Answer Sheet

1		14	$8\bar{i} + 6\bar{j}$
2	113 miles	15	$500\bar{i}$
3	$6\bar{i} - 5\bar{j}$	16	$-21\bar{i} + 25\bar{j}$
4	$-\bar{i} + 2\bar{j}$	17	$479\bar{i} + 25\bar{j}$
5	$-5\bar{i} + 2\bar{j}$	18	480 mph
6	$-\bar{i} - 16\bar{j}$	19	N 87° E
7	$5\bar{i} + 12\bar{j}$	20	$2\sin(x + 330°)$
8	$\sqrt{13}$	21	Use Your Own Paper
9	2	22	Use Your Own Paper
10	$\sqrt{17}$	23	Use Your Own Paper
11	$\sqrt{5}$	24	Use Your Own Paper
12	$\sqrt{29}$	25	Use Your Own Paper
13	13		44

Find the horizontal and vertical components of the vector and write in $\vec{\imath}$ and $\vec{\jmath}$ form.

14. $|\vec{v}| = 10$ and $\theta = 35^o$

True Velocity of a Jet

A pilot heads her jet due East traveling at 500 mph. A wind is blowing at 32 MPH at a heading of $N40^oW$.

15. Express the velocity of the plane as a vector without the wind.
16. Express the velocity of the wind as a vector.
17. Express the velocity of the jet with the wind. That is, the resultant vector.
18. What is the true speed of the jet with the wind?
19. What is the true bearing of the jet with the wind?

Write the expression as a single sine.

20. $\sqrt{3}\sin(x) - \cos(x)$

Prove the following Identities

21. $2cos^2(\theta) - 1 = 1 - 2sin^2(\theta)$

22. $\frac{sin(\theta)}{1+cos(\theta)} = \frac{1-cos(\theta)}{\sin(\theta)}$

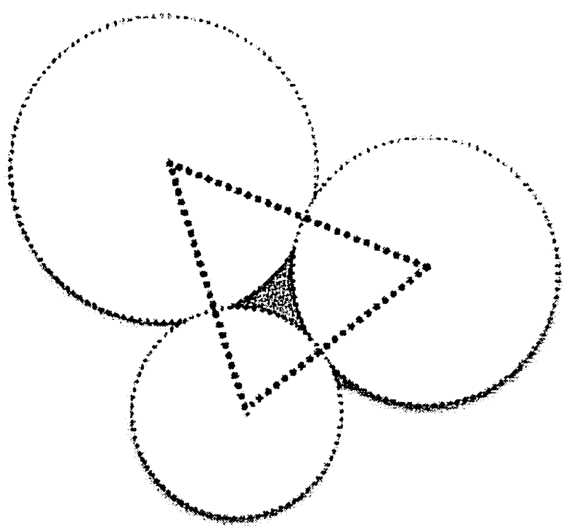
23. $tan^2(\theta) - sin^2(\theta) = tan^2(\theta)sin^2(\theta)$

24. $cos(x-\pi) = -cos(x)$

25. $sin(x+y) - sin(x-y) = 2cos(x)sin(y)$

East Los Angeles College
Department of Mathematics
Math 241
Test 3

1. The circles are barely touching one another and have the following radii's 2, cm, 3 cm, and 4 cm from smallest to largest. Determine the shaded area.



Navigation

Two cars leave the same intersection at noon. Car 1 travels at 75 mph in the direction of $N25^oE$ while car 2 travels at 60 mph in the direction of $N55^oE$.

2. How far apart are the cars at 3:00 PM?

Let P and Q be two points in a plane. Determine the coordinate vector $\overrightarrow{PQ}$

3. $P = (3, -1)$ and $Q = (-2,5)$

Let $\vec{u} = -3\vec{i} + 2\vec{j}$ and $\vec{v} = 2\vec{i}$ and $\vec{w} = \vec{i} - 4\vec{j}$. Determine:

