\left(\frac{1}{10}\right) = \log_{10}\left(\frac{1}{10}\right)$

let $t = \log_{10}\left(\frac{1}{10}\right)$

$10^t = \frac{1}{10}$, $10^t = 10^{-1}$

$t = -1$

as $x \to \frac{1}{10}$; $\log(x) \to -1$

(17) $\lim_{x \to 8} \log_2(x)$

$y = \log_2(x)$ is continuous at $x = 8$

ie $\lim_{x \to 8} \log_2(x) = \log_2(8)$

let $t = \log_2(8)$; $2^t = 8$

$2^t = 2^3$; $t = 3$

ie, as $x \to \infty$; $\log_2(x) \to \boxed{3}$

(19) $\lim_{x \to 25} \log_5(x)$

$y = \log_5(x)$ is continuous at $x = 8$

ie, $\lim_{x \to 25} \log_5(x) = \log_5(25)$

let $t = \log_5(25)$; $5^t = 25$;

$5^t = 5^2$;

$t = 2$

as $x \to 25$; $\log_5(x) \to \boxed{2}$

(21) $\lim_{x \to 3} \log_9 (x)$

$y = \log_9 (x)$ is continuous at $x = 3$

ie, $\lim_{x \to 3} \log_9 (x) = \log_9 (3)$

<u>note</u> let $t = \log_9 (3)$, $9^t = 3$

$(3^2)^t = 3$; $3^{2t} = 3^1$; $2t = 1$

$t = 1/2$

as $x \to 3$; $\log_9 (x) \to \boxed{\frac{1}{2}}$

(23) $\lim_{x \to 4} \log_4 (x)$

$y = \log_4 (x)$ is continuous at $x = 4$

ie, $\lim_{x \to 4} \log_4 (x) = \log_4 (4)$

<u>note</u> let $t = \log_4 (4)$, $4^t = 4$

$t = 1$

ie, as $x \to 4$; $\log_4 (x) \to \boxed{1}$

(25) $\lim_{x \to \frac{1}{4}} \log_4(x)$

$y = \log_4(x)$ is continuous at $x = \frac{1}{4}$

ie, $\lim_{x \to \frac{1}{4}} \log_4(x) = \log_4\left(\frac{1}{4}\right)$

let $t = \log_4\left(\frac{1}{4}\right)$; $4^t = \frac{1}{4}$

$4^t = 4^{-1}$;

