

Graphing Sine and Cosine with Phase Shift

What about phase shift (Horizontal Shifting)?

$$\begin{aligned}y &= a\sin[k(x - b)] \\y &= a\cos[k(x - b)] \\k &> 0\end{aligned}$$

$$\begin{aligned}\text{Amp} &= |a| \text{ and Period } p = \frac{2\pi}{k} \\ \text{Interval of One Cycle} &= [b, b + p] \\ \text{Phase Shift } &b\end{aligned}$$

Determine the amplitude, period, interval of one cycle, phase shift, x-intercepts, max value, min value, and use this information to sketch the curve.

1. $y = \sin\left(x - \frac{\pi}{3}\right)$

2. $y = \sin\left(x - \frac{\pi}{4}\right)$

3. $y = \cos\left(x + \frac{\pi}{3}\right)$

4. $y = \cos\left(x + \frac{\pi}{4}\right)$

5. $y = \sin\left[2\left(x + \frac{\pi}{2}\right)\right]$

6. $y = \sin\left[3\left(x - \frac{\pi}{4}\right)\right]$

7. $y = 3\cos[2(x - \pi)]$

8. $y = 2\cos[4(x - \pi)]$

9. $y = \frac{3}{2}\sin\left[4\left(x + \frac{\pi}{3}\right)\right]$

10. $y = 5\sin\left[2\left(x + \frac{\pi}{6}\right)\right]$

11. $y = -\cos\left[2x - \frac{\pi}{3}\right]$

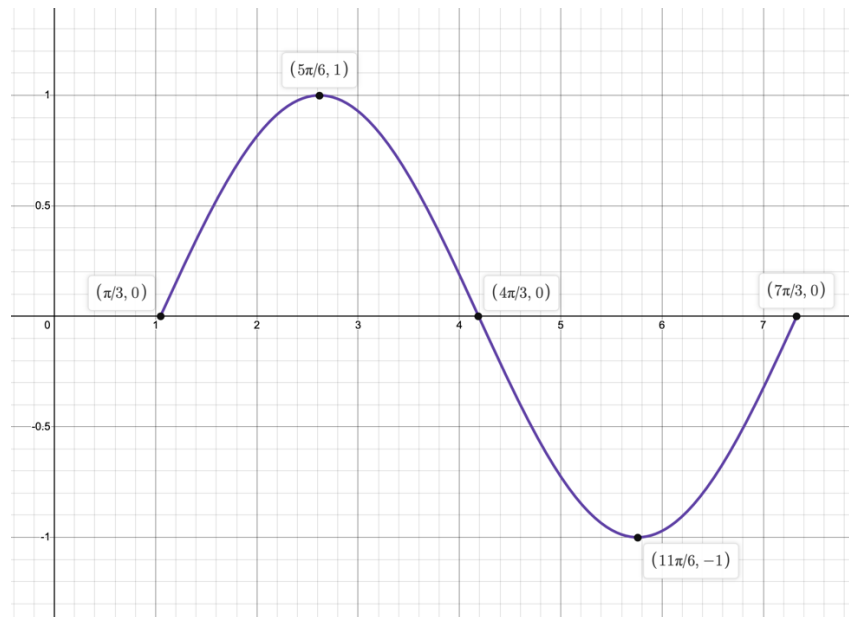
12. $y = -\cos\left[2x - \frac{\pi}{4}\right]$

13. $y = -5\sin\left[3x + \frac{\pi}{2}\right]$

14. $y = -4\sin\left[3x + \frac{\pi}{6}\right]$

Answers

1.

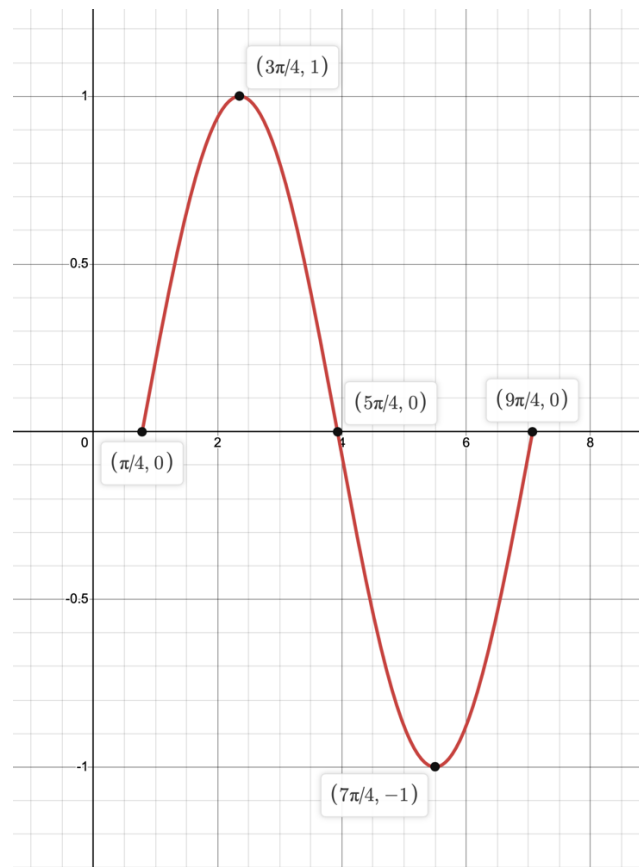


$$\text{amp} = 1, \text{period} = 2\pi, \text{phase shift} = \frac{\pi}{3}, \text{Interval} = \left[\frac{\pi}{3}, \frac{7\pi}{3}\right],$$

