L'Hospitals Rule

(1) $\lim\limits_{x \to 2} \dfrac{x^3 - 8}{x^4 + 2x - 20}$ $\qquad \dfrac{0}{0}$

$\underset{H}{=} \lim\limits_{x \to 2} \dfrac{3x^2}{4x^3 + 2} = \dfrac{3 \cdot 2^2}{4 \cdot 2^3 + 2}$

$= \dfrac{12}{34} \quad \boxed{\dfrac{6}{17}}$

(3) $\lim\limits_{x \to 0} \dfrac{e^x - x - 1}{\cos x - 1}$ $\qquad \dfrac{e^0 - 0 - 1}{\cos 0 - 1}$ $\qquad \dfrac{0}{0}$

$\underset{H}{=} \lim\limits_{x \to 0} \dfrac{e^x - 1}{-\sin x}$ $\qquad \dfrac{e^0 - 1}{-\sin 0}$ $\qquad \dfrac{0}{0}$

$\underset{H}{=} \lim\limits_{x \to 0} \dfrac{e^x}{-\cos x}$ $\qquad \dfrac{e^0}{-\cos 0}$ $\qquad \dfrac{1}{-1} \quad \boxed{-1}$

(5) $\lim\limits_{x \to 3} \dfrac{2x^2 - 5x - 3}{x - 4}$ $\qquad \dfrac{2 \cdot 3^2 - 5 \cdot 3 - 3}{3 - 4}$ $\qquad \dfrac{0}{-1}$

$\boxed{0}$

(7) $\lim\limits_{x \to 9} \dfrac{x^{1/2} + x - 6}{x^{3/2} - 27}$ $\qquad \dfrac{0}{0}$

(9) $\lim\limits_{x \to \infty} \dfrac{x^2}{e^x}$ $\qquad \dfrac{\infty}{\infty}$

$\underset{H}{=} \lim\limits_{x \to \infty} \dfrac{2x}{e^x}$ $\qquad \dfrac{\infty}{\infty}$

$\underset{H}{=} \lim\limits_{x \to \infty} \dfrac{2}{e^x}$

$\lim\limits_{x \to \infty} 2e^{-x} =$

$2 \cdot \lim\limits_{x \to \infty} e^{-x}$

$2 \cdot 0$

$\boxed{0}$

(11) $\lim\limits_{x \to -\infty} \dfrac{\ln(x^4+1)}{x}$ $\quad \dfrac{\infty}{-\infty}$

$\overset{=}{H} \lim\limits_{x \to -\infty} \dfrac{\dfrac{1}{x^4+1} \; 4x^3}{1}$

$= \lim\limits_{x \to -\infty} \dfrac{4x^3}{x^4+1}$ $\quad \dfrac{-\infty}{\infty}$

$\overset{=}{H} \lim\limits_{x \to -\infty} \dfrac{12x^2}{4x^3}$ $\quad \dfrac{3}{x}$

$\lim\limits_{x \to -\infty} \dfrac{3}{x} = 3 \cdot \lim\limits_{x \to -\infty} \dfrac{1}{x} = 3 \cdot 0 = \boxed{0}$

(13) $\lim\limits_{x \to 0} \dfrac{\tan(x)}{x}$ $\quad \dfrac{0}{0}$

$\overset{=}{H} \lim\limits_{x \to 0} \dfrac{\sec^2(x)}{1} = \lim\limits_{x \to 0} \sec^2(x)$

$= \sec^2(0) = [\sec(0)]^2$

$= \left[\dfrac{1}{\cos(0)}\right]^2 = 1^2 = \boxed{1}$

$0 \cdot (-\infty)$

(15) $\lim\limits_{x \to 0+} x \ln(x)$ IP

$= \lim\limits_{x \to 0+} \dfrac{\ln(x)}{1/x}$ $\quad \dfrac{-\infty}{\infty}$

$\overset{=}{H} \lim\limits_{x \to 0+} \dfrac{1/x}{-1/x^2}$ $\quad \dfrac{1}{x} \cdot \dfrac{-x^2}{1}$ $\quad -x$

$\lim\limits_{x \to 0+} (-x)$

$- \lim\limits_{x \to 0+} x$

$-1 \cdot 0 \quad \boxed{0}$

(17) $\lim_{x \to 1} \tan\left(\frac{\pi x}{2}\right) \ln(x)$ $\qquad \tan\left(\frac{\pi}{2}\right) \cdot \ln(1)$

$\infty \cdot 0$

IP

$= \lim_{x \to 1} \frac{\ln(x)}{\frac{1}{\tan\left(\frac{\pi x}{2}\right)}} = \lim_{x \to 1} \frac{\ln(x)}{\cot\left(\frac{\pi x}{2}\right)} \quad \frac{0}{0}$

