Goodness of Fit

Discrete Random Variables

Does a sample set of data that consists of **observed frequency counts** arranged in a one-way frequency table (row or column) align with the **expected frequency counts** from the known distribution (uniform distribution, normal distribution, or any other distribution). To make that determination, we need to perform a **"Goodness of Fit Test".**

Goodness of Fit Test- Is a Hypothesis Test that and observed frequency distribution fits (conforms) some claimed distribution.

A Goodness of Fit Test is also known as a Chi Square Test of a Distribution.

Consider the experiment that a bag on M and M's has an equal distribution of colors in every package. That is, the distribution of red, orange, yellow, green, blue, and brown is **uniform**. This is a vital requirement for the Goodness of Fit Test. **We need a random variable x of data and an assumed probability distribution.**

We require the following:

- Data is randomly selected.
- Sample data consists of frequency counts for each of the categories of data.
- Every category needs an expected frequency that is at least 5.

We need to Create the Following Hypothesis Test.

$$H_0: p_1 = p_2 = p_3 = p_4 = p_5 = p_6$$ Uniform Distribution as our claim

$$H_1: \text{ At least one } p_i \text{ is not equal.}$$

$$\alpha = 5\%$$

What we are saying is that the assumption in H_0 is that the frequency counts are the same while H_1 indicates that the frequency counts are not the same.

The following is our sample information as well as the expected frequencies. Recall, that $p_i = 1/6$ for all i.

M and M Candies		
Sample Size=900	Observed	Expected
Color	**O**	**E**
Red	134	150
Orange	152	150
Yellow	126	150
Green	168	150
Blue	162	150
Brown	158	150
Total	**900**	**900**

Test Statistic

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

Where **E** is the expected frequency of an outcome found by the assumption of the distribution.

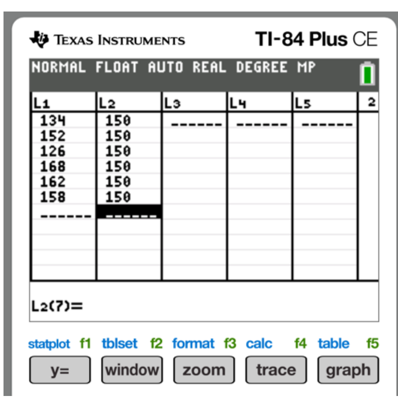
If the expected frequencies are **all equal** as in a uniform distribution, then $\boldsymbol{E = n/k}$

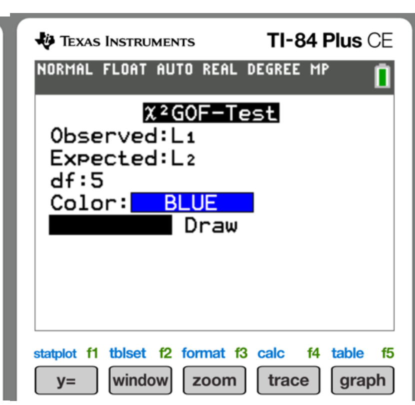
If the expected frequencies are **not all equal**, then $\boldsymbol{E = np_i}$ for all i each group.

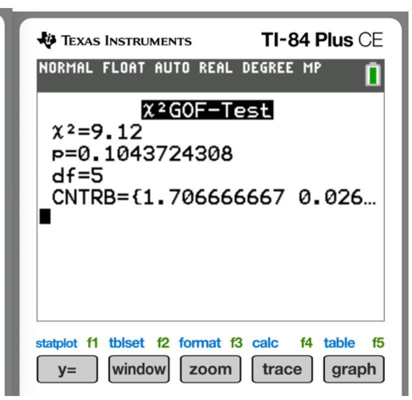
$$\boldsymbol{E = \frac{n}{k} = \frac{800}{6} = 150}$$

M and M Candies					
Sample Size=900	Observed	Expected			
Color	**O**	**E**	**O-E**	**(O-E)^2**	**((O-E)^2)/E**
Red	134	150	-16	256	1.7067
Orange	152	150	2	4	0.0267
Yellow	126	150	-24	576	3.8400
Green	168	150	18	324	2.1600
Blue	162	150	12	144	0.9600
Brown	158	150	8	64	0.4267
Total	**900**	**900**		1368	**9.12**

$$\boldsymbol{\chi^2 \approx 9.12}$$







$\boldsymbol{p \approx 0.104; p \nless \alpha}$; **Accept** $\boldsymbol{H_0}$
The Sample Supports the Claim
The Distribution is Uniform

