East Los Angeles College

Department of Mathematics

Math 261

Test 1

42 ✓

Show Your Work for Credit

Evaluate the following limits

1. $\lim_{x \to 2^+} \frac{4}{x+2}$

$\boxed{1}$

✓

2. $\lim_{x \to \frac{\pi}{2}^-} \tan(x)$

$\boxed{\infty}$

✓

See Scratch

3. $\lim_{x \to 2} \left(\frac{4x+1}{x^2-3x+2} \right)$

y

as $x \to 2^-$

$y \to -\infty$ ✓

as $x \to 2^+$ ✓

$y \to \infty$

✓

as $x \to 2$

$y \to$ DNE

6 ✓

4. $\lim_{x \to 0} (x^3 + 7x - 3)$

✓

$\boxed{-3}$

5. $\lim_{x \to -3} \left(\frac{|x+3|}{x+3} \right)$

See Scratch

y

as $x \to -3^-$ ✓

$y \to -1$

as $x \to -3^+$

$y \to 1$ ✓

as $x \to -3$ ✓

$y \to$ DNE

3 ✓

$$\text{Let } f(x) = \begin{cases} x^2+3 & for\ x \geq 1 \\ 3x+1 & 0 < x < 1 \\ \frac{1}{x-2} & x < 0 \end{cases}$$

Answer the following questions.

6. $\lim_{x \to 1} f(x)$

$\boxed{4}$ ✓

7. $f(1)$

$\boxed{4}$ ✓

8. Is the function continuous at $x = 1$, explain why or why not?

$\boxed{\text{yes}}$ ✓; $\lim_{x \to 1} f(x) = f(1)$

9. $\lim_{x \to 0} f(x)$ $= \boxed{\text{DNE}}$ ✓

$x \to 0^-$; $y \to -\frac{1}{2}$

$x \to 0^+$; $y \to 1$

10. $f(0)$ $= \boxed{\text{und}}$ ✓

11. Is the function continuous at $x = 0$, explain why or why not?

$\boxed{\text{No}}$ ✓; $\lim_{x \to 0} f(x) \neq f(0)$

12. $\lim_{x \to -5} f(x)$ $= \boxed{-\frac{1}{7}}$ ✓

13. $f(-5)$ $= \boxed{-\frac{1}{7}}$ ✓

14. Is the function continuous at -5, explain why or why not?

$\boxed{\text{yes}}$ ✓ $\lim_{x \to -5} f(x) = f(-5)$ ✓

15. $\lim_{x \to 2} f(x)$ $= \boxed{7}$ ✓

16. $f(2)$ $= \boxed{7}$

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17. Is the function continuous at $x = 2$, explain why or why not?

$\boxed{\text{yes}}$, ✓ $\lim_{x \to 2} f(x) = f(2)$

18. Show that $x^3 = 2x - 5$ has a solution in the interval $(-4, 0)$

Hint: Use the Intermediate Value Theorem

let $f(x) = x^3 - 2x + 5$

$f(0) = 5$ $\cancel{\geq}$ +

$f(-4) = -51$ $<$ -

as f is continuous over $[-4, 0]$

by IVT there is a $r \in (-4, 0)$

such that $f(r) = 0$.

ie, ~~of~~ f has a root.

aka Solution

Determine the interval of continuity for the following functions.

19. $f(x) = \frac{\sqrt{x}}{x^2-4}$ See Scratch

f is continuous over $[0, 2) \cup (2, \infty)$

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Use the definition of derivative (mtan) to determine the equation of the line tangent to the curve for the following functions at the indicated points.

20. $f(x) = x^3 + 1$ at $P(1,2)$ See Scratch

$m_{tan} = 3$ ✓ ; $y - y_1 = m(x - x_1)$

$\boxed{y = 3x - 1}$ ✓ ✓

5 ✓

21. $f(x) = \frac{2}{\sqrt{x+3}}$ at $P(1,1)$ See Scratch

$m_{tan} = -\frac{1}{8}$ ✓ ;

$y - y_1 = m(x - x_1)$

$$\boxed{y = -\frac{1}{8}x + \frac{9}{8}}$$

✓ ✓

5 ✓

Use the definition of derivative to differentiate the following function.