$\underset{H}{=} \lim_{x \to 1} \frac{1/x}{-\csc^2\left(\frac{\pi x}{2}\right) \cdot \frac{\pi}{2}}$

$= \lim_{x \to 1} \frac{1}{-\frac{\pi}{2} x \csc^2\left(\frac{\pi x}{2}\right)}$

$= \lim_{x \to 1} \frac{\sin^2\left(\frac{\pi x}{2}\right)}{-\frac{\pi}{2} x} \qquad \frac{\sin^2\left(\frac{\pi}{2}\right)}{-\frac{\pi}{2}} \quad \frac{1^2}{-\pi/2}$

$\boxed{-\frac{2}{\pi}}$

$0 \cdot \infty$ IP

(19) $\lim_{x \to \infty} e^{-x}(x^3 - x^2 + 9)$

$= \lim_{x \to \infty} \frac{x^3 - x^2 + 9}{e^x} \quad \frac{\infty}{\infty}$

$\underset{H}{=} \lim_{x \to \infty} \frac{3x^2 - 2x}{e^x} \quad \frac{\infty}{\infty}$

$\underset{H}{=} \lim_{x \to \infty} \frac{6x - 2}{e^x} \quad \frac{\infty}{\infty}$

$\underset{H}{=} \lim_{x \to \infty} \left(\frac{6}{e^x}\right) \quad 6e^{-x}$

$\lim_{x \to \infty} 6e^{-x}$

$6 \lim_{x \to \infty} e^{-x}$

$6 \cdot 0$

$\boxed{0}$

$$= \lim_{x \to 4} \left[\frac{x - 4\sqrt{x} + 4}{x\sqrt{x} - 2x - 4\sqrt{x} + 8} \right]$$

$$= \lim_{x \to 4} \left[\frac{x - 4\sqrt{x} + 4}{x^{3/2} - 2x - 4x^{1/2} + 8} \right]$$

$$\underset{H}{=} \lim_{x \to 4} \left[\frac{1 - \frac{4}{2\sqrt{x}}}{\frac{3}{2}x^{1/2} - 2 - 4 \cdot \frac{1}{2} x^{-1/2}} \right]$$

$$= \lim_{x \to 4} \left[\frac{1 - \frac{2}{\sqrt{x}}}{\frac{3}{2}\sqrt{x} - 2 - \frac{2}{\sqrt{x}}} \right] \cdot \frac{\sqrt{x}}{\sqrt{x}}$$

simplify

$$= \lim_{x \to 4} \left[\frac{\sqrt{x} - 2}{\frac{3}{2}x - 2\sqrt{x} - 2} \right] \qquad \frac{0}{0}$$

$$\underset{H}{=} \lim_{x \to 4} \left[\frac{\frac{1}{2\sqrt{x}}}{\frac{3}{2} - \frac{2}{2\sqrt{x}}} \right] = \frac{\frac{1}{2\sqrt{4}}}{\frac{3}{2} - \frac{1}{\sqrt{4}}}$$

$$\frac{\frac{1}{2 \cdot 2}}{\frac{3}{2} - \frac{1}{2}} \qquad \frac{\frac{1}{4}}{1}$$

$$\boxed{\frac{1}{4}}$$

(21) $\lim_{x\to\infty} \frac{e^{2x} - 1 - x}{x^2}$ $\quad \frac{\infty - \infty}{\infty}$ indeterminant

$\lim_{x\to\infty} \frac{e^{2x}}{x^2} - \frac{1}{x^2} - \left(\frac{x}{x^2}\right) \frac{1}{x}$

$\lim_{x\to\infty} \frac{e^{2x}}{x^2} - \frac{1}{x^2} - \frac{1}{x}$

$\lim_{x\to\infty} \frac{e^{2x}}{x^2} - \cancel{\lim_{x\to\infty} \frac{1}{x^2}}^{0} - \cancel{\lim_{x\to\infty} \frac{1}{x}}^{0}$

$\frac{\infty}{\infty}$

$\overset{=}{H} \lim_{x\to\infty} \frac{e^{2x} \cdot 2}{2x} = \lim_{x\to\infty} \frac{e^{2x}}{x} \quad \frac{\infty}{\infty}$

$\overset{=}{H} \lim_{x\to\infty} \frac{e^{2x} \cdot 2}{1} = \lim_{x\to\infty} 2e^{2x}$

$= 2 \lim_{x\to\infty} e^{2x} \quad 2 \cdot \infty$

$\boxed{\infty}$

(23) $\lim_{x\to 4} \left[\frac{1}{\sqrt{x} - 2} - \frac{4}{x - 4}\right] \quad \infty - \infty$

algebra

$= \lim_{x\to 4} \left[\frac{x - 4 - 4(\sqrt{x} - 2)}{(\sqrt{x} - 2)(x - 4)}\right]$

