Answer Sheet

1	-3	12	22
2	1	13	Speeding up
3	$6t^2 - 14t + 4$	14	$0, -10$
4	4	15	See Scratch
5	16	16	$m_{tan} = 60$ $y = 60x + 14$
6	Right	17	$y' = 2\cos(2x) - 2\sin(2x)$ $\pi/8,\ 5\pi/8,\ 9\pi/8,\ 13\pi/8$
7	16	18	$m_{tan} = 0$; $y = \frac{\sqrt{2}}{2}$
8	$t = 1/3,\ t = 2$	19	$y' = \dfrac{\cos(x)}{3\sqrt[3]{(4+\sin x)^2}}$
9	$[0, 1/3) \cup (2, \infty)$	20	$y' = \dfrac{3x+6}{\sqrt{x+3}}$
10	$(\frac{1}{3}, 2)$	21	$y' = -\dfrac{x+4}{2\sqrt{x}\,(x-4)^2}$
11	$12t - 14$	22	$y' = \dfrac{\sec(\sqrt{x})\tan(\sqrt{x})}{2\sqrt{x}}$

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East Los Angeles College
Department of Mathematics
Math 261
Test 2 Study Guide

Show Work for Credit

Let $s(t) = 2t^3 - 7t^2 + 4t + 1$ be a position function measured in meters where t is measured in seconds and $t \geq 0$

1. Determine the average velocity over the interval [1,2]
2. Determine the initial position.
3. Determine the velocity function.
4 Determine the initial velocity.
5. Determine the velocity at t=3 seconds.
6. Determine the direction of travel at t=3 seconds.
7. Determine the speed at t=3 seconds.
8. At what time t does the particle stop?
9. For what time interval t is the particle moving to the right?
10. For what time interval t is the particle moving to the left?
11. Determine the acceleration function.
12. What is the acceleration at t=3 seconds?
13. Is the particle speeding up or slowing down at t=3 seconds?
14. Determine the acceleration of the particle when velocity is 0.

15. Show that $f(x) = |x + 4|$ is not differentiable at $x = -4$

16. Determine the equation of the line tangent to the curve at the indicated point.

$$f(x) = 4(3x + 1)^5 \text{ at } (0,4)$$

17. Determine the points of horizontal tangents for $y = \cos(2x) + \sin(2x)$ over $0 \leq x \leq 2\pi$

18. Use implicit differentiation to find the equation of the line tangent to the curve at the indicated point.

$$x^2 + 2y^2 = 1 \text{ at } \left(0, \frac{\sqrt{2}}{2}\right)$$

Differentiate the following functions.

19. $f(x) = \sqrt[3]{4 + \sin(x)}$

20. $f(x) = 2x\sqrt{x + 3}$

21. $f(x) = \frac{\sqrt{x}}{x-6}$

22. $f(x) = \sec(\sqrt{x})$

math 261 Test 2

(1) $s(t) = 2t^3 - 7t^2 + 4t + 1$

$$\underset{[1,2]}{\text{avg}} = \frac{s(2) - s(1)}{2-1}$$

$$= \frac{(2\cdot 2^3 - 7\cdot 2^2 + 4\cdot 2 + 1) - (2\cdot 1^3 - 7\cdot 1^2 + 4\cdot 1 + 1)}{2-1}$$

