East Los Angeles College
Department of Mathematics
Math 261
Test 3

50 ✓

Determine the absolute max and absolute min for the following functions over the given interval.

1. $f(x) = (x^2 + 2x)^3 \;\; over \;\; [-2,1]$

$f'(x) = 6(x^2+2x)^2(x+1)$ ✓ ✓

CV $x = 0, \; x = -1, \; x = -2$ ✓ ✓ ✓

$f(1) = 27$ max ✓

✓

$f(-1) = -1$ min

8 ✓

Determine the intervals of increasing/decreasing, local max and local min, intervals of concavity, inflection points (if any).

2. $f(x) = \frac{1}{1-x^2}$

✓

$f'(x) = \frac{2x}{(1-x^2)^2}$; CV $x = 0, x = 1, x = -1$ ✓ ✓ ✓

f is decreasing $(-\infty, -1) \cup (-1, 0)$ ✓ ✓

f is increasing $(0, 1) \cup (1, \infty)$ ✓ ✓

Rel min $= f(0) = 1$ ✓

$f''(x) = \frac{2(1+3x^2)}{(1-x^2)^3}$ ✓

f is CU $(-\infty, -1) \cup (1, \infty)$ ✓ ✓

f is CD $(-1, 1)$ ✓

no inflection points

13 ✓

3. $f(x) = x\sqrt{2+x}$

$$f'(x) = \frac{3x+4}{\sqrt{2+x}}$$ ✓ cv $x=-1$, $x=-2$ ✓ ✓

✓

f is increasing on $(-\frac{4}{3}, \infty)$

f is decreasing on $(-2, -\frac{4}{3})$ ✓

Rel min at $f(-\frac{4}{3}) = -\frac{4\sqrt{6}}{9}$ ✓

$$f''(x) = \frac{3x+8}{2(2+x)\sqrt{2+x}}$$ ✓

f is CU $(-2, \infty)$ ✓

8 ✓

no inflection points.

Determine the following limits at infinity

4. $\lim_{x\to\infty} \frac{1-3x^2}{2x^3-x+1}$ $\boxed{0}$

✓ ✓

5. $\lim_{x\to\infty} \frac{5x-2}{4x+1}$ $\boxed{\frac{5}{4}}$

✓ ✓

8 ✓

6. $\lim_{x\to\infty} \left(\sqrt{4x^2+3x}-2x\right)$ $\boxed{\frac{3}{4}}$

✓ ✓

7. $\lim_{x\to-\infty} (x^3-x^7)$ $\boxed{\infty}$

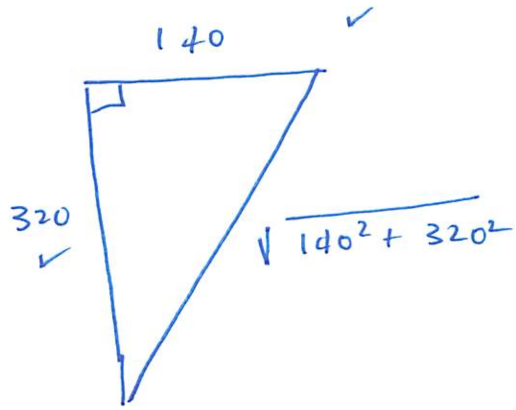
✓ ✓

8. Two cars start moving from the same point. One travels South at 80 miles per hour and the other travels West at 35 miles per hour. At what rate is the distance between the cars increasing 4 hours later?

$$d^2 = x^2 + y^2$$

$$d \frac{dd}{dt} = x \frac{dx}{dt} + y \frac{dy}{dt}$$ ✓

$$\frac{dx}{dt} = 35 \qquad \frac{dy}{dt} = 80$$



4 ✓

$$\frac{dd}{dt} = 87.3 \text{ mph}$$

✓

9. A particle moves along a curve the curve at $y = 2\cos\left(\frac{\pi x}{2}\right)$. As the particle passes through the point $(0,2)$ its x-coordinate is increasing at a rate of $\sqrt{10}$ cm/sec. How fast is the distance from the particle to the origin changing at this instant?

$$d^2 = x^2 + \cos^2\left(\frac{\pi}{2}x\right)$$

$$d\,\frac{dd}{dt} = \frac{dx}{dt}\left[x - 2\pi \cos\left(\frac{\pi x}{2}\right) \sin\left(\frac{\pi x}{2}\right)\right]$$

$$d = 2 \;;$$

$$2\,\frac{dd}{dt} = \sqrt{10}\left[0 - 2\pi \cos(0) \sin(0)\right]$$

$$\frac{dd}{dt} = 0$$

4

Determine the linearization function at the indicated points for the following functions.