22. $f(x) = \frac{5}{x^2}$ at a

See Scratch

✓ ✓ ✓

$f'(a) = -\frac{10}{a^3}$

5✓

math 261 Test 1

(1) $\lim_{x \to 2^+} \dfrac{4}{x+2} = \boxed{1}$

let $x = 2.001$

(2) $\lim_{x \to \frac{\pi}{2}^-} \tan(x) = \boxed{\infty}$

(3) $\lim_{x \to 2} \dfrac{4x+1}{x^2-3x+2} = \boxed{\text{DNE}}$

$\lim_{x \to 2^-} \dfrac{4x+1}{(x-2)(x-1)} = -\infty$

$x = +1.999$

$\lim_{x \to 2^+} \dfrac{4x+1}{(x-2)(x-1)} = \infty$

$x = 2.001$

(4) $\lim_{x \to 0} (x^3+7x-3) = 0^3+7.0-3$

$= \boxed{-3}$

(5) $\lim_{x \to -3} \left(\dfrac{|x+3|}{|x+3} \right)$

$$\frac{|x+3|}{x+3} = \begin{cases} \dfrac{\cancel{x+3}}{\cancel{x+3}} \; 1 & x+3 \geq 0 \\ \dfrac{\cancel{x+3}}{-(\cancel{x+3})} \; -1 & x+3 < 0 \end{cases}$$

$$= \begin{cases} 1, & x \geq -3 \quad \text{Right} \\ -1, & x < -3 \quad \text{left} \end{cases}$$

$$\lim_{x \to -3^-} \frac{|x+3|}{x+3} = \lim_{x \to -3} -1 = -1$$

$$\lim_{x \to -3^+} \frac{|x+3|}{x+3} = \lim_{x \to -3^+} 1 = 1$$

$$\lim_{x \to -3} \frac{|x+3|}{x+3} = \boxed{\text{DNE}}$$

(6) $\lim_{x \to 1} f(x) = \boxed{4}$

$$\lim_{x \to 1^-} f(x) = 3 \cdot 1 + 1 = 4$$

$$\lim_{x \to 1^+} f(x) = 1^2 + 3 = 4$$

(7) $f(1) = 1^2 + 3 = 4$

$\boxed{f(1) = 4}$

(8) $\boxed{\lim_{x \to 1} f(x) = f(1)}$

(9) $\lim_{x \to 0^-} f(x) = \frac{1}{0-2} = -\frac{1}{2}$

$\lim_{x \to 0^+} f(x) = 3 \cdot 0 + 1 = 1$

$\lim_{x \to 0} f(x) = \boxed{DNE}$

(10) $f(0) =$ undefined,

(11) No, $\boxed{\lim_{x \to 0} f(x) \neq f(0)}$

(12) $\lim_{x \to -5^-} f(x) = \frac{1}{-5-2} = -\frac{1}{7}$

$\lim_{x \to -5^+} f(x) = \frac{1}{-5-2} = -\frac{1}{7}$

$\lim_{x \to -5} f(x) = \boxed{-\frac{1}{7}}$

$f(-5) = \frac{1}{-5-2}$; (13) $\boxed{f(-5) = -\frac{1}{7}}$

(14) $\lim_{x \to -5} f(x) = f(-5)$ yes

(15) $\lim_{x \to 2} f(x) = \boxed{7}$

as $\lim_{x \to 2^-} f(x) = 2^2 + 3 = 7$

$\lim_{x \to 2^+} f(x) = 2^2 + 3 = 7$

(16) $f(2) = 2^2 + 3$; $\boxed{f(2) = 7}$

(17) continuous as $\boxed{\lim_{x \to 2} f(x) = f(2)}$

(18) $x^3 = 2x - 5$ $(-4, 0)$

let $f(x) = x^3 - 2x + 5$

$f(-4) = (-4)^3 - 2(-4) + 5$

$= -64 + 8 + 5$

$= -51$

$f(0) = 0^3 - 2 \cdot 0 + 5$; $f(0) = 5$

$= 5$

since f is continuous over $[-4, 0]$.

by IVT there is a $c \in (-4, 0)$

such that $f(r) = 0$

ie, there is a root.

