East Los Angeles College
Department of Mathematics
Math 262
Test 1

57 ✓

last page is extra credit!

Perform the following limits

1. $\lim_{x\to 3} e^{2\pi x}$

$= e^{2\pi \cdot 3} = e^{6\pi}$ ✓ ✓ ✓

2. $\lim_{x\to\infty} e^{-x^3}$; $t = -x^3$ ✓

as $x \to \infty$

$t \to -\infty$ ✓

$e^t \to \boxed{0}$ ✓

3. $\lim_{x\to 0^-} e^{\frac{1}{x}}$; $t = \frac{1}{x}$ ✓

as $x \to 0^-$ ✓

$t \to -\infty$

$e^t \to \boxed{0}$ ✓

4. $\lim_{x\to 0^+} 10^{-\frac{1}{x}}$; $t = -\frac{1}{x}$ ✓

as $x \to 0^+$

$t \to -\infty$ ✓

$10^t \to \boxed{0}$ ✓

5. $\lim_{x\to -\infty} e^{-x^2}$; $t = -x^2$ ✓

as $x \to -\infty$ ✓

$t \to -\infty$

$e^t \to \boxed{0}$ ✓

6. $\lim_{x\to 125} log_5\,(x)$

$= \log_5 (125)$ ✓

$= \boxed{3}$ ✓ ✓

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7. $\lim_{x\to\infty} log\left(\frac{1}{x^3}\right)$; $t = \frac{1}{x^3}$

as $x \to \infty$

$t \to 0$

$\log(t) \to -\infty$

8. $\lim_{x\to\infty} log\left(\frac{x^2-4x+3}{x^2+10x+1}\right)$

$= \log\left[\lim_{x\to\infty} \frac{x^2-4x+3}{x^2+10x+1}\right]$

$= \log\left[\lim_{x\to\infty} \frac{1-4/x+3/x^2}{1+10/x+1/x^2}\right]$

$= \log(1) = \boxed{0}$

Differentiate the following

9. $f(x) = xe^{-\pi x}$

$f'(x) = -\pi x e^{-\pi x} + e^{-\pi x}$

10. $f(x) = ln[\sec(x) + \tan(x)]$

$$f'(x) = \frac{\sec(x)\tan(x) + \sec^2(x)}{\sec(x) + \tan(x)}$$

$f'(x) = \sec(x)$

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11. $f(x) = x - tan^{-1}(x^2)$

$$f'(x) = 1 - \frac{2x}{1+x^4}$$ ✓

$$f'(x) = \frac{x^4 - 2x + 1}{1+x^4}$$ ✓ ✓

12. $f(x) = x^{\cos(x)}$; $y = x^{\cos(x)}$; $\ln(y) = \cos(x)\ln(x)$ ✓

$$\frac{y'}{y} = \frac{\cos(x)}{x} - \ln(x)\sin(x)$$ ✓

$$y' = x^{\cos(x)}\left[\frac{\cos(x)}{x} - \ln(x)\sin(x)\right]$$

✓

13. $f(x) = \frac{x^2\sqrt{2x+5}}{(3x-2)^4}$; $y = \frac{x^2\sqrt{2x+5}}{(3x-2)^4}$; ✓

$$\ln(y) = 2\ln(x) + \frac{1}{2}\ln(2x+5) - 4\ln(3x-2)$$

$$\frac{y'}{y} = \frac{2}{x} + \frac{1}{2x+5} - \frac{12}{3x-2}$$ ✓

$$y' = \frac{x^2\sqrt{2x+5}}{(3x-2)^4}\left[\frac{2}{x} + \frac{1}{2x+5} - \frac{12}{3x-2}\right]$$ ✓

9 ✓

14. $f(x) = sin^{-1}(\sqrt{x})$

$$f'(x) = \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

15. $f(x) = e^{\sqrt{x}+xtan(x)}$

$$f'(x) = e^{\sqrt{x}+x\tan(x)}\left[\frac{1}{2\sqrt{x}} + x\sec^2(x) + \tan(x)\right]$$