$t = -1$

ie, as $x \to \frac{1}{4}$; $\log_4(x) \to \boxed{-1}$

(27) $\lim_{x \to e} \ln(x)$

$y = \ln(x)$ is continuous at $x = e$

ie, $\lim_{x \to e} \ln(x) = \ln(e) \to \boxed{1}$

as $x \to e$; $\ln(x) \to \boxed{1}$

(29) $\lim_{x \to e^3} \ln(x)$

$y = \ln(x)$ is continuous at $x = e^3$

ie, $\lim_{x \to e^3} \ln(x) = \ln(e^3) = \boxed{3}$

note let $t = \ln(e^3)$

$t = \log_e(e^3)$

$e^t = e^3$; $t = 3$

as $x \to e^3$; $\ln(x) \to \boxed{3}$

(31) $\lim_{x \to 1} \ln(x)$

$y = \ln(x)$ is continuous at $x = 1$

ie, $\lim_{x \to 1} \ln(x) = \ln(1) \to 0$

as $x \to 1$; $\ln(x) \to \boxed{0}$

(33) $\lim_{x \to 3} \ln(x)$

$y = \ln(x)$ is continuous at $x = 3$

ie, $\lim_{x \to 3} \ln(x) = \ln(3)$

as $x \to 3$; $\ln(x) \to \boxed{\ln(3)}$

(35) $\lim_{x \to \frac{1}{e}} \ln(x)$

$y = \ln(x)$ is continuous at $x = \frac{1}{e}$

ie, $\lim_{x \to \frac{1}{e}} \ln(x) = \ln(\frac{1}{e})$

<u>note</u> let $t = \ln(\frac{1}{e})$; $t = \log_e(\frac{1}{e})$

$e^t = \frac{1}{e}$; $e^t = e^{-1}$; $t = -1$

ie, as $x \to \frac{1}{e}$, $\ln(x) \to \boxed{-1}$

(37) $\lim_{x \to \frac{1}{e^4}} \ln(x)$

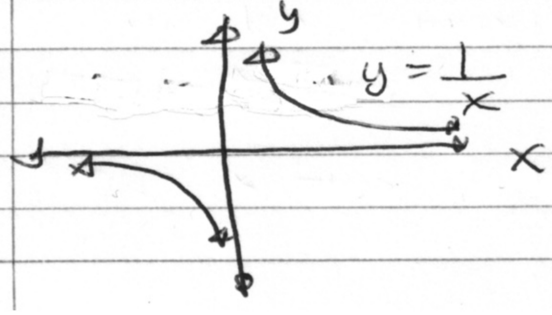
$y = \ln(x)$ is continuous at $x = \frac{1}{e^4}$

ie, $\lim_{x \to \frac{1}{e^4}} \ln(x) = \ln(\frac{1}{e^4})$

<u>note</u> $\ln(\frac{1}{e^4}) = \ln(e^{-4})$

$= -4 \cancel{\ln(e)}^{1} = -4$

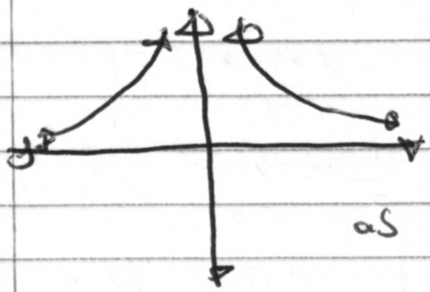
(39) $\lim_{x \to \infty} \log(\frac{1}{x})$



as $x \to \infty$, $\frac{1}{x} \to 0$

$\log(\frac{1}{x}) \to -\infty$

(41) $\lim\limits_{x \to \infty} \log\left(\frac{1}{x^2}\right)$

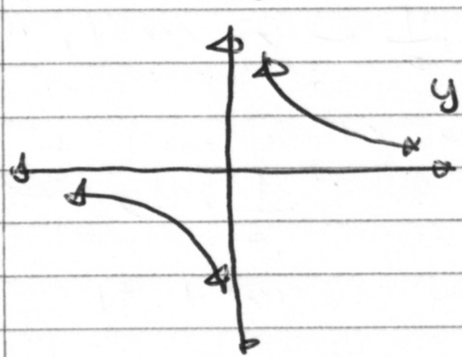


$y = \frac{}{x^2}$

as $x \to \infty$; $\frac{1}{x^2} \to 0$

$\log\left(\frac{1}{x^2}\right) \to -\infty$

(43) $\lim\limits_{x \to \infty} \ln\left(\frac{1}{x^3}\right)$



$y = \frac{1}{x^3}$ as $x \to \infty$;

$\frac{1}{x^3} \to 0$

$\ln\left(\frac{1}{x^3}\right) \to -\infty$

(45) $\lim\limits_{x \to \infty} \ln\left(\frac{x^2+10}{x^2+2}\right)$

let $t = \frac{x^2+10}{x^2+2}$

$t = \frac{1 + 10/x^2}{1 + 2/x^2}$ as $x \to \infty$;

$t \to 1$

$\ln(t) \to \ln(1)$

$\boxed{0}$

(47) $\lim_{x \to 0} \log\left(\frac{x^2+10}{x^2+2}\right)$

$= \log\left[\lim_{x \to 0}\left(\frac{x^2+10}{x^2+2}\right)\right]$

$= \log\left[\lim_{x \to 0}\left(\frac{0^2+10}{0^2+2}\right)\right] = \log(5)$

(49) $\lim_{x \to \infty} \ln\left(\frac{4x^2+x+3}{x^2+2x+1}\right)$

$= \ln\left[\lim_{x \to \infty}\left(\frac{4x^2+x+3}{x^2+2x+1}\right)\right]$

$= \ln\left[\lim_{x \to \infty}\left(\frac{4+\frac{1}{x}+3/x^2}{1+\frac{2}{x}+1/x^2}\right)\right]$

$= \ln\left[\frac{4}{1}\right] = \ln(4)$