10. $f(x) = \tan(x)\ at\ a = \frac{\pi}{6}$

$$m_{tan} = \sec^2(x) \Big|_{x=\frac{\pi}{6}} = \sec^2\left(\frac{\pi}{6}\right) = \frac{4}{3}$$

$$y - \tan\left(\frac{\pi}{6}\right) = \frac{4}{3}\left(x - \frac{\pi}{6}\right)$$

$$y - \frac{1}{\sqrt{3}} = \frac{4}{3}\left(x - \frac{\pi}{6}\right)$$

$$y = \frac{4}{3}x - \frac{2\pi}{9} + \frac{1}{\sqrt{3}}$$

$$L(x) = \frac{4}{3}x - \frac{2\pi}{9} + \frac{1}{\sqrt{3}}$$

Use the linearization functions above to approximate the following expressions.

11. tan (0.48) see 10

$$L(0.48) \approx 0.519$$

S.

Correct	Points
50	125
49	123
48	120
47	118
46	115
45	113
44	110
43	108
42	105
41	103
40	100
39	98
38	95
37	93
36	90
35	88
34	85
33	83
32	80
31	78
30	75
29	73
28	70
27	68
26	65
25	63
24	60
23	58
22	55
21	53
20	50
19	48
18	45
17	43
16	40
15	38
14	35
13	33
12	30
11	28
10	25
9	23
8	20
7	18

math 261 Test 3

(1) $f(x) = (x^2+2x)^3$ over $[-2, 1]$

closed Interval method

$$f'(x) = \frac{d}{dx}\left[(x^2+2x)^3\right]$$

$$= 3(x^2+2x)^2 \frac{d}{dx}(x^2+2x)$$

$$= 3(x^2+2x)^2 \cdot (2x+2)$$

$$= 3(x^2+2x)^2 \cdot 2 \cdot (x+1)$$

$$f'(x) = 6(x^2+2x)^2(x+1)$$

(CV) $f'(x) = 0$

$$6(x^2+2x)^2(x+1) = 0$$

$$(x^2+2x)^2(x+1) = 0$$

$(x^2+2x)^2 = 0$	$(x+1) = 0$
$x^2+2x = 0$	$x = -1$
$x(x+2) = 0$	$\boxed{CV}$
$x = 0$ | $x+2=0$	
$x = -2$	

$f(1) = (1^2+2\cdot1)^3 = 3^2 = 27$ abs max

$f(-2) = [(-2)^2 + 2(-2)]^3$

$= [4-4]^3 = 0^3 = \boxed{0}$

$f(-1) = [(-1)^2 + 2(-1)]^3$

$= [1-2]^3 = (-1)^3 = \boxed{-1}$ min

$f(0) = [0^2 + 2\cdot 0]^3 = 0^3 = \boxed{0}$

(2) $f(x) = \dfrac{1}{1-x^2}$

$f'(x) = \dfrac{d}{dx}\left(\dfrac{1}{1-x^2}\right)$

$= \dfrac{(1-x^2)\overbrace{\frac{d}{dx}(1)}^{0} - 1\overbrace{\frac{d}{dx}(1-x^2)}^{-2x}}{(1-x^2)^2}$

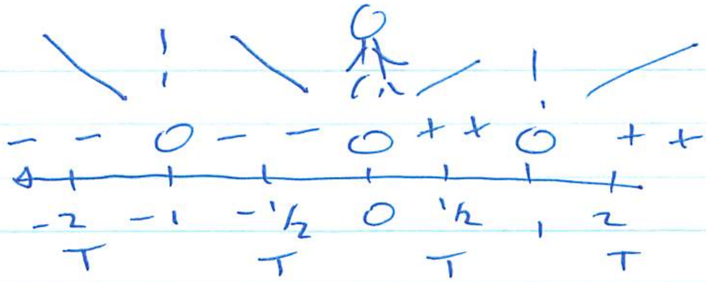
$f'(x) = \dfrac{2x}{(1-x^2)^2}$ sign analysis