Integrate the following

16. $\int \tan(x)\, dx$

$$\int \frac{\sin(x)}{\cos(x)}\, dx = -\int \frac{1}{u}\, du = -\ln|u| + C$$

$u = \cos(x)$

$$= \ln|\sec(x)| + C$$

$du = -\sin(x)\, dx$

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17. $\int \frac{1}{e^{2\pi x}} dx$ $= \int e^{-2\pi x} dx = -\frac{1}{2\pi} \int e^u du$ ✓

$u = -2\pi x$

✓

$du = -2\pi dx$

$= -\frac{1}{2\pi} e^u + C$ ✓

$= -\frac{1}{2\pi} e^{-2\pi x} + C$

✓

18. $\int \frac{1}{4x-3} dx$ $= \int \frac{1}{u} \frac{du}{4} = \frac{1}{4} \int \frac{1}{u} du$

$u = 4x - 3$ ✓

$du = 4 dx$

$= \frac{1}{4} \ln|u| + C$

✓

$= \frac{1}{4} \ln|4x-3| + C$

19. $\int \frac{x^2+x-3}{x} dx$ $= \int \left(x + 1 - \frac{3}{x}\right) dx$

✓ ✓ ✓

$= \frac{x^2}{2} + x - 3\ln|x| + C$

9 ✓

20. $\int \frac{1}{9+x^2} dx = \int \frac{1}{3^2+x^2} dx \quad ; \quad a = 3$

$$= \boxed{\frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + C}$$

21. $\int \frac{1}{1+4x^2} dx = \int \frac{1}{1+(2x)^2} dx \quad ; \quad u = 2x$

$du = 2\,dx$

$$= \int \frac{1}{1+u^2} \frac{du}{2} = \frac{1}{2}\int \frac{1}{1+u^2} du$$

$$= \frac{1}{2} \tan^{-1}(u) + C = \boxed{\frac{1}{2} \tan^{-1}(2x) + C}$$

22. $\int \frac{1}{\sqrt{16-x^2}} dx = \int \frac{1}{\sqrt{4^2-x^2}} dx \quad ; \quad a = 4$

$$= \boxed{\sin^{-1}\left(\frac{x}{4}\right) + C}$$

23. $\int \frac{1}{\sqrt{1-25x^2}} dx = \int \frac{1}{\sqrt{1-(5x)^2}} dx \quad ; \quad u = 5x$

$du = 5\,dx$

$$= \frac{1}{5} \int \frac{1}{\sqrt{1-u^2}} du = \frac{1}{5} \sin^{-1}(u) + C$$

$$= \boxed{\frac{1}{5} \sin^{-1}(5x) + C}$$

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math 262 Test 1

(1) $\lim_{x \to 3} e^{2\pi x} = e^{2\pi \cdot 3} = \boxed{e^{6\pi}}$

as $e^{2\pi x}$ is continuous everywhere

(2) $\lim_{x \to \infty} e^{-x^3}$

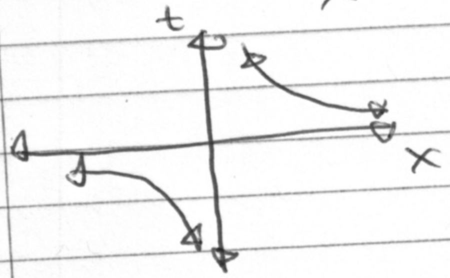
let $t = -x^3$; as $x \to \infty$; $t \to -\infty$