$\lim_{x\to 4} \left[\frac{x - 4\sqrt{x} + 4}{(\sqrt{x} - 2)(x - 4)}\right]$

$\frac{0}{0}$

$= \lim_{x\to 4} \left[\frac{x - 4 - 4\sqrt{x} + 8}{(\sqrt{x} - 2)(x - 4)}\right]$

(25) $\lim_{x\to 0}\left[\frac{1}{\sin x}-\frac{1}{x}\right]$ $\quad \infty-\infty$

algebra

$$= \lim_{x\to 0}\left[\frac{x-\sin x}{x\sin x}\right] \qquad \frac{0-\sin 0}{0\sin 0} \quad \frac{0-0}{0\cdot 0} \quad \frac{0}{0}$$

$$\underset{H}{=} \lim_{x\to 0}\frac{1-\cos x}{x\cos x+\sin x} \qquad \frac{1-\cos 0}{0\cos 0+\sin 0} \quad \frac{1-1}{0\cdot 1+0} \quad \frac{0}{0}$$

$$\underset{H}{=} \lim_{x\to 0}\frac{\sin x}{-x\sin x+\cos x+\cos x} \qquad \frac{0}{-0\cdot\sin 0+\cos 0+\cos 0} \quad \frac{0}{2}$$

$$\boxed{0}$$

(27) $\lim_{x\to 0^+} x^x$ $\quad 0^0$ $\quad$ ± power

let $y = x^x$; $\ln y = \ln(x^x)$; $\ln(y) = x\ln x$

$$\lim_{x\to 0^+}\ln(y) = \lim_{x\to 0^+} x\ln x \qquad 0\cdot -\infty$$

$$y = \lim_{x\to 0^+}\frac{\ln x}{1/x} \qquad \frac{-\infty}{\infty} \quad \pm Q$$

$$\underset{H}{=} \lim_{x\to 0^+}\frac{1/x}{-1/x^2} \qquad \frac{1}{x}\cdot -x^2 \quad -x$$

$$= \lim_{x\to 0^+} -x = 0$$

as $x \to 0^+$; $\ln(y) \to 0$; $e^{\ln(y)} \to e^0 = \boxed{1}$

$y \to \boxed{1}$

(29) $\lim_{x \to \infty} x^{1/x^2}$ $\quad \infty^0$

let $y = x^{\frac{1}{x^2}}$; $\ln y = \ln\left(x^{\frac{1}{x^2}}\right)$

$\ln(y) = \frac{1}{x^2} \ln(x)$

$\lim_{x \to \infty} \ln(y) = \lim_{x \to \infty} \frac{\ln x}{x^2}$ $\quad \frac{\infty}{\infty}$

$\overset{H}{=} \lim_{x \to \infty} \left(\frac{\frac{1}{x}}{2x}\right)$ $\quad \frac{1}{x} \div 2x$

$\frac{1}{2x^2}$

$\lim_{x \to \infty} \frac{1}{2x^2}$

$\frac{1}{2} \lim_{x \to \infty} \frac{1}{x^2}$ $\quad \frac{1}{2} \cdot 0$ $\quad 0$

as $x \to \infty$; $\ln(y) \to 0$; $e^{\ln(y)} \to e^0$

$y \to \boxed{1}$

(31) $\lim_{x \to 0} [\cos(x)]^{3/x^2}$ $\quad 1^\infty$

let $y = [\cos(x)]^{3/x^2}$; $\ln(y) = \ln\left[[\cos(x)]^{3/x^2}\right]$

$\ln(y) = \frac{3}{x^2} \ln[\cos(x)]$

$$\lim_{x\to 0} \frac{3\ \ln[\cos x]}{x^2} \qquad \frac{0}{0}$$

$$\overset{H}{=} \lim_{x\to 0} \frac{3 \cdot \frac{1}{\cos x} - \sin x}{2x}$$

$$= \lim_{x\to 0} - \frac{3\ \tan x}{2x} \qquad \frac{0}{0}$$

$$\overset{H}{=} \lim_{x\to 0} - \frac{3\ \sec^2 x}{2} = -\frac{3}{2} \lim_{x\to 0} \sec^2 x$$

$$= -\frac{3}{2}[\sec 0]^2 = -\frac{3}{2}\left[\frac{1}{\cos 0}\right]^2$$

$$= -\frac{3}{2}$$

as $x \to 0$, $\ln y \to -\frac{3}{2}$

$$e^{\ln y} \to e^{-3/2}$$

$$y \to \boxed{\frac{1}{e^{3/2}}}$$