Critical Values

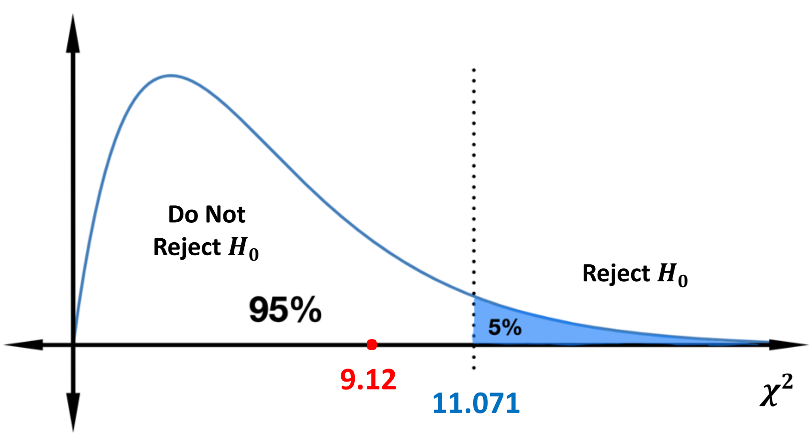
Chi-Square Distribution where $df = k - 1$

k is the number of categories

Always a **Right Tail Test**

Table A-4 Chi-Square (χ^2) Distribution

Degrees of Freedom	Area to the Right of the Critical Value									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169



Conclusion

Do Not Reject the Null

The Distribution is Uniform

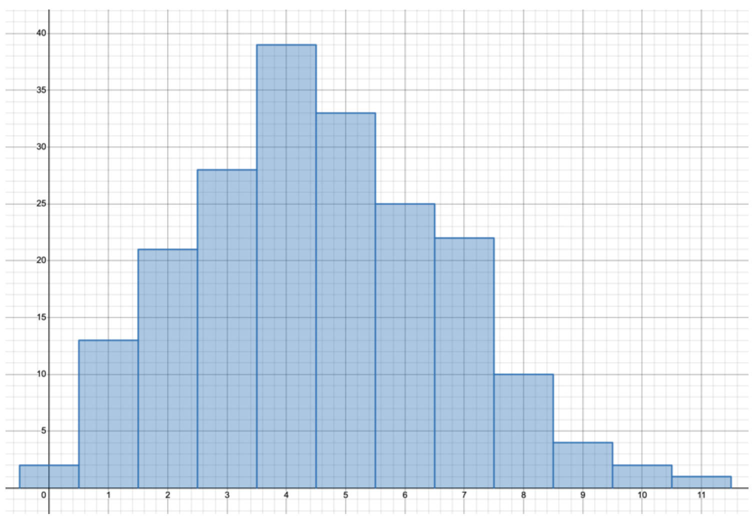
Let us consider another experiment that consists of Poisson Probability Distribution. If we let x represent the number of 911 emergency calls in an hour where it is known that there is 4.6 calls per hour, the distribution will look as follows.

$$x = 0, 1, 2, 3, 4, \dots, 11$$

Let us now assume that the distribution is uniform and that we should see every outcome as equally likely.

$$p_i = \frac{1}{12} \text{ for all } i$$

However, when we randomly collect 200 data values (Poisson Distribution) we see the following histogram. We will conduct a **Goodness of Fit Test** and verify that the distribution is **not Uniform**.