$$\text{xint: } x = \frac{\pi}{3}, x = \frac{4\pi}{3}, x = \frac{7\pi}{3}$$

$$\text{min} = -1, \text{max} = 1$$

2.

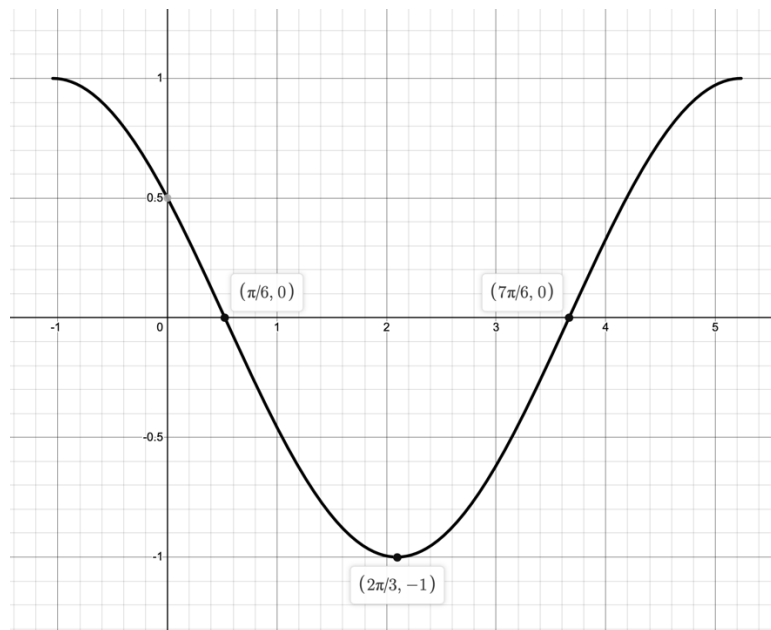


$$amp = 1, period = 2\pi, phase\ shift = \frac{\pi}{4}, Interval = \left[\frac{\pi}{4}, \frac{9\pi}{4}\right]$$

$$xint: x = \frac{\pi}{4}, x = \frac{5\pi}{4}, x = \frac{9\pi}{4}$$

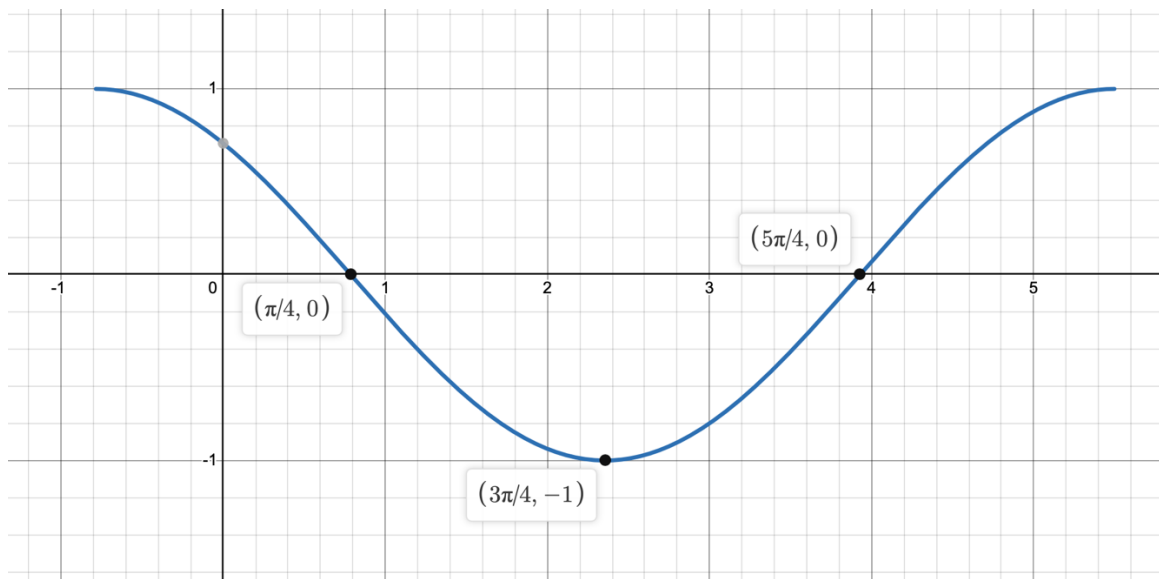
$$min = -1, max = 1$$

3.



$$\begin{aligned} \text{amp} &= 1, \text{period} = 2\pi, \text{phase shift} = -\frac{\pi}{3}, \text{Interval} = \left[-\frac{\pi}{3}, \frac{5\pi}{3}\right], \\ \text{xint: } x &= \frac{\pi}{6}, x = \frac{7\pi}{6} \\ \text{min} &= -1, \text{max} = 1 \end{aligned}$$

4.