$2x = 0$; $(1-x^2)^2 = 0$

$x = 0$

$1-x^2 = 0$; $x^2 = 1$

$x = \pm 1$



$$f'(x) = \frac{2x}{(1-x^2)^2}$$

f is decreasing on $(-\infty, -1) \cup (-1, 0)$

f is increasing on $(0, 1) \cup (1, \infty)$

f has a rel min at $x = 0$

$$f(0) = \frac{1}{1-0^2} = 1$$

Rel min

(3) $f(x) = x\sqrt{2+x}$

Domain

$2 + x \geq 0$

$x \geq -2$

$$f'(x) = \frac{d}{dx}\left(x(2+x)^{1/2}\right)$$

$$= x \frac{d}{dx}(2+x)^{1/2} + (2+x)^{1/2} \frac{d}{dx}(x)$$

$$= x \cdot \frac{1}{2}(2+x)^{\frac{1}{2}-1} \frac{d}{dx}(2+x)$$

$$+ (2+x)^{1/2} \cdot 1$$

concavity (3)

$$f'(x) = \frac{3x+4}{\sqrt{2+x}}$$

$(2+x)^{1/2}$

$$f''(x) = \frac{\sqrt{2+x}\,\frac{d}{dx}(3x+4) - (3x+4)\,\frac{d}{dx}(\sqrt{2+x})}{(\sqrt{2+x})^2}$$

$$= \frac{\sqrt{2+x}\cdot 3 - (3x+4)\,\frac{1}{2}\,(2+x)^{-1/2}}{2+x}$$

$$= \frac{3\sqrt{2+x} - \dfrac{3x+4}{2\sqrt{2+x}}}{2+x}$$

$$= \frac{\dfrac{3\sqrt{2+x}\cdot 2\sqrt{2+x}}{2\sqrt{2+x}} - \dfrac{3x+4}{2\sqrt{2+x}}}{2+x}$$

$$= \frac{\dfrac{6(2+x)}{2\sqrt{2+x}} - \dfrac{3x+4}{2\sqrt{2+x}}}{2+x}$$

$$= \frac{12+6x-3x-4}{2\sqrt{2+x}} \div (2+x)$$

$$\boxed{f''(x) = \frac{3x+8}{2(2+x)\sqrt{2+x}}}$$

Sign Analysis

$3x + 8 = 0$

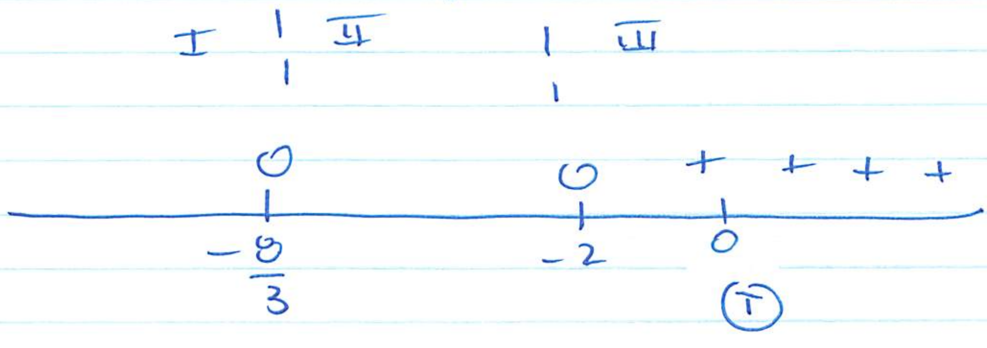
$\boxed{x = -8/3}$

$2(2+x)\sqrt{2+x} = 0$

$\to 2+x = 0$;

$\boxed{x = -2}$

Domain = $\{x \mid x \geq -2\}$



$$f''(x) = \frac{3x+8}{2(2+x)\sqrt{2+x}} > 0$$

f is cu $(-2, \infty)$

No inflection points

(4) $\lim_{x \to \infty} \dfrac{1 - 3x^2}{2x^3 - x + 1}$

$$\lim_{x \to \infty} \frac{\frac{1}{x^3} - \frac{3x^2}{x^3}}{\frac{2x^3}{x^3} - \frac{x}{x^3} + \frac{1}{x^3}}$$

$$\lim_{x \to \infty} \left(\frac{\frac{1}{x^3} - \frac{3}{x}}{2 - \frac{1}{x^2} + \frac{1}{x^3}} \right)$$

$$= \frac{0 - 0}{2 - 0 + 0} \quad \frac{0}{2} = \boxed{0}$$

(5) $\lim_{x \to \infty} \dfrac{5x - 2}{4x + 1}$

$$\lim_{x \to \infty} \frac{\frac{5x}{x} - \frac{2}{x}}{\frac{4x}{x} + \frac{1}{x}}$$

$$\lim_{x \to \infty} \frac{5 - \frac{2}{x}}{4 + \frac{1}{x}} = \frac{5 - 0}{4 + 0} = \boxed{\frac{5}{4}}$$