$H_0: p_1 = p_2 = p_3 = p_4 = p_5 = p_6$ **Uniform Distribution as our claim**

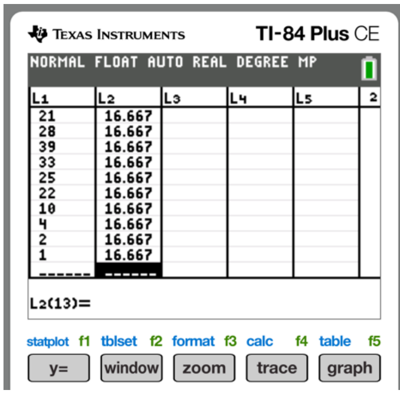
H_1: **At least one p_i is not equal.**

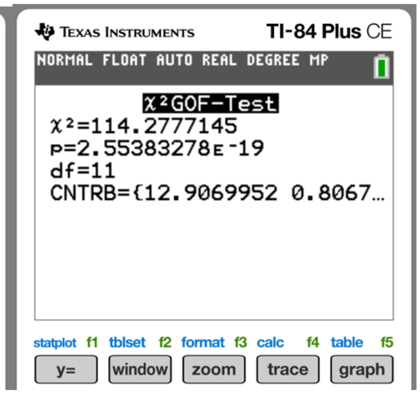
Using the sample information below we can proceed with $E = np_i$ for all i with $n = 200$.

$$\alpha = 5\%$$

n=200	Observed	Expected			
x	O	E	O-E	(O-E)^2	(O-E)^2/E
0	2	16.667	-14.667	215.121	12.907
1	13	16.667	-3.667	13.447	0.807
2	21	16.667	4.333	18.775	1.126
3	28	16.667	11.333	128.437	7.706
4	39	16.667	22.333	498.763	29.925
5	33	16.667	16.333	266.767	16.006
6	25	16.667	8.333	69.439	4.166
7	22	16.667	5.333	28.441	1.706
8	10	16.667	-6.667	44.449	2.667
9	4	16.667	-12.667	160.453	9.627
10	2	16.667	-14.667	215.121	12.907
11	1	16.667	-15.667	245.455	14.727
Total	200	200.004		1904.667	**114.278**

$$\chi^2 \approx 114.278$$





$p \approx 0.000$; $p < \alpha$; H_0 is too low, has to go!

The Sample Does Not Support the Claim

The Distribution is not as Claimed

Critical Values

Chi-Square Distribution where $df = k - 1$

k is the number of categories

Always a **Right Tail Test**

Table A-4 Chi-Square (χ^2) Distribution

Degrees of Freedom	Area to the Right of the Critical Value									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
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13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
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16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
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19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
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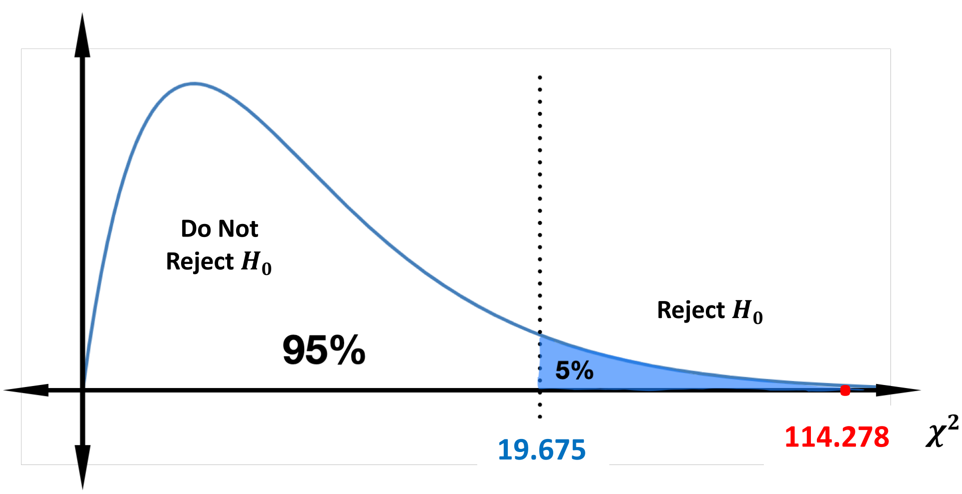
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169



Conclusion

Reject the Null

The Distribution is not as Claimed

Let us revisit the Poisson Distribution of a mean 4.6 office hour arrivals in an hour. We will use a **Goodness to Fit Test** with the **assumption that the distribution is a Poisson Distribution**. Let's see how Goodness to Fit addresses works in the case where each group has assigned probabilities using the **5% level of significance**.