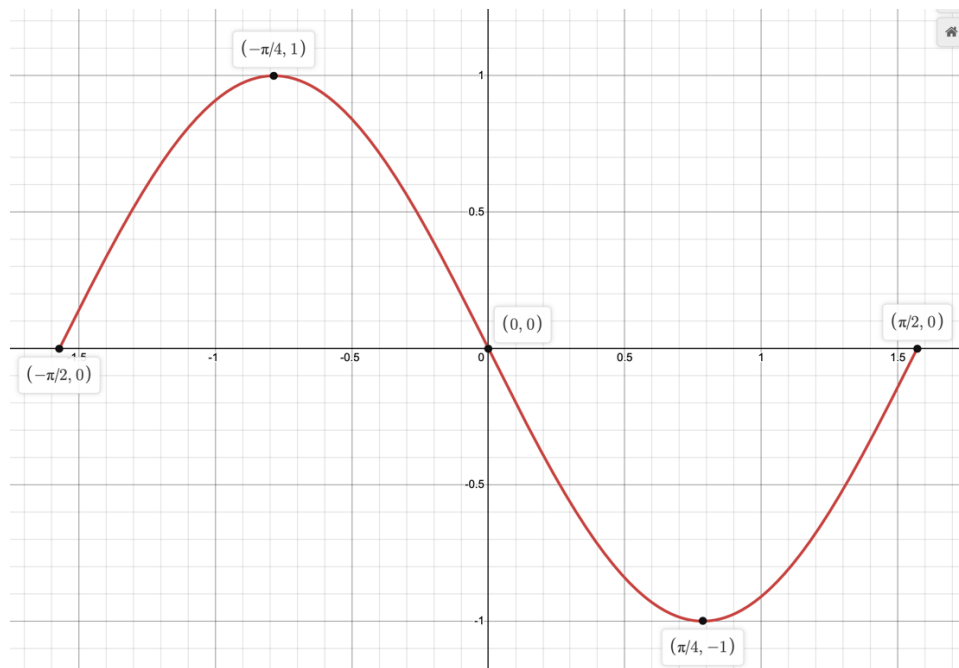


$$\text{amp} = 1, \text{period} = 2\pi, \text{phase shift} = -\frac{\pi}{4}, \text{Interval} = \left[-\frac{\pi}{4}, \frac{7\pi}{4}\right]$$

$$\text{xint: } x = \frac{\pi}{4}, x = \frac{5\pi}{4}$$

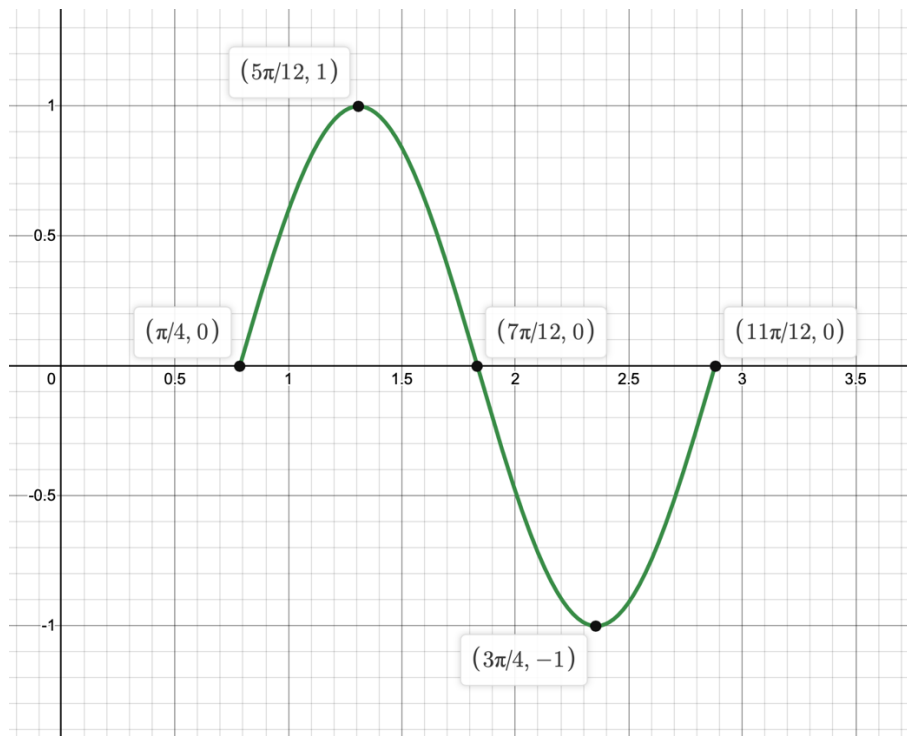
$$\text{min} = -1, \text{max} = 1$$

5.