Rel min at $x = -1$

$$f\left(-\frac{4}{3}\right) = -\frac{4}{3}\sqrt{2 - \frac{4}{3}}$$

$$= -\frac{4}{3}\sqrt{\frac{6}{3} - \frac{4}{3}}$$

$$= -\frac{4}{3}\sqrt{\frac{2}{3}} = -\frac{4}{3}\frac{\sqrt{2}}{\sqrt{3}} = -\frac{4\sqrt{2}\sqrt{3}}{3\sqrt{3}\sqrt{3}}$$

concavity (2)

$$= \boxed{-\frac{4\sqrt{6}}{9} \text{ Rel min}}$$

$$f'(x) = \frac{2x}{(1-x^2)^2}$$

$$f''(x) = \frac{d}{dx}\left[\frac{2x}{(1-x^2)^2}\right]$$

$$= \frac{(1-x^2)^2 \frac{d}{dx}(2x) - 2x \frac{d}{dx}(1-x^2)^2}{(1-x^2)^4}$$

$$= \frac{(1-x^2)^2 \cdot 2 - 2x \cdot 2(1-x^2)(-2x)}{(1-x^2)^4}$$

$$= \frac{2(1-x^2)^2 + 8x^2(1-x^2)}{(1-x^2)^4}$$

$$= \frac{2(1-x^2)\left[(1-x^2) + 4x^2\right]}{(1-x^2)^4}$$

$$f''(x) = \frac{2(1-x^2)(1+3x^2)}{(1-x^2)^4}$$

$$f''(x) = \frac{2(1+3x^2)}{(1-x^2)^3}$$

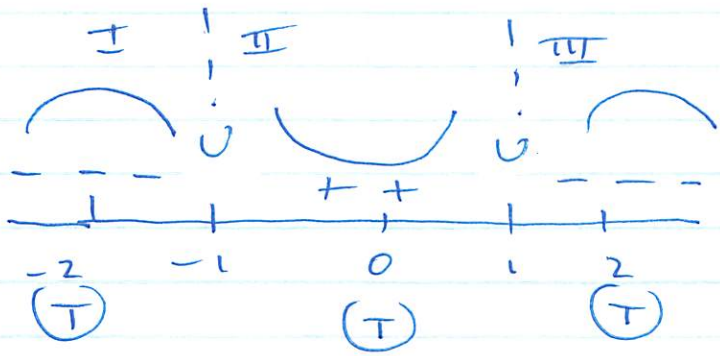
Sign Analysis $f''(x)$

$2(1+3x^2) = 0$; $(1-x^2)^3 = 0$

never happens! $x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$



$f''(x)$

$$f''(x) = \frac{2(1+3x^2)}{(1-x^2)^3}$$

f is C.D on $(-\infty, -1) \cup (1, \infty)$

f is CU on $(-1, 1)$

$$f'(x) = \frac{x}{2\sqrt{2+x}} + \sqrt{2+x} \quad .$$

$$= \frac{x}{2\sqrt{2+x}} + \sqrt{2+x}\ \frac{2\sqrt{2+x}}{2\sqrt{2+x}}$$

$$= \frac{x + 2(2+x)}{2\sqrt{2+x}} = \frac{x + 4 + 2x}{2\sqrt{2+x}}$$

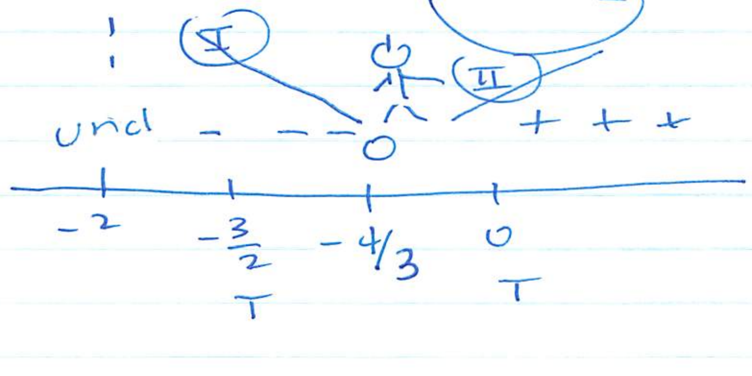
$$= \frac{3x+4}{2\sqrt{2+x}} \quad ;$$

$$f'(x) = \frac{3x+4}{\sqrt{2+x}} \quad ; \quad \text{Sign analysis}$$

$3x + 4 = 0$ $\qquad$ $\sqrt{2+x} = 0$

$x = -4/3$ $\qquad$ $2 + x = 0$

$x = -2$



$$f'(x) = \frac{3x+4}{\sqrt{2+x}}$$

f is decreasing on $(-2, -\frac{4}{3})$ $\qquad$ f is increasing on $(\frac{4}{3}, \infty)$