$$x = 0, 1, 2, 3, 4, \dots, 11$$

x	p(x)
0	0.010
1	0.046
2	0.106
3	0.163
4	0.188
5	0.173
6	0.132
7	0.087
8	0.050
9	0.026
10	0.012
11	0.005
12	0.002
Total	**0.999**

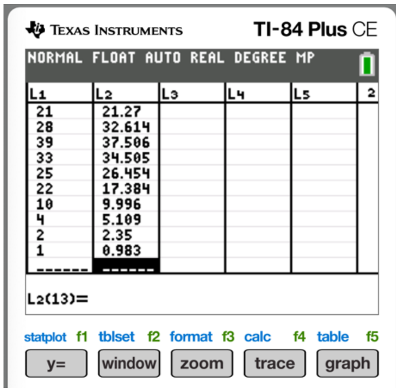
$$H_0: p_1 = 0.010,\ p_2 = 0.046,\ p_3 = 0.163,\ p_4 = 0.188,\ p_5 = 0.173,\ p_6 = 0.132, p_7 = 0.087, p_8 = 0.050, p_9 = 0.026, p_{10} = 0.012, p_{11} = 0.005, p_{12} = 0.002$$ Claim

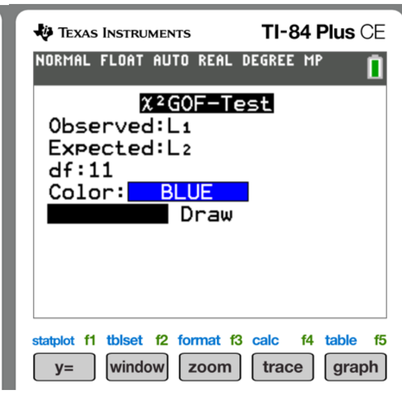
H_1: **At least one p_i is not equal to what is in the claim.**

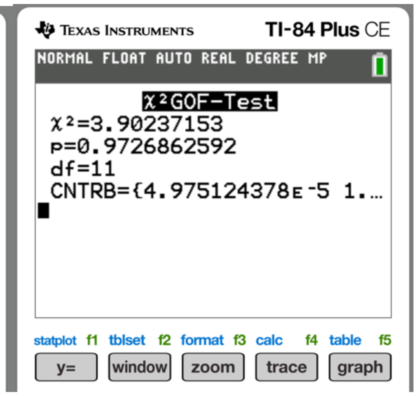
Using the original sample information below we can proceed with $E = np_i$ for all i with $n = 200$.

n=200	Observed	Expected			
x	**O**	**E**	**O-E**	**(O-E)^2**	**(O-E)^2/E**
0	2	2.010	-0.010	0.000	0.000
1	13	9.248	3.752	14.080	1.523
2	21	21.270	-0.270	0.073	0.003
3	28	32.614	-4.614	21.285	0.653
4	39	37.506	1.494	2.233	0.060
5	33	34.505	-1.505	2.265	0.066
6	25	26.454	-1.454	2.114	0.080
7	22	17.384	4.616	21.307	1.226
8	10	9.996	0.004	0.000	0.000
9	4	5.109	-1.109	1.230	0.241
10	2	2.350	-0.350	0.123	0.052
11	1	0.983	0.017	0.000	0.000
Total	200	199.4274733		64.710	**3.903**