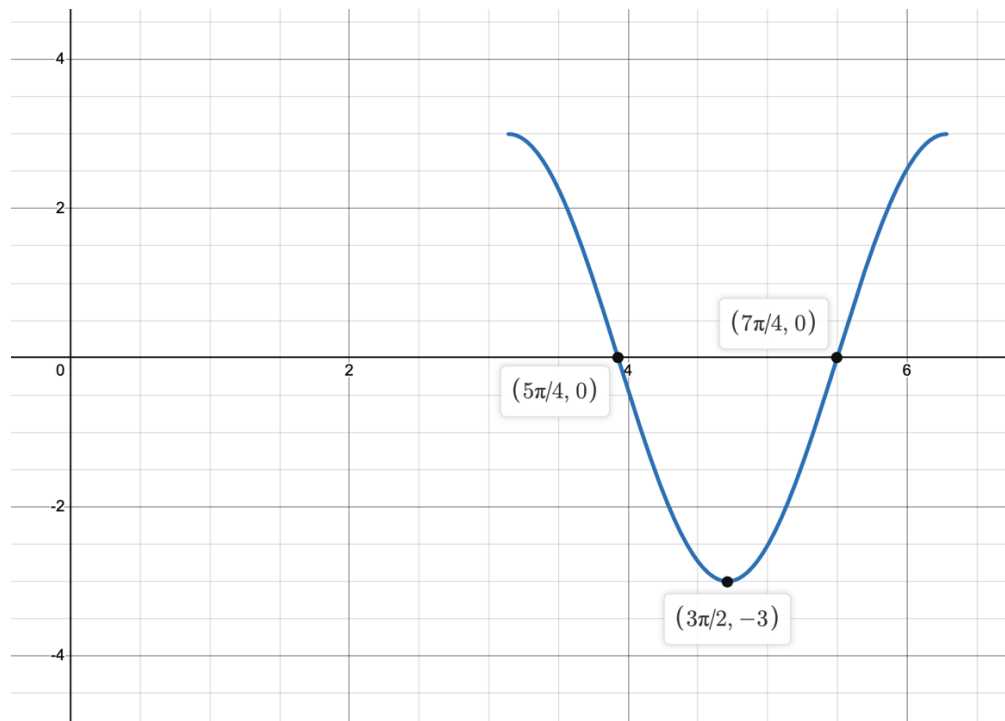
$$\begin{aligned} \text{amp} &= 1, \text{period} = \pi, \text{phase shift} = -\frac{\pi}{2}, \text{Interval} = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \text{xint: } x &= -\frac{\pi}{2}, x = 0, x = \frac{\pi}{2} \\ \text{min} &= -1, \text{max} = 1 \end{aligned}$$

6.



$$\begin{aligned} \text{amp} &= 1, \text{period} = \frac{2\pi}{3}, \text{phase shift} = \frac{\pi}{4}, \text{Interval} = \left[\frac{\pi}{4}, \frac{11\pi}{12}\right], \\ \text{xint: } x &= \frac{\pi}{4}, x = \frac{7\pi}{12}, x = \frac{11\pi}{12} \\ \text{min} &= -1, \text{max} = 1 \end{aligned}$$

7.