4. $\vec{u} + \vec{v}$

5. $\vec{u} - \vec{v}$

6. $2\vec{u} + 5\vec{w}$

7. $4\vec{v} - 3\vec{w}$

8. $|\vec{u}|$

9. $|\vec{v}|$

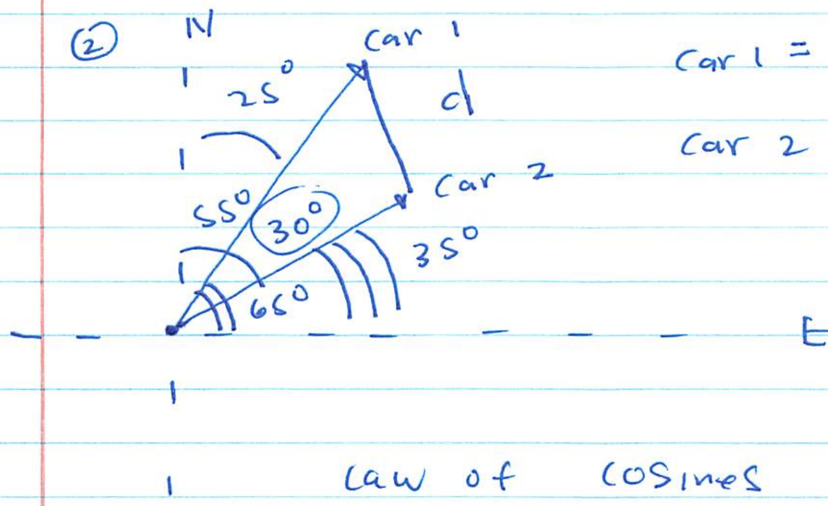
10. $|\vec{w}|$

11. $|\vec{u} + \vec{v}|$

12. $|\vec{u} - \vec{v}|$

13. $|4\vec{v} - 3\vec{w}|$

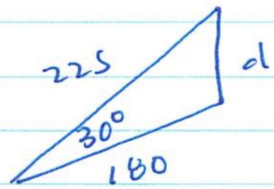
(2)



car 1 $= 75 \cdot 3 = 225$ miles

car 2 $= 60 \cdot 3 = 180$ miles

law of cosines



$$d^2 = 225^2 + 180^2 - 2 \cdot 225 \cdot 180 \cos(30^\circ)$$

$$d = \sqrt{225^2 + 180^2 - 2 \cdot 225 \cdot 180 \cdot \cos(30^\circ)}$$

$$\boxed{d \approx 113 \text{ miles}}$$

(3) $P = (3,-1)$; $Q = (-2, 5)$

$\overrightarrow{PQ} = \langle -2-3,\ 5 - -1 \rangle$;

$= \langle -5,\ 5+1 \rangle$

$= \langle -5,\ 6 \rangle$

$\boxed{\overrightarrow{PQ} = -5i + 6\bar{j}}$

(4) $\bar{u} + \bar{v} = -3\bar{i} + 2\bar{j} + 2\bar{i}$

$\boxed{\bar{u} + \bar{v} = -\bar{i} + 2\bar{j}}$

(5) $\bar{u} - \bar{v} = (-3\bar{i} + 2\bar{j}) - (2\bar{i})$

$= -3\bar{i} + 2\bar{j} - 2i$

$\boxed{\bar{u} - \bar{v} = -5\bar{i} + 2\bar{j}}$

(6) $2\bar{u} + 5\bar{w} = 2(-3\bar{i} + 2\bar{j})$

$+ 5(\bar{i} - 4\bar{j})$

$= -6\bar{i} + 4\bar{j} + 5\bar{i} - 20\bar{j}$

$= \boxed{-\bar{i} - 16\bar{j}}$

(7) $4\bar{v} - 3\bar{w}$

$= 4(2\bar{i}) - 3(\bar{i} - 4\bar{j})$

$= 8\bar{i} - 3\bar{i} + 12\bar{j}$

$= \underline{5\bar{i} + 12\bar{j}}$

(8) $|\bar{u}| = \sqrt{(-3)^2 + 2^2}$

$= \sqrt{9 + 4}$

$= \sqrt{13}$

(9) $|\bar{v}| = \sqrt{2^2 + 0^2}$

$= \sqrt{4} = 2$

(10) $|\bar{w}| = \sqrt{1^2 + (-4)^2}$

$= \sqrt{1 + 16} = \sqrt{17}$

(11) $|\bar{u} + \bar{v}| = \sqrt{(-1)^2 + 2^2}$

$= \sqrt{1 + 4}$

$= \sqrt{5}$

(12) $|\bar{u} - \bar{v}| = \sqrt{(-5)^2 + 2^2}$

$= \sqrt{25 + 4} = \sqrt{29}$

(13) $|4\bar{v} - 3\bar{w}| = \sqrt{5^2 + 12^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169} = 13$

(14) $\bar{v} = |\bar{v}| \cos\theta \,\bar{i} + |\bar{v}| \sin\theta \,\bar{j}$

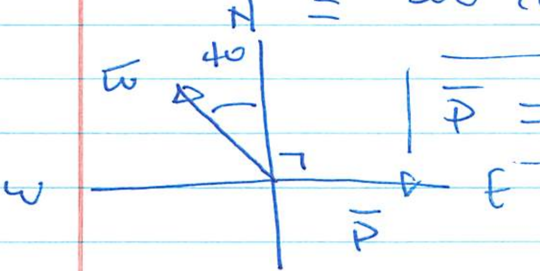
$= 10 \cos(35^\circ)\bar{i} + 10 \sin(35^\circ)\bar{j}$