$$\chi^2 \approx 3.903$$







$p \approx 0.973; p \nless \alpha$; **Accept** H_0

The Sample Supports the Claim

The Distribution is as Claimed

Critical Values

Chi-Square Distribution where $df = k - 1$

k is the number of categories

Always a **Right Tail Test**

TABLE A-4 Chi-Square (χ^2) Distribution

Degrees of Freedom	Area to the Right of the Critical Value									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
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2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
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8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
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17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
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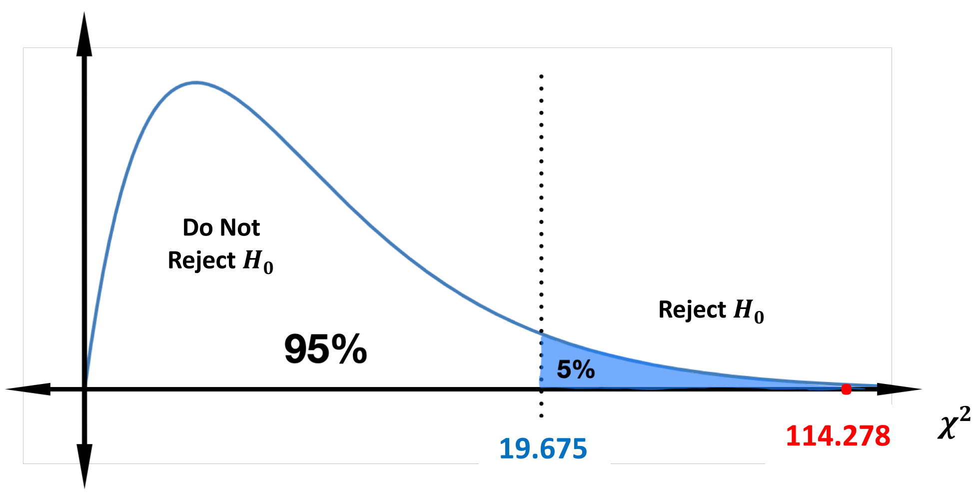
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169



Conclusion

Reject The Null

The Distribution is not as Claimed.

Police Calls

St. Vegas released the number of Police Calls for every day of the week in the month of October for 31 days. Monday (128), Tuesday (158), Wednesday (172), Thursday (176), Friday (382), Saturday (398), Sunday (348).
Use the 1% **level of significance** to test the **claim that the distribution is not Uniform.**

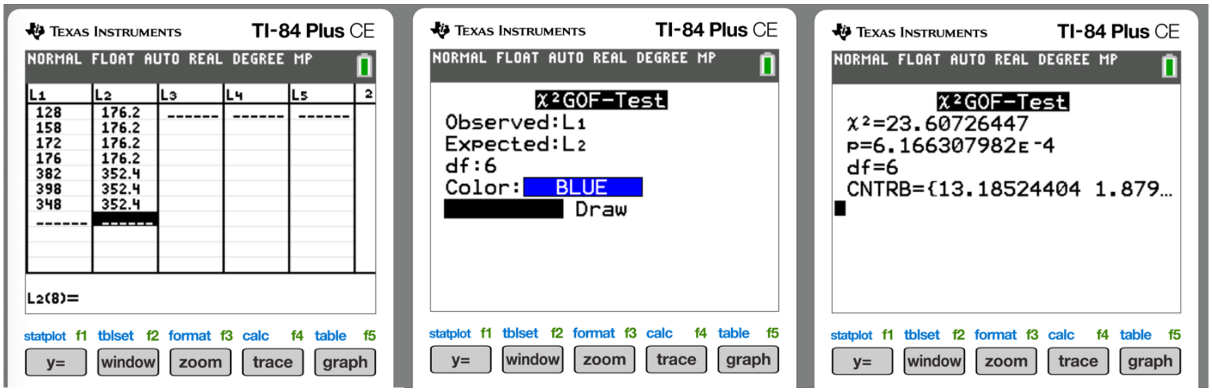
$$H_0: p_1 = 0.1,\ p_2 = 0.1,\ p_3 = 0.1,\ p_4 = 0.1,\ p_5 = 0.2,\ p_6 = 0.2,\ p_7 = 0.2$$ **Claim**

H_1: **At least one p_i is not equal to what is in the claim.**

Using the original sample information below we can proceed with $E = np_i$ for all i with $n = 1762$

x	p(x)	O	E	O-E	(O-E)^2	(O-E)^2/E
Monday	0.1	128	176.2	-48.2	2323.24	13.185
Tuesday	0.1	158	176.2	-18.2	331.24	1.880
Wednesday	0.1	172	176.2	-4.2	17.64	0.100
Thursday	0.1	176	176.2	-0.2	0.04	0.000
Friday	0.2	382	352.4	29.6	876.16	2.486
Saturday	0.2	398	352.4	45.6	2079.36	5.901
Sunday	0.2	348	352.4	-4.4	19.36	0.055
Total	**1**	**1762**				**23.607**
		Sample Size				

$$\chi^2 \approx 23.607$$



$p \approx 0.000; p < \alpha;$ p value is low, H_0 has to go!