so, $e^{t} \to \boxed{0}$

(3) $\lim_{x \to 0^-} e^{\frac{1}{x}}$

let $t = \frac{1}{x}$; as $x \to 0^-$

$t \to -\infty$



$e^{t} \to \boxed{0}$

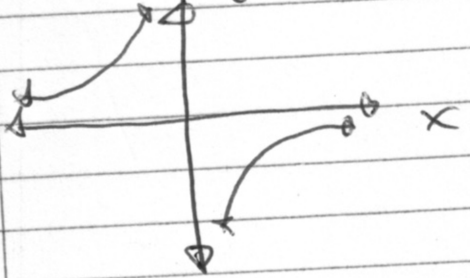
(4) $\lim_{x \to 0^+} 10^{-\frac{1}{x}}$

as $x \to 0^+$

$t \to -\infty$

let $t = -\frac{1}{x}$

$10^{t} \to \boxed{0}$



(5) $\lim_{x \to -\infty} e^{-x^2}$

let $t = -x^2$

as $x \to -\infty$; $t \to -\infty$; $e^t \to \boxed{0}$

(6) $\lim_{x \to 125} \log_5 (x) = \log_5 (125)$

as log is continuous at 125

as $x \to 125$; log

let $t = \log_5 (125) \leftrightarrow 5^t = 125$

$t = 3$

as $x \to 125$; $\log_5 (x) \to \boxed{3}$

(7) $\lim_{x \to \infty} \log \left(\frac{1}{x^3}\right)$

let $t = \frac{1}{x^3}$ as $x \to \infty$

$t \to 0$

$\log(t) \to -\infty$

(8) $\lim_{x \to \infty} \log \left(\frac{x^2 - 4x + 3}{x^2 + 10x + 1}\right)$

$= \log \left[\lim_{x \to \infty} \frac{x^2 - 4x + 3}{x^2 + 10x + 1}\right]$

$$\lim_{x \to \infty} \left(\frac{\frac{x^2}{x^2} - \frac{4x}{x^2} + \frac{3}{x^2}}{\frac{x^2}{x^2} + \frac{10x}{x^2} + \frac{1}{x^2}} \right)$$

$$\lim_{x \to \infty} \left(\frac{1 - 4/x + 3/x^2}{1 + 10/x + 1/x^2} \right)$$

$$= \frac{1}{1} = 1 \quad ; \text{ let } t = \frac{x^2 - 4x + 3}{x^2 + 10x + 1}$$

ie, as $x \to \infty$; $t \to 1$

$$\log(t) \to \cancel{\log(1)} \to \boxed{0}$$

(9) $f(x) = x e^{-\pi x}$

$$f'(x) = \frac{d}{dx}\left(x e^{-\pi x}\right)$$

$$= x \frac{d}{dx}\left(e^{-\pi x}\right) + e^{-\pi x} \cancel{\frac{d}{dx}(x)} \to 1$$