$\infty - \infty$ indeterminant

(6) $\lim\limits_{x \to \infty} \left(\sqrt{4x^2+3x} - 2x\right) \dfrac{\left(\sqrt{4x^2+3x} + 2x\right)}{\left(\sqrt{4x^2+3x} + 2\right)}$

$$\lim_{x \to \infty} \frac{4x^2 + 3x - 4x^2}{\sqrt{4x^2+3x} + 2}$$

$$\lim_{x \to \infty} \frac{3x}{\sqrt{4x^2+3x} + 2} \qquad \frac{\infty}{\infty} \text{ indeterminant}$$

Divide by $\sqrt{x^2} = x > 0$

$$\lim_{x \to \infty} \frac{3x / x}{\dfrac{\sqrt{4x^2+3x}}{\sqrt{x^2}} + \dfrac{2x}{x}}$$

$$\lim_{x \to \infty} \frac{3}{\sqrt{\dfrac{4x^2}{x^2} + \dfrac{3x}{x^2}} + 2}$$

$$\lim_{x \to \infty} \frac{3}{\sqrt{4 + \dfrac{3}{x}} + 2} = \frac{3}{\sqrt{4} + 2}$$

$\frac{3}{2+}$ $\boxed{\frac{3}{4}}$

(7) $\lim_{x \to -\infty} (x^3 - x^7) = -\infty - (-\infty)$

$-\infty + \infty$

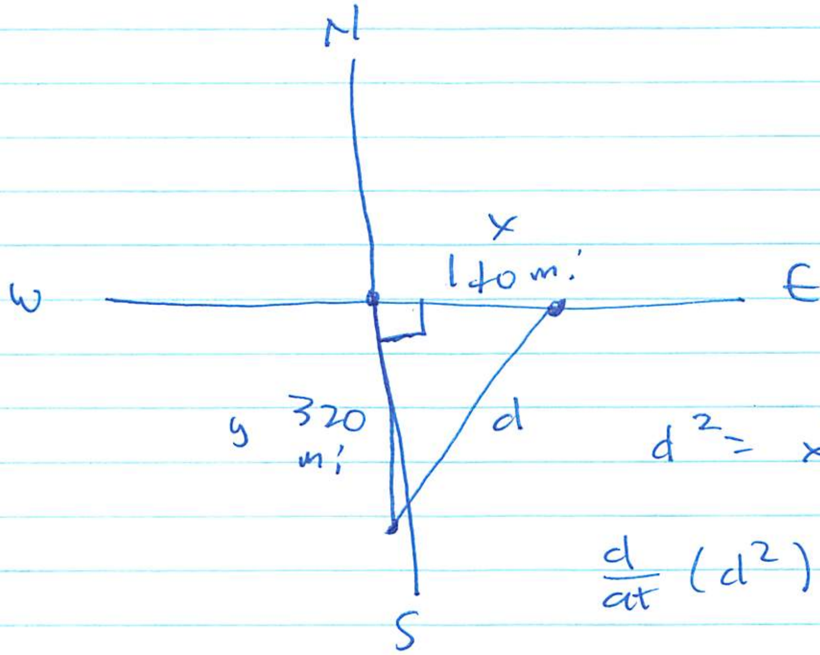
Indeterminant

$\lim_{x \to -\infty} x^3 (1 - x^4)$

$-\infty (1 - \infty)$

$-\infty (-\infty)$ $\boxed{\infty}$

(8)