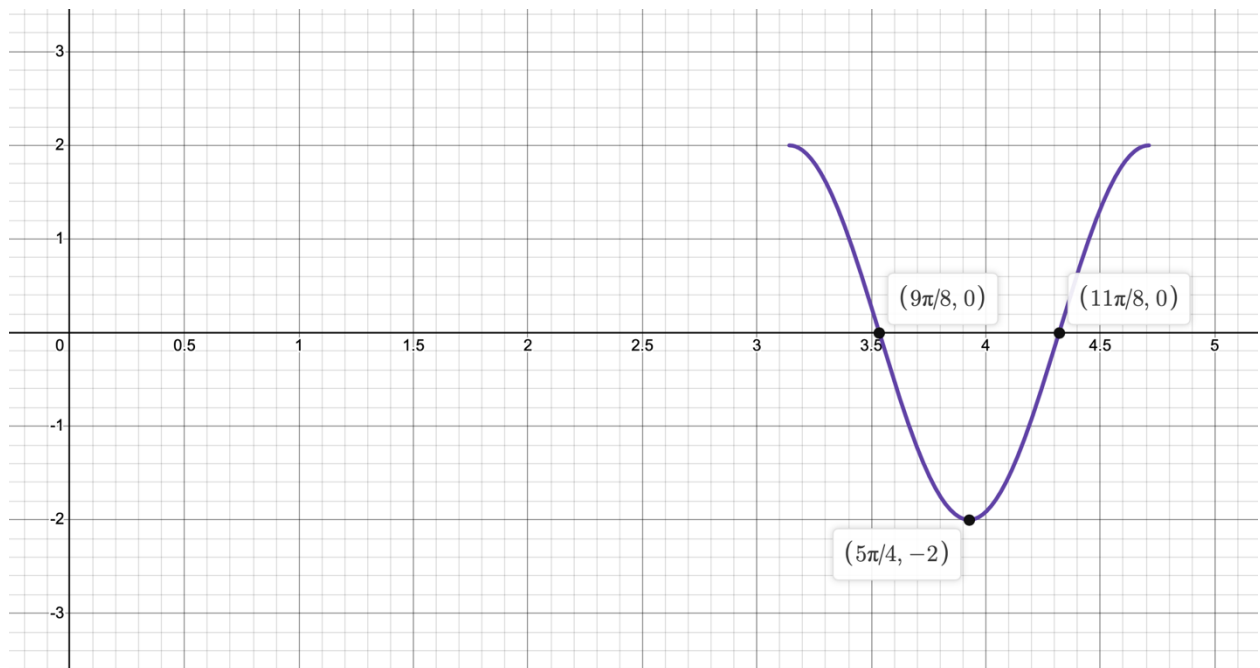


$$amp = 3, period = \pi, phase\ shift = \pi, Interval = [\pi, 2\pi]$$

$$xint: x = \frac{5\pi}{4}, x = \frac{7\pi}{4}$$

$$min = -3, max = 3$$

8.

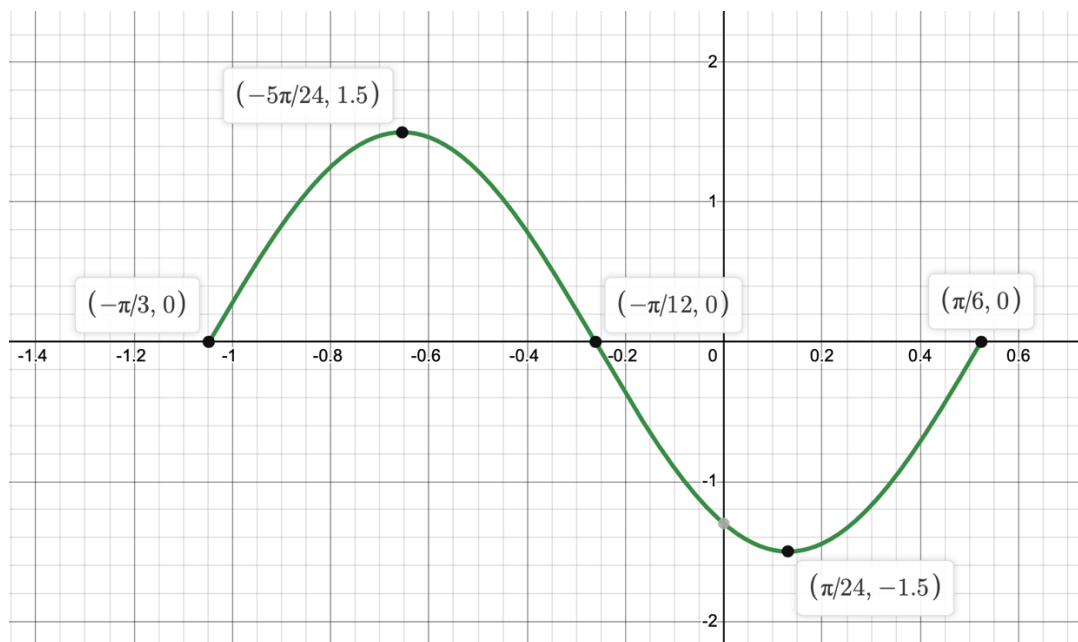


$$amp = 2, period = \frac{\pi}{2}, phase\ shift = \pi, Interval = \left[\pi, \frac{3\pi}{2} \right]$$

$$xint: x = \frac{9\pi}{8}, x = \frac{11\pi}{8}$$

$$min = -2, max = 2$$

9.