The Sample Does Not Support the Claim

The Distribution is not as Claimed

Critical Values

Chi-Square Distribution where $df = k - 1$

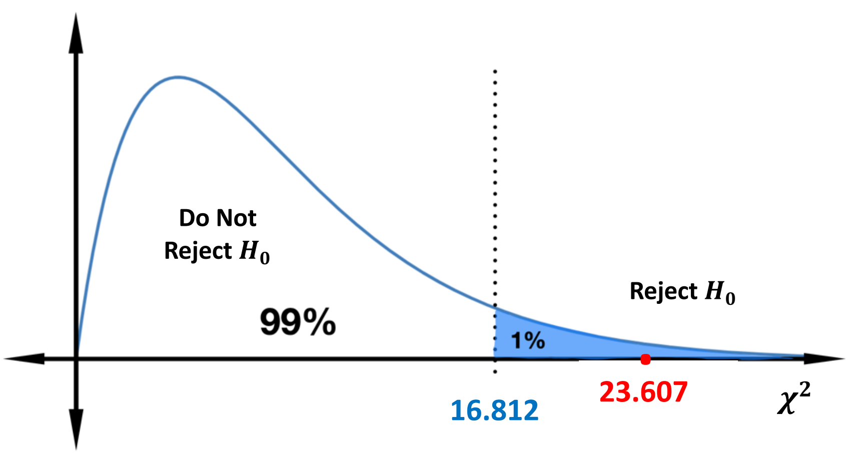
k is the number of categories

Always a **Right Tail Test**

$\alpha = 1\%$ **and** $df = 6$

TABLE A-4 Chi-Square (χ^2) Distribution

Degrees of Freedom	Area to the Right of the Critical Value									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169



Conclusion

Reject the Null

The Distribution is not as Claimed.

World Series Games

The table below illustrates the number of games played in 105 Major League Baseball World Series. The table also includes the expected proportions for the number of games played in the MLB World Series, assuming that in each series each team has the same chance of winning. Use the **1% level of significance** to test the claim that the actual number of games fit the distribution indicated by the expected proportions.

Games Played	World Series Contests	Expected Proportion
4	21	0.125
5	23	0.25
6	23	0.3125
7	38	0.3125
Total	105	1

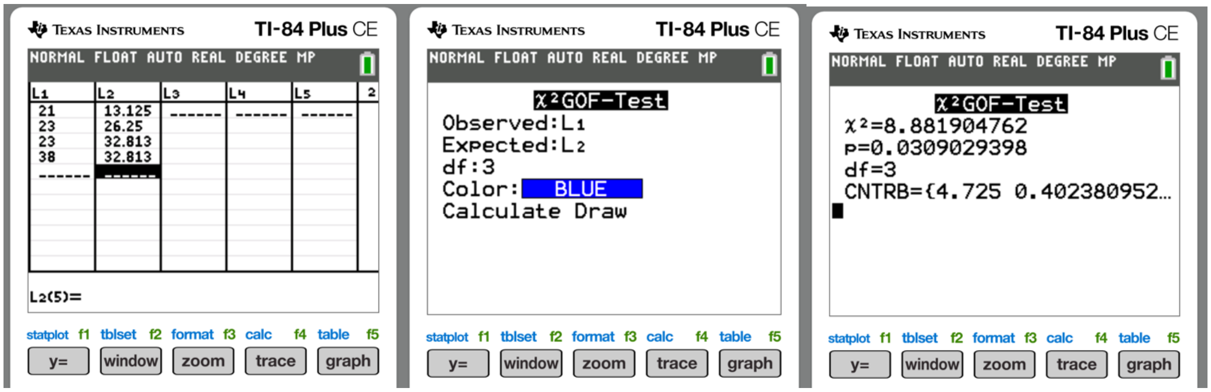
$$\boldsymbol{H_0: p_1 = 0.125,\ p_2 = 0.25,\ p_3 = 0.3125,\ p_4 = 0.3125}$$ **Claim**

H_1: At least one p_i is not equal to what is in the claim.