$$e^{-\pi x} \frac{d}{dx}(-\pi x) \to -\pi$$

$$\boxed{f'(x) = -\pi x e^{-\pi x} + e^{-\pi x}}$$

(10) $$f(x) = \ln\left[\sec(x) + \tan(x)\right]$$

$$f'(x) = \frac{d}{dx}\left[\ln\left[\sec(x) + \tan(x)\right]\right]$$

$$= \frac{1}{\sec(x) + \tan(x)} \frac{d}{dx}\left[\sec(x) + \tan(x)\right]$$

$$\sec(x)\tan(x) + \sec^2(x)$$

$$f'(x) = \frac{\sec(x)\tan(x) + \sec^2(x)}{\sec(x) + \tan(x)}$$

(11) $$f(x) = x - \tan^{-1}(x^2)$$

$$f'(x) = \frac{d}{dx}\left[x - \tan^{-1}(x^2)\right]$$

$$= \frac{d}{dx}[x] - \frac{d}{dx}\left[\tan^{-1}(x^2)\right]$$

$$= 1 - \frac{1}{1+(x^2)^2} \cancel{\frac{d}{dx}(x^2)} \; 2x$$

$$= 1 - \frac{2x}{1+x^4}$$

$$= \frac{1+x^4}{1+x^4} - \frac{2x}{1+x^4}$$

$$= a$$

$$\boxed{f'(x) = \frac{x^4 - 2x + 1}{1 + x^4}}$$

(12) $f(x) = x^{\cos(x)}$

Let $y = x^{\cos(x)}$; $\ln(y) = \ln\left[x^{\cos(x)}\right]$

$$\ln(y) = \cos(x)\ln(x)$$

$$\frac{d}{dx}[\ln(y)] = \frac{d}{dx}[\cos(x)\ln(x)]$$

$$\frac{1}{y}\cdot y' = \cos(x)\cancelto{\frac{1}{x}}{\frac{d}{dx}(\ln(x))} + \ln(x)\frac{d}{dx}(\cancelto{-\sin(x)}{\cos(x)})$$

$$\frac{y'}{y} = \frac{\cos(x)}{x} - \ln(x)\sin(x)$$

$$y' = y\left[\frac{\cos(x)}{x} - \ln(x)\sin(x)\right]$$

$$\boxed{y' = x^{\cos(x)}\left[\frac{\cos(x)}{x} - \ln(x)\sin(x)\right]}$$

(13) $f(x) = \dfrac{x^2\sqrt{2x+5}}{(3x-2)^4}$

Let $y = \dfrac{x^2\,(2x+5)^{1/2}}{(3x-2)^4}$

$$\ln(y) = \ln\left[\frac{x^2\,(2x+5)^{1/2}}{(3x-2)^4}\right]$$

$$\ln(y) = \ln\left[x^2\,(2x+5)^{1/2}\right] - \ln\left[(3x-2)^4\right]$$

$$\ln(y) = \ln(x^2) + \ln\left[(2x+5)^{1/2}\right] - \ln\left[(3x-2)^4\right]$$

$$\ln(y) = 2\ln(x) + \frac{1}{2}\ln(2x+5) - 4\ln(3x-2)$$

$$\frac{d}{dx}\left[\ln(y)\right] = \frac{d}{dx}\left[2\ln(x) + \frac{1}{2}\ln(2x+5) - 4\ln(3x-2)\right]$$

$$\frac{1}{y}y' = 2\frac{d}{dx}\left[\ln(x)\right] + \frac{1}{2}\frac{d}{dx}\left[\ln(2x+5)\right] - 4\frac{d}{dx}\left[\ln(3x-2)\right]$$

$$\frac{y'}{y} = 2\cdot\frac{1}{x} + \frac{1}{2}\,\frac{1}{(2x+5)}\,\cancel{\frac{d}{dx}(2x+5)}\ 2 - 4\,\frac{1}{3x-2}\,\cancel{\frac{d}{dx}(3x-2)}\ 3$$

$$\frac{y'}{y} = \frac{2}{x} + \frac{1}{2x+5} - \frac{12}{3x-2}$$

$$y' = y\left[\frac{2}{x} + \frac{1}{2x+5} - \frac{12}{3x-2}\right]$$

$$\boxed{y' = \frac{x^2\sqrt{2x+5}}{(3x-2)^4}\left[\frac{2}{x} + \frac{1}{2x+5} - \frac{12}{3x-2}\right]}$$

(14) $f(x) = \sin^{-1}(\sqrt{x})$

$$f'(x) = \frac{d}{dx}\left[\sin^{-1}(\sqrt{x})\right]$$

$$= \frac{1}{\sqrt{1-(\sqrt{x})^2}} \; \cancel{\frac{d}{dx}(\sqrt{x})} \quad \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$$