$$= \frac{2\cdot 8 - 7\cdot 4 + 8 + 1 - (2 - 7 + 4 + 1)}{1}$$

$$= 16 - 28 + 9 - 0$$

$$\boxed{\text{avg} = -3 \text{ m/sec}}$$ ✓

(2) $s(0) = 2\cdot 0^3 - 7\cdot 0^2 + 4\cdot 0 + 1$

$$\boxed{s(0) = 1 \text{ m}}$$ ✓

(3) $s'(t) = \frac{ds}{dt} = \frac{d}{dt}(2t^3 - 7t^2 + 4t + 1)$

$$\boxed{v(t) = 6t^2 - 14t + 4}$$ ✓ ✓ ✓

(4) $v(0) = 6\cdot 0^2 - 14\cdot 0 + 4$

$$\boxed{v(0) = 4 \text{ m/sec}}$$ ✓

(5) $v(3) = 6 \cdot 3^2 - 14 \cdot 3 + 4$

$= 6 \cdot 9 - 42 + 4$

$= 54 - 42 + 4$

$\boxed{v(3) = 16 \text{ m/sec}}$ ✓

(6) Right ✓

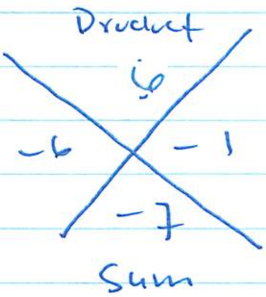
(7) Speed $= |v(3)| = |16|$

$\boxed{\text{Speed} = 16 \text{ m/sec}}$ ✓

(8) $v(t) = 0$; $6t^2 - 14t + 4 = 0$

$2(3t^2 - 7t + 2) = 0$

$3t^2 - 7t + 2 = 0$



$(t - \frac{1}{3})(t - \frac{6}{3}) = 0$

$(3t - 1)(t - 2) = 0$

✓	✓
$3t - 1 = 0$	$t - 2 = 0$
$3t = 1$	$t = 2$
$t = 1/3$	

$s'(t)$ sign analysis

$$s'(t) = 6(3t-1)(t-2)$$

I | II | III

$+ \; + \; 0 \; - \; - \; - \; 0 \; + \; + \; +$ $\qquad s'(t)$

$0 \quad 1/4 \quad 1/3 \quad 1 \quad 2 \quad 3$

(9) Right ; $[0, 1/3) \cup (2, \infty)$

(10) left ; $(1/3, 2)$

(11) $v(t) = 6t^2 - 14t + 4$

$$a(t) = v'(t) = \frac{d}{dt}(6t^2 - 14t + 4)$$

$$\boxed{a(t) = 12t - 14}$$

(12) $a(3) = 12 \cdot 3 - 14$
$= 36 - 14$; $\boxed{a(3) = 22 \text{ m/sec}^2}$

(13) Speeding up as the particle is moving to the right and $a > 0$.
ie, booth signs are +

(14) $a(1/3) = 12 \cdot \frac{1}{3} - 14 = \boxed{-10 \text{ m/sec}^2}$

$a(2) = 12 \cdot 2 - 14 = \boxed{0 \text{ m/sec}^2}$

(16) $f(x) = 4(3x+1)^3$ at $(0, 4)$

$$m_{tan} = f'(0)$$

$$= 12(3x+1)^2 \cdot 3$$

$$= 36(3x+1)^2 \Big|_{x=0}$$

$$= 36 \cdot (3 \cdot 0 + 1)^2$$

$$= \boxed{36}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 36(x - 0)$$

$$y - 4 = 36x$$

$$\boxed{y = 36x + 4}$$

(17) $y = \cos(2x) + \sin(2x)$

$$y' = \frac{d}{dx}\left[\cos(2x) + \sin(2x)\right]$$

$$= \frac{d}{dx}\left[\cos(2x)\right] + \frac{d}{dx}\left[\sin(2x)\right]$$

$$= -\sin(2x)\,\cancel{\frac{d}{dx}(2x)}^{\,2} + \cos(2x)\,\cancel{\frac{d}{dx}(2x)}^{\,2}$$

$$\boxed{y' = -2\sin(2x) + 2\cos(2x)}$$

$y' = 0$; $-2\sin(2x) + 2\cos(2x) = 0$

$\to \cancel{2}\cos(2x) = 2\sin(2x)$

$\to \tan(2x) = 1$

ie, $\tan(\underset{2x}{\theta}) = 1$ or $\underset{2x}{\theta} = \frac{\pi}{4} + n\pi$

so, $2x = \frac{\pi}{4} + n\pi$

$$\boxed{x = \frac{\pi}{8} + \frac{n\pi}{2}}$$

$n=0$; $x = \pi/8$ ✓ ; $y = \cos(2\cdot\frac{\pi}{8}) + \sin(2\cdot\frac{\pi}{8})$