Using the original sample information below we can proceed with $\boldsymbol{E = np_i}$ for all i with $\boldsymbol{n = 105}$

Games Played	O	Expected Proportion	E	O-E	(O-E)^2	(O-E)^2/E
4	21	0.125	13.125	7.875	62.0156	4.7250
5	23	0.25	26.25	-3.25	10.5625	0.4024
6	23	0.3125	32.8125	-9.8125	96.2852	2.9344
7	38	0.3125	32.8125	5.1875	26.9102	0.8201
Total	**105**	**1**				**8.8819**

$$\chi^2 \approx \mathbf{8.882}$$



$p \approx 0.031$; $p \nless \alpha$; Accept H_0

The Sample Supports the Claim

The Distribution is as Claimed

Critical Values

Chi-Square Distribution where $df = k - 1$

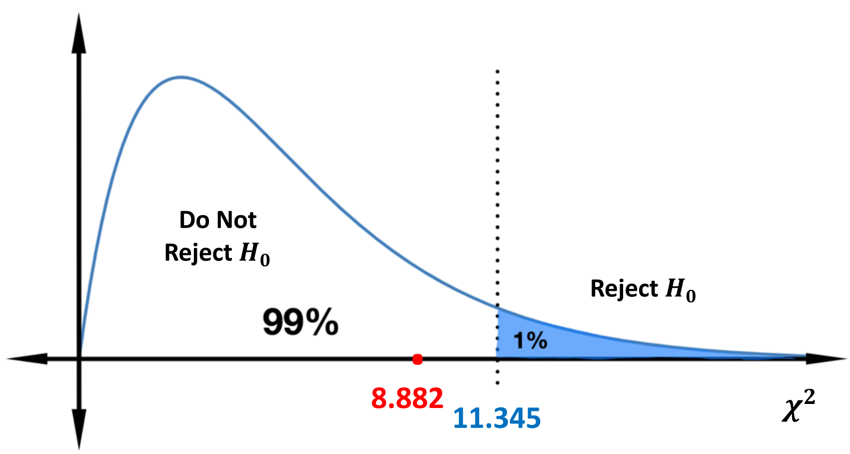
k is the number of categories

Always a **Right Tail Test**

$\alpha = 1\%$ **and** $df = 3$

TABLE A-4 Chi-Square (χ^2) Distribution

Degrees of Freedom	Area to the Right of the Critical Value									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169



Conclusion

Do Not Reject the Null

The Distribution is as Claimed.

IQ Scores

We know **IQ Scores are Normally Distributed** ($\mu = 100; \sigma = 15$) and the "Goodness of Fit Test" applies to discrete random variables. The Normal Distribution is a continuous random variable that can be made to approximate a discrete random variable when we look at the outcomes over mutually exclusive (non-intersecting) intervals. Consider the following example where we shall consider the **1% level of significance** to Test the Claim **IQ Scores** fit the distribution by expected proportions.

	Expected Proportion
x	**p(x)**
Less than 70	0.023
Between 70 and 79.99	0.068
Between 80 and 89.99	0.161
Between 90 and 99.99	0.247
Between 100 and 109.99	0.247
Between 110 and 119.9	0.161
Between 120 and 129.9	0.068
At least than 130	0.023
Total	**0.998**