$\boxed{\bar{v} = 8.2\bar{i} + 5.7\bar{j}}$

(15) $\bar{p} = |p| \cos\theta \,\bar{i} + |p| \sin\theta \,\bar{j}$

$= 500 \cos(0^\circ)\bar{i} + 500 \sin(0^\circ)\bar{j}$

$\boxed{\bar{p} = 500\bar{i}}$



(16) $\bar{w} = |\bar{w}| \cos\theta \,\bar{i} + |\bar{w}| \sin\theta \,\bar{j}$

$= 32 \cos(130^\circ)\bar{i} + 32 \sin(130^\circ)\bar{j}$

$\boxed{\bar{w} = -21\bar{i} + 25\bar{j}}$

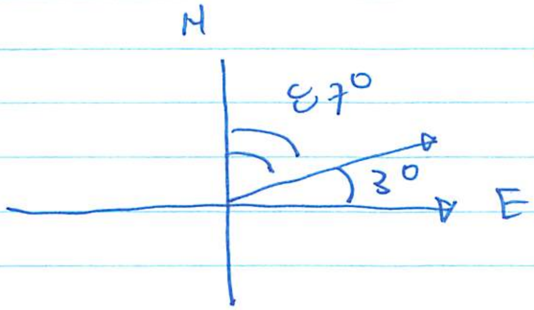
(17) $\bar{p} + \bar{w} = 500\bar{i} + -21\bar{i} + 25\bar{j}$

$\bar{p} + \bar{w} = \boxed{479\bar{i} + 25\bar{j}}$

(18) $|\vec{P} + \vec{w}| = \sqrt{479^2 + 25^2}$

$= \boxed{480 \text{ mph}}$

(19) $\tan\theta = \frac{25}{479}$; $\theta = \tan^{-1}\left(\frac{25}{479}\right)$



$\theta \approx 3°$

$\boxed{N\ 87° E}$

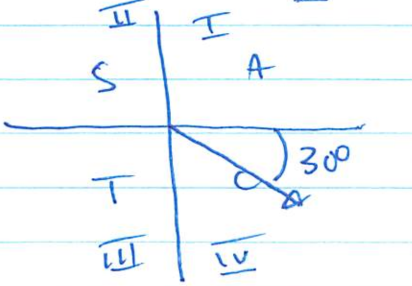
(20) $\overset{A}{\sqrt{3}} \sin(x) \overset{B}{-} \cos(x)$

$k = \sqrt{A^2 + B^2}$

$= \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{3+1}$

$= \sqrt{4} = 2$

$\cos\phi = \frac{\sqrt{3}}{2}$; $\sin\phi = -\frac{1}{2}$



$\bar{\phi} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\bar{\phi} = 30°$

ie, $\phi = 360 - 30$

$\phi = 330°$

$\boxed{2\sin(x + 330°)}$

(21) $2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

LHS = RHS

$$\begin{aligned}
LHS &= 2\cos^2\theta - 1 \\
&= 2[1 - \sin^2\theta] - 1 \\
&= 2 - 2\sin^2\theta - 1 \\
&= 1 - 2\sin^2\theta \\
&= RHS
\end{aligned}$$

(22) $\dfrac{\sin(\theta)}{1+\cos(\theta)} = \dfrac{1-\cos(\theta)}{\sin(\theta)}$

LHS = RHS

$$\begin{aligned}
LHS &= \frac{\sin(\theta)}{1+\cos(\theta)} \\
&= \frac{\sin\theta}{1+\cos\theta} \cdot \frac{(1-\cos\theta)}{(1-\cos\theta)} \\
&= \frac{\sin\theta\,((1-\cos\theta)}{1-\cos^2\theta} \\
&= \frac{\cancel{\sin\theta}\,(1-\cos\theta)}{\sin^{\cancel{2}}\theta} \\
&= \frac{1-\cos\theta}{\sin(\theta)} \\
&= RHS
\end{aligned}$$

(23) $\tan^2(\theta) - \sin^2(\theta) = \tan^2(\theta)\sin^2(\theta)$

LHS $\qquad\qquad$ = RHS

$$\text{LHS} = \tan^2\theta - \sin^2(\theta)$$
$$= \frac{\sin^2\theta}{\cos^2(\theta)} - \sin^2\theta$$
$$= \sin^2\theta\left[\frac{1}{\cos^2\theta} - 1\right]$$
$$= \sin^2\theta\left[\sec^2\theta - 1\right]$$
$$= \sin^2\theta\tan^2\theta$$
$$= \text{RHS}$$

(24) $\cos(x-\pi) = \cos(x)\underbrace{\cancel{\cos(\pi)}}_{-1} + \sin(x)\underbrace{\cancel{\sin(\pi)}}_{0}$

$$= -\cos(x)$$

(25) $\sin(x+y) - \sin(x-y) = 2\cos(x)\cos(y)$

LHS $\qquad\qquad$ = RHS

$$\text{LHS} = \sin(x+y) - \sin(x-y)$$
$$= \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$- [\sin(x)\cos(y) - \cos(x)\sin(y)]$$

$$\begin{aligned}
LHS &= \sin(x)\cos(y) + \cos(x)\sin(y) \\
&\quad - \sin(x)\cos(y) + \cos(x)\sin(y) \\
&= 2\cos(x)\sin(y) \\
&= RHS
\end{aligned}$$