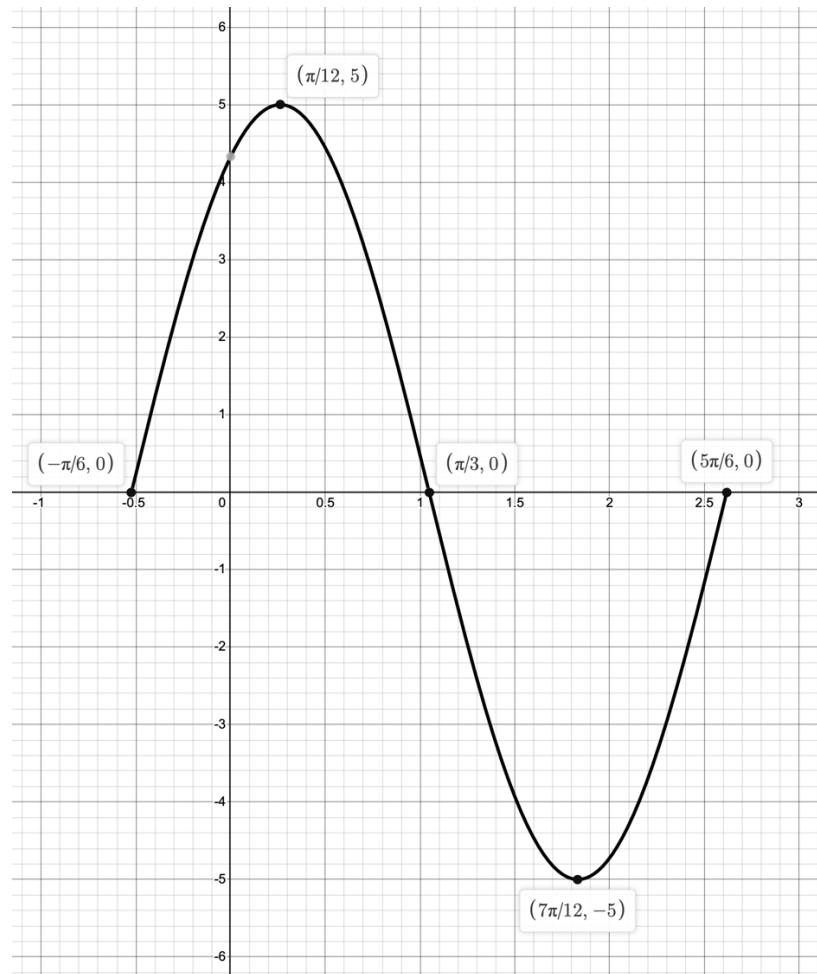


$$\text{amp} = \frac{3}{2}, \text{period} = \frac{\pi}{2}, \text{phase shift} = -\frac{\pi}{3}, \text{Interval} = \left[-\frac{\pi}{3}, \frac{7\pi}{6}\right]$$

$$\text{xint: } x = -\frac{\pi}{3}, x = -\frac{\pi}{12}, x = \frac{\pi}{6}$$

$$\text{min} = -\frac{3}{2}, \text{max} = \frac{3}{2}$$

10.

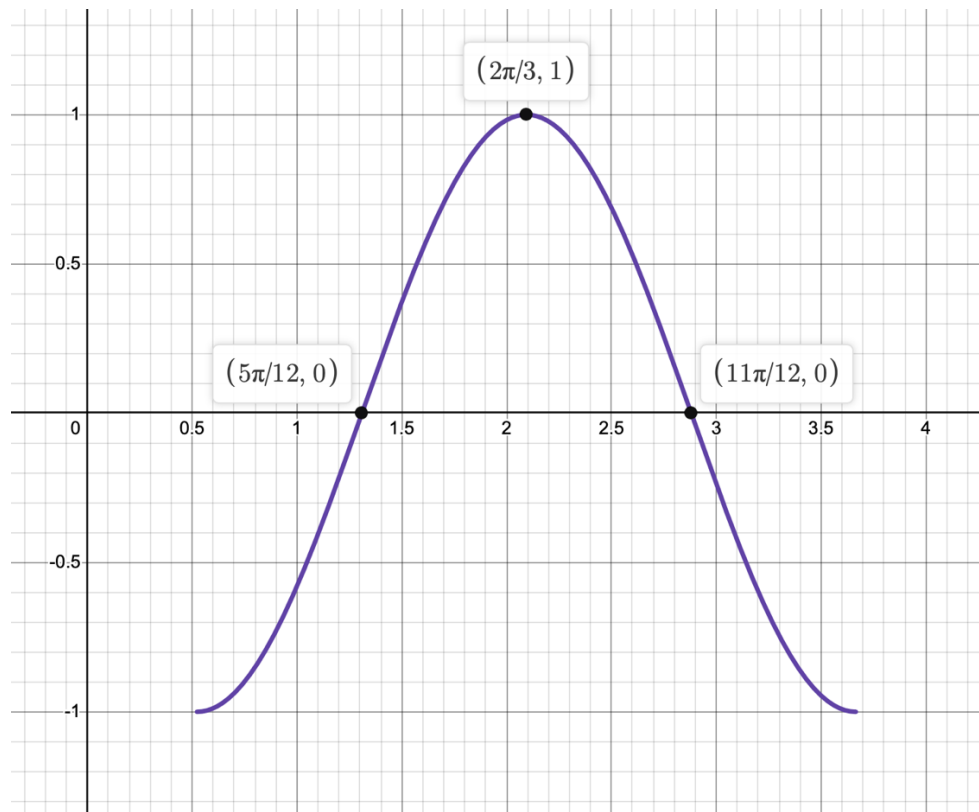


$$\text{amp} = 5, \text{period} = \pi, \text{phase shift} = -\frac{\pi}{6}, \text{Interval} = \left[-\frac{\pi}{6}, \frac{5\pi}{6}\right]$$

$$\text{xint: } x = -\frac{\pi}{6}, x = \frac{\pi}{3}, x = \frac{5\pi}{6}$$

$$\text{min} = -5, \text{max} = 5$$

11.