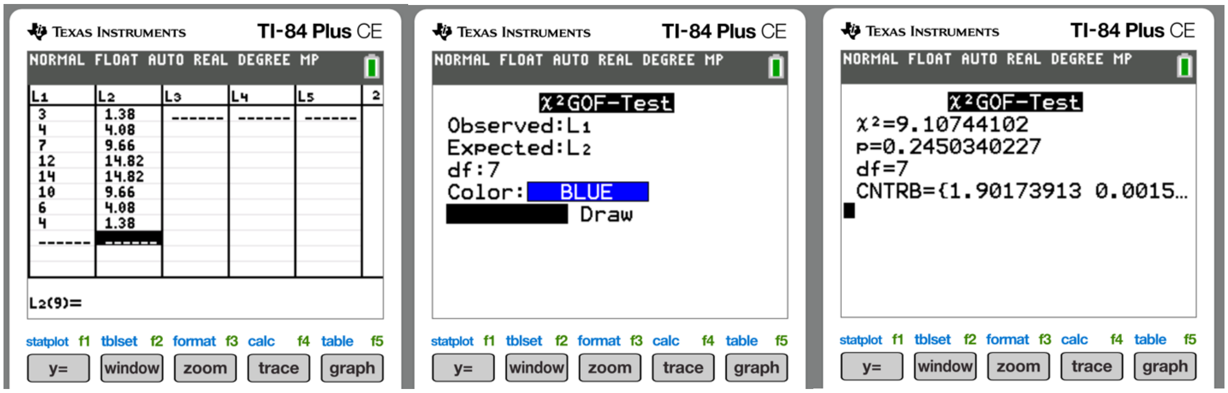
$$H_0: p_1 = 0.023, p_2 = 0.068,\ p_3 = 0.161,\ p_4 = 0.247$$
$$p_5 = 0.247,\ p_6 = 0.161,\ p_7 = 0.068,\ p_8 = 0.023$$ **Claim**

H_1: **At least one p_i is not equal to what is in the claim.**

Using the original sample information below we can proceed with $E = np_i$ for all i with $n = 60$

	Expected Proportion					
x	**p(x)**	**O**	**E**	**O-E**	**(O-E)^2**	**(O-E)^2/E**
Less than 70	0.023	3	1.380	1.620	2.624	1.902
Between 70 and 79.99	0.068	4	4.080	-0.080	0.006	0.002
Between 80 and 89.99	0.161	7	9.660	-2.660	7.076	0.732
Between 90 and 99.99	0.247	12	14.820	-2.820	7.952	0.537
Between 100 and 109.99	0.247	14	14.820	-0.820	0.672	0.045
Between 110 and 119.9	0.161	10	9.660	0.340	0.116	0.012
Between 120 and 129.9	0.068	6	4.080	1.920	3.686	0.904
At least than 130	0.023	4	1.380	2.620	6.864	4.974
Total	**0.998**	60	60			**9.107**

$$\chi^2 \approx 9.107$$



$p \approx 0.245; p \nless \alpha$; **Accept** H_0

The Sample Supports the Claim

The Distribution is as Claimed

Critical Values

Chi-Square Distribution where $df = k - 1$

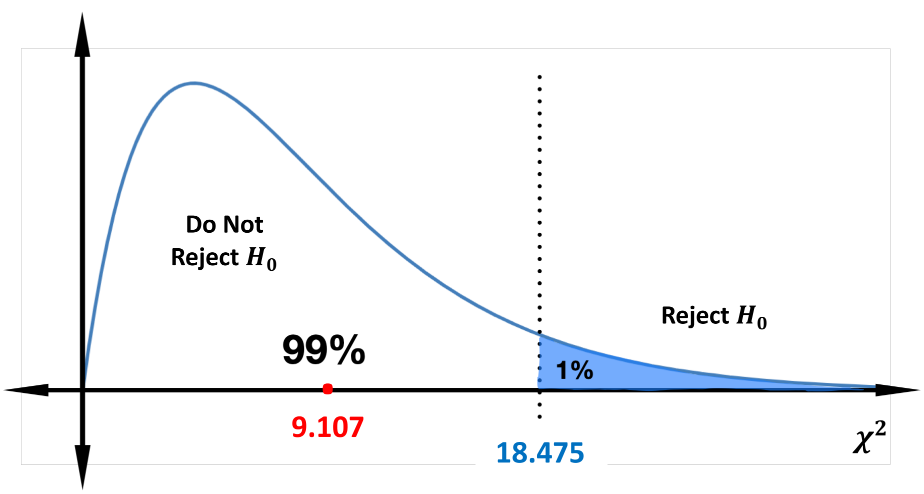
k is the number of categories

Always a **Right Tail Test**

$\alpha = 1\%$ **and** $df = 7$

TABLE A-4 Chi-Square (χ^2) Distribution

Degrees of Freedom	Area to the Right of the Critical Value									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169



Conclusion

Do Not Reject the Null

The Distribution is as Claimed