$n=1$; $x = 5\pi/8$ ✓

$n=2$; $x = 9\pi/8$ ✓ $= \cos(\frac{\pi}{4}) + \sin(\pi/4)$

$n=3$; $x = 13\pi/8$ ✓

✓

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}} = \sqrt{2}$

(18) $x^2 + 2y^2 = 1$ at $(0, \sqrt{2}/2)$

$\frac{d}{dx}(x^2 + 2y^2) = \frac{d}{dx}(1)$

$\frac{d}{dx}(x^2) + 2\frac{d}{dx}(y^2) = 0$

$2x + 2\cdot 2y\cdot y' = 0$

(15) $f(x) = |x+4|$

$$\lim_{x \to -4} \frac{f(x) - f(-4)}{x - -4} = \lim_{x \to -4} \frac{|x+4| - |-4+4|}{x+4}$$

$$= \lim_{x \to -4} \frac{|x+4|}{x+4} = \underline{|\text{DNE}|}$$

as $\lim_{x \to -4^-} \frac{|x+4|}{x+4} = \lim_{x \to -4^-} (-1) = -1$

and $\lim_{x \to -4^+} \frac{|x+4|}{x+4} = \lim_{x \to -4^+} (1) = 1$

$$2x + 4y \cdot y' = 0$$

$$4y \cdot y' = -2x \quad ; \quad y' = -\frac{2x}{4y}$$

$$y' = -\frac{x}{2y} \quad ; \quad m_{tan} = -\frac{x}{2y}\Bigg|_{(0,\, \sqrt{2}/2)}$$

$$m_{tan} = -\frac{0}{2\sqrt{2}/2} = -\frac{0}{\sqrt{2}}$$

$$m_{tan} = 0$$

$$y - \frac{\sqrt{2}}{2} = 0(x - 0) \quad ;$$

$$\boxed{y = \frac{\sqrt{2}}{2}}$$

(14) $$y = \sqrt[3]{4 + \sin(x)} = (4 + \sin(x))^{1/3}$$

$$y' = \frac{1}{3}(4 + \sin(x))^{\frac{1}{3}-1} \frac{d}{dx}(4 + \sin(x))$$

$$= \frac{1}{3}(4 + \sin(x))^{-2/3} \cdot \cos(x)$$

$$= \frac{\cos(x)}{3(4 + \sin(x))^{2/3}}$$

$$\boxed{y' = \frac{\cos(x)}{3\sqrt[3]{(4 + \sin(x))^2}}}$$

(20) $f(x) = 2x\sqrt{x+3}$

$$y' = \frac{d}{dx}\left(2x\,(x+3)^{1/2}\right)$$

$$= 2x\,\frac{d}{dx}(x+3)^{1/2} + (x+3)^{1/2}\,\frac{d}{dx}(2x)$$

$$= 2x \cdot \frac{1}{2}(x+3)^{-1/2} \cdot 1 + (x+3)^{1/2} \cdot 2$$

$$= \frac{x}{\sqrt{x+3}} + 2\sqrt{x+3} \cdot \frac{\sqrt{x+3}}{\sqrt{x+3}}$$

$$= \frac{x}{\sqrt{x+3}} + \frac{2(x+3)}{\sqrt{x+3}}$$

$$\boxed{y' = \frac{3x+6}{\sqrt{x+3}}}$$ ✓ ✓

(21) $f(x) = \frac{\sqrt{x}}{x-4}$

$$f'(x) = \frac{(x-4)\frac{d}{dx}(x^{1/2}) - x^{1/2}\frac{d}{dx}(x-4)}{(x-4)^2}$$

$$= \frac{(x-4)\frac{1}{2\sqrt{x}} - \sqrt{x} \cdot 1}{(x-4)^2}$$

$$= \frac{\frac{(x-4)}{2\sqrt{x}} - \sqrt{x} \cdot \frac{2\sqrt{x}}{2\sqrt{x}}}{(x-4)^2}$$

$$y' = \frac{\frac{x-4}{2\sqrt{x}} - \frac{2x}{2\sqrt{x}}}{(x-4)^2}$$

$$y' = \frac{x-4-2x}{2\sqrt{x}} \div (x-4)^2$$

$$\boxed{y' = \frac{-x-4}{2\sqrt{x}\,(x-4)^2}}$$

(22) $f(x) = \sec(\sqrt{x})$

$$y' = \frac{d}{dx}\left(\sec(\sqrt{x})\right)$$

$$= \sec(\sqrt{x})\tan(\sqrt{x})\cancel{\frac{d}{dx}(\sqrt{x})}\ \frac{1}{2\sqrt{x}}$$

$$\boxed{y' = \frac{\sec(\sqrt{x})\tan(\sqrt{x})}{2\sqrt{x}}}$$

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