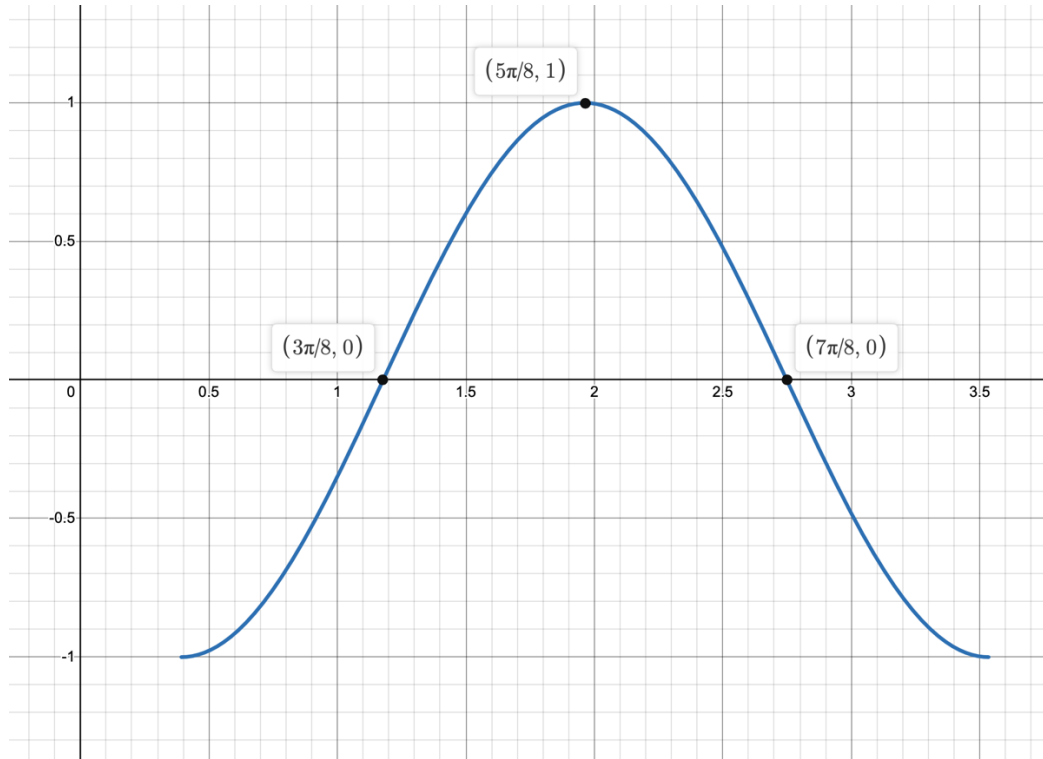


$$amp = 1, period = \pi, phase\ shift = \frac{\pi}{6}, Interval = \left[\frac{\pi}{6}, \frac{7\pi}{6}\right]$$