$d^2 = x^2 + y^2$

$\frac{d}{dt}(d^2) = \frac{d}{dt}(x^2 + y^2)$

Car 1 $= 80 \cdot 4$

$= 320$

; $\frac{dx}{dt} = 85$

$\frac{dy}{dt} = 80$

Car 2 $= 35 \cdot 4$

$= 140$

$2 \cdot \cdot \frac{\cdot}{dt} =$

$$\frac{d}{dt}(d^2) = \frac{d}{dt}(x^2 + y^2)$$

$$\frac{d}{dt}(d^2) = \frac{d}{dt}(x^2) + \frac{d}{dt}(y^2)$$

$$2d\,\frac{dd}{dt} = 2x\,\frac{dx}{dt} + 2y\,\frac{dy}{dt}$$

$$\cancel{2}d\,\frac{dd}{dt} = \cancel{2}x\,\frac{dx}{dt} + \cancel{2}y\,\frac{dy}{dt}$$

$$d\,\frac{dd}{dt} = x\,\frac{dx}{dt} + y\,\frac{dy}{dt}$$

$$\sqrt{140^2 + 320^2}\,\frac{dd}{dt} = 140 \cdot 35 + 80 \cdot 320$$

$$\frac{dd}{dt} = \frac{140 \cdot 35 + 80 \cdot 320}{\sqrt{140^2 + 320^2}}$$

$$\boxed{\frac{dd}{dt} = 87.3 \text{ mph}}$$

(9) $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$d^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

$(0,2)_1$ and $(x,y)_2$

$d^2 = (x-0)^2 + (y-0)^2$; $d^2 = x^2 + (2\cos(\frac{\pi x}{2}))^2$

$d^2 = x^2 + 4\cos^2(\frac{\pi x}{2})$

$\frac{d}{dt}(d^2) = \frac{d}{dt}\left[x^2 + 4\cos^2(\frac{\pi x}{2})\right]$

$\frac{d}{dt}(d^2) = \frac{d}{dt}(x^2) + 4\frac{d}{dt}\left[\cos^2(\frac{\pi x}{2})\right]$

$2d\frac{dd}{dt} = 2x\frac{dx}{dt} + 4 \cdot 2\cos(\frac{\pi x}{2})\left[-\sin(\frac{\pi x}{2})\right]\frac{\pi}{2}\frac{dx}{dt}$

$2d\frac{dd}{dt} = 2x\frac{dx}{dt} - 8\cos(\frac{\pi x}{2})\sin(\frac{\pi x}{2}) \cdot \frac{\pi}{2}\frac{dx}{dt}$

$d\frac{dd}{dt} = x\frac{dx}{dt} - 2\pi\cos(\frac{\pi x}{2})\sin(\frac{\pi x}{2})\frac{dx}{dt}$

$d\frac{dd}{dt} = \frac{dx}{dt}\left[x - 2\pi\cos(\frac{\pi x}{2})\sin(\frac{\pi x}{2})\right]$

note $(0,0)_1$ and $(0,2)_1$;

$d^2 = (0-0)^2 + (2-0)^2$; $d^2 = 4$; $d = 2$

$2 \cdot \frac{dd}{dt} = \sqrt{10}\left[0 - 2\pi\cos(0)\sin(0)\right]$

$2\frac{dd}{dt} = 0$; $\boxed{\frac{dd}{dt} = 0}$

(10) $y - y_1 = \cancel{m} \, \overset{4/3}{m_{tan}} \, (x - x_1)$

$\qquad \tan(\pi/6) \qquad\qquad\qquad \pi/6$

$y = \tan(x) \quad ; \quad y' = \sec^2(x) \Big|_{\pi/6}$

$m_{tan} = \sec^2\left(\frac{\pi}{6}\right)$

$= \left[\sec\left(\frac{\pi}{6}\right)\right]^2$

$= \left(\frac{2}{\sqrt{3}}\right)^2$

$= \frac{4}{3}$

$y - \cancel{\tan\left(\frac{\pi}{6}\right)} \, \frac{1}{\sqrt{3}} = \frac{4}{3}\left(x - \frac{\pi}{6}\right)$

$y - \frac{1}{\sqrt{3}} = \frac{4}{3}\left(x - \frac{\pi}{6}\right)$

$$y - \frac{1}{\sqrt{3}} = \frac{4}{3}\left(x - \frac{\pi}{6}\right)$$

$$y = \frac{4}{3}x - \frac{4\pi}{18} + \frac{1}{\sqrt{3}}$$

$$\boxed{y = \frac{4}{3}x - \frac{2\pi}{9} + \frac{1}{\sqrt{3}}}$$

or

$$L(x) = \frac{4}{3}x - \frac{2\pi}{9} + \frac{1}{\sqrt{3}}$$

(11) $L(0.48) = \frac{4}{3}0.48 - \frac{2 \cdot \pi}{9} + \frac{1}{\sqrt{3}}$

φ

radian

$$L(0.48) \approx \boxed{0.519}$$