(19) $f(x) = \frac{\sqrt{x}}{x^2 - 4}$

$x \geq 0$; $x^2 - 4 \neq 0$

ie, $x \neq \pm \sqrt{4}$

$x \neq \pm 2$

ie, f is continuous over $[0, 2) \cup (2, \infty)$

(20) $f(x) = x^3 + 1$; $(\overset{a}{1}, \underset{f(a)}{2})$

$$m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{(1+h)^3 + 1 - 2}{h}$$

note: $(1+h)^3 = (1+h)(1+h)(1+h)$

$= (1 + 2h + h^2)(1+h)$

$$(1+h)^3 = 1 + 2h + h^2$$
$$\quad + h + 2h^2 + h^3$$

$$= 1 + 3h + 3h^2 + h^3$$

i.e.,

$$m_{tan} = \lim_{h \to 0} \frac{\cancel{1} + 3h + h^2 + h^3 + \cancel{1 - 2}}{h}$$

$$= \lim_{h \to 0} \frac{3h + h^2 + h^3}{h}$$

$$= \lim_{h \to 0} \frac{3h}{h} + \frac{h^2}{h} + \frac{h^3}{h}$$

$$= \lim_{h \to 0} 3 + h + h^2$$

$$= 3 + 0 + 0^2 \quad ; \quad m_{tan} = 3$$

$$y - y_1 = m(x - x_1) \qquad (1, 2) \; \begin{matrix} x & y \end{matrix}$$

$$y - 2 = 3(x - 1)$$

$$y - 2 = 3x - 3 \quad ; \quad \boxed{y = 3x - 1}$$
$$+2 \qquad\quad +2$$

(21) $f(x) = \dfrac{2}{\sqrt{x+3}}$ $\quad (\underset{a}{1}, 1)$

$$m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2}{\sqrt{1+h+3}} - 1}{h}$$

note : $$\frac{\frac{2}{\sqrt{1+h+3}} - \frac{1}{1}}{h}$$

$$= \frac{2 - \sqrt{1+h+3}}{\sqrt{1+h+3}} \div h$$

$$= \frac{(2 - \sqrt{1+h+3})}{h\sqrt{1+h+3}} \cdot \frac{(2 + \sqrt{1+h+3})}{(2 + \sqrt{1+h+3})}$$

$$\frac{4-(1+h+3)}{h\sqrt{1+h+3}\,(2+\sqrt{1+h+3})}$$

$$\frac{\cancel{4}-\cancel{1}-\cancel{h}-\cancel{3}}{\cancel{h}\sqrt{1+h+3}\,(2+\sqrt{1+h+3})}$$

$$\lim_{h\to 0}\frac{-1}{\sqrt{1+h+3}\,(2+\sqrt{1+h+3})}$$

$$\frac{-1}{\sqrt{1+0+3}\,(2+\sqrt{1+0+3})}$$

$$\frac{-1}{\sqrt{4}\,(2+\sqrt{4})} \qquad \frac{-1}{2(2+2)}$$

$$= -\frac{1}{8}$$

$$y-y_1 = m(x-x_1)$$

$(1,1)$
$x \; y$

$$y-1 = -\frac{1}{8}(x-1)$$

$$y-1 = -\frac{1}{8}x+\frac{1}{8} \qquad \boxed{y = -\frac{1}{8}x+\frac{9}{8}}$$

$+1 \qquad\qquad +1$

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$$f(x) = \frac{5}{x^2} \quad \text{at } a$$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{5}{(a+h)^2} - \frac{5}{a^2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{5a^2 - 5(a+h)^2}{(a+h)^2 \cdot a^2}}{h}$$

note: $\frac{5a^2 - 5(a+h)^2}{(a+h)^2 \cdot a^2} \div h$

foil

$$\frac{5a^2 - 5(a+h)^2}{(a+h)^2 \cdot a^2 \cdot h}$$

$$\frac{5a^2 - 5(a^2 + 2ah + h^2)}{(a+h)^2 \cdot a^2 \cdot h}$$

$$\frac{\cancel{5a^2} - \cancel{5a^2} - 10ah - 5h^2}{(a+h)^2 a^2 h}$$

$$\frac{-10ah - 5h^2}{(a+h)^2 a^2 h}$$

$$\frac{(-10a - 5h)\cancel{h}}{(a+h)^2 a^2 \cancel{h}}$$

$$m_{tan} = \lim_{h \to 0} \frac{-10a - 5h}{(a+h)^2 a^2}$$

$$= \frac{-10a - 5 \cdot 0}{(a+0)^2 a^2}$$

$$= \frac{-10a}{a^2 a^2}$$

$$= -\frac{10a}{a^4}, \quad m_{tan} = -\frac{10}{a^3}$$

ie, $\boxed{f'(a) = -\frac{10}{a^3}}$