$$x_{int}: x = \frac{5\pi}{12}, x = \frac{11\pi}{12}$$

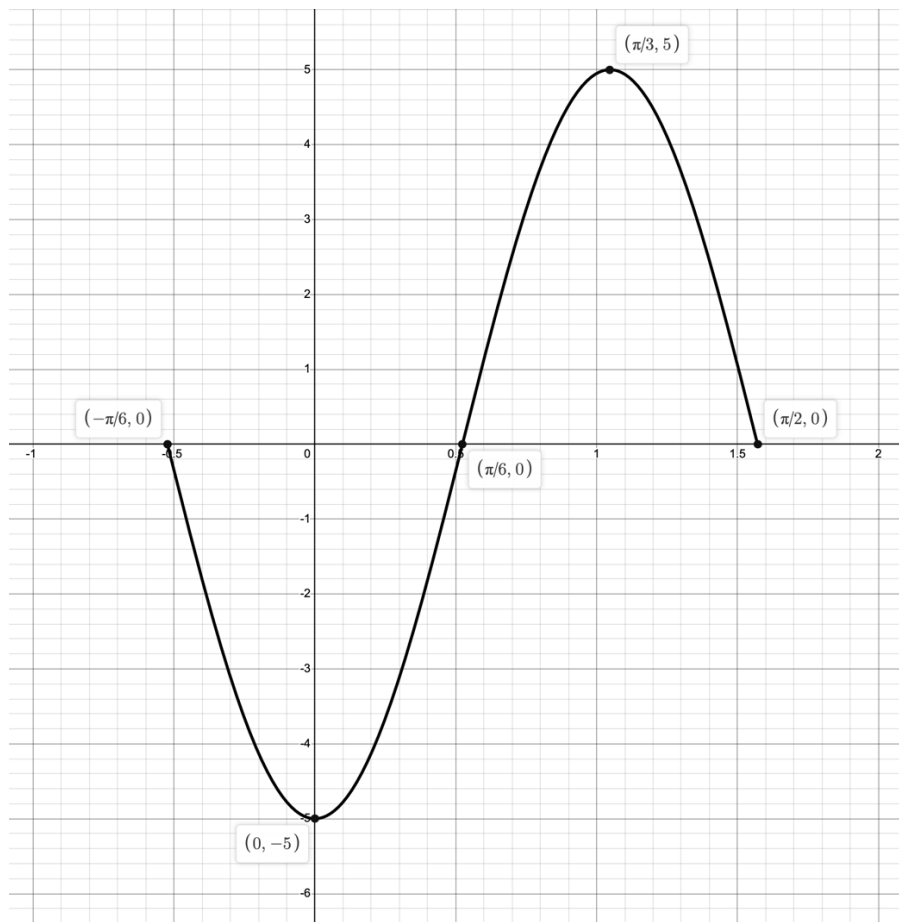
$$min = -1, max = 1$$

12.



$$\begin{aligned} \text{amp} &= 1, \text{period} = \pi, \text{phase shift} = \frac{\pi}{8}, \text{Interval} = \left[\frac{\pi}{8}, \frac{9\pi}{8}\right] \\ \text{xint: } x &= \frac{3\pi}{8}, x = \frac{7\pi}{8} \\ \text{min} &= -1, \text{max} = 1 \end{aligned}$$

13.

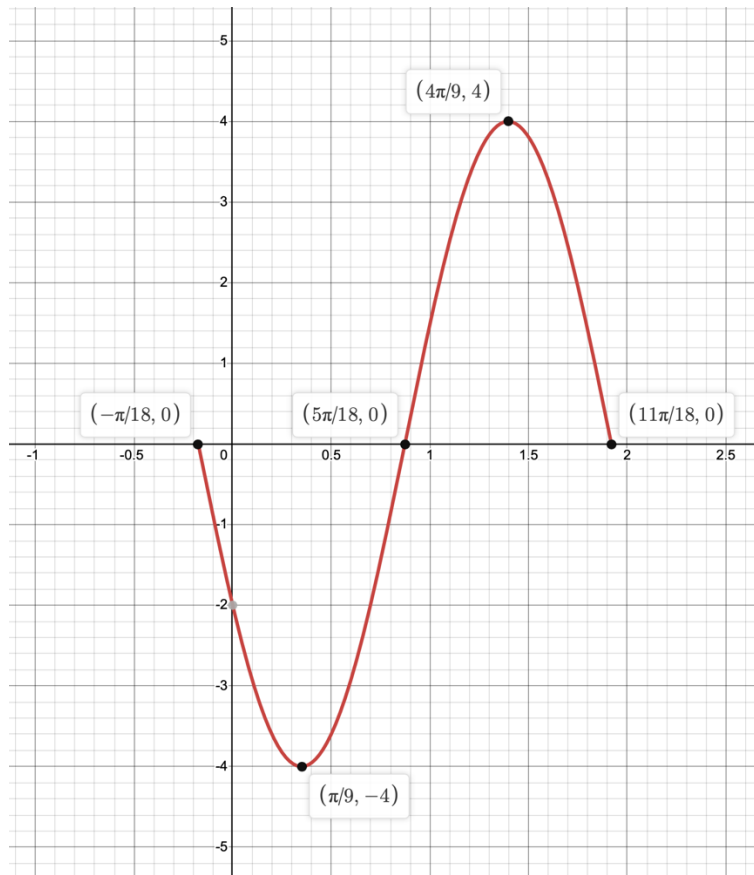


$$\text{amp} = 5, \text{period} = \frac{2\pi}{3}, \text{phase shift} = -\frac{\pi}{6}, \text{Interval} = \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$$

$$x_{\text{int}}: x = -\frac{\pi}{6}, x = \frac{\pi}{6}, x = \frac{\pi}{2}$$

$$\text{min} = -5, \text{max} = 5$$

14.



$$\text{amp} = 4, \text{period} = \frac{2\pi}{3}, \text{phase shift} = -\frac{\pi}{18}, \text{Interval} = \left[-\frac{\pi}{18}, \frac{11\pi}{18}\right],$$

$$x_{\text{int}}: x = -\frac{\pi}{18}, x = \frac{5\pi}{18}, x = \frac{11\pi}{18}$$

$$\text{min} = -4, \text{max} = 4$$