$$\boxed{f'(x) = \frac{1}{2\sqrt{x}\sqrt{1-x}}}$$

(15) $f(x) = e^{\sqrt{x} + x\tan(x)}$

$f'(x) = \frac{d}{dx}\left[e^{\sqrt{x} + x\tan(x)}\right]$

$= e^{\sqrt{x} + x\tan(x)} \quad \frac{d}{dx}\left[\sqrt{x} + x\tan(x)\right]$

$\frac{d}{dx}(\sqrt{x}) + \frac{d}{dx}(x\tan(x))$

$\frac{1}{2\sqrt{x}} \qquad x\frac{d}{dx}(\tan(x)) + \tan(x)\frac{d}{dx}(x)$

$\sec^2(x) \qquad 1$

$f'(x) = e^{\sqrt{x} + x\tan(x)}\left[\frac{1}{2\sqrt{x}} + x\sec^2(x) + \tan(x)\right]$

(16) $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx$

let $u = \cos(x)$

$\frac{du}{dx} = -\sin(x)$; $dx = \frac{du}{-\sin(x)}$

$\int \frac{\cancel{\sin(x)}}{u}\,\frac{du}{-\cancel{\sin(x)}} = -\int \frac{1}{u}\,du$

$-1 \cdot [\ln|u|] + C$;

$-\ln|\cos(x)| + C$

$\boxed{\ln|\sec(x)| + C}$

(17) $\int \frac{1}{e^{2\pi x}}\,dx = \int e^{-2\pi x}\,dx$

Let $u = -2\pi x$

$\frac{du}{dx} = -2\pi$; $dx = \frac{du}{-2\pi}$

$\int e^u \frac{du}{-2\pi} = -\frac{1}{2\pi}\int e^u\,du$

$= -\frac{1}{2\pi} e^u + C$; $\boxed{-\frac{1}{2\pi} e^{-2\pi x} + C}$

(18) $\int \frac{1}{4x-3}\,dx$; $u = 4x-3$

$\frac{du}{dx} = 4$;

$dx = \frac{du}{4}$

$\int \frac{1}{u}\frac{du}{4}$

$\frac{1}{4}\int \frac{1}{u}\,du = \frac{1}{4}\ln|u| + C$

$\frac{1}{4}\ln|4x-3| + C$

(19) $\int \frac{x^2+x-3}{x}\,dx$

$\int\left(\frac{x^2}{x}+\frac{x}{x}-\frac{3}{x}\right)dx$

$\int\left(x+1-\frac{3}{x}\right)dx$

$\int x\,dx+\int dx-3\int\frac{1}{x}\,dx$

$\frac{x^2}{2}+x-3\ln|x|+C$

(20) $\int\frac{1}{a+x^2}\,dx=\int\frac{1}{3^2+x^2}\,dx$

$a=3$

$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$;

$\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C$

(21) $\int\frac{1}{1+4x^2}\,dx=\int\frac{1}{1+(2x)^2}\,dx$

Let $u=2x$

$\frac{du}{dx}=2$;

$dx=\frac{du}{2}$

$\int\frac{1}{1+u^2}\frac{du}{2}$

$\frac{1}{2}\int\frac{1}{1+u^2}\,du$

$$\frac{1}{2}\int \frac{1}{1+u^2}\,du$$

$$\frac{1}{2}\tan^{-1}(u) + C \quad ; \quad \boxed{\frac{1}{2}\tan^{-1}(2x) + C}$$

(22) $$\int \frac{1}{\sqrt{16-x^2}}\,dx = \int \frac{1}{\sqrt{4^2-x^2}}\,dx$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C \qquad a = 4$$

$$\sin^{-1}\left(\frac{x}{4}\right) + C$$

(23) $$\int \frac{1}{\sqrt{1-25x^2}}\,dx = \int \frac{1}{\sqrt{1-(5x)^2}}\,dx$$

Let $u = 5x$

$\frac{du}{dx} = 5$; $$\int \frac{1}{\sqrt{1-u^2}}\,\frac{du}{5}$$

$dx = \frac{du}{5}$

$$\frac{1}{5}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{5}\sin^{-1}(u) + C$$

$$\frac{1}{5}\sin^{-1}(5x) + C$$