East Los Angeles College
Department of Mathematics
Math 261
Test 3

solutions

Determine the critical value for the following functions.

1. $f(x) = x^{1/3} - x^{-2/3}$

$f'(x) = \dfrac{x+2}{3x^{5/3}}$; CV $x=0$; $x=-2$

Determine the absolute max and absolute min for the following functions over the given interval.

2. $f(x) = 2x^3 - 3x^2 - 12x + 1 \quad over \quad [-2,3]$

$f'(x) = 6x^2 - 6x - 12$; CV $x=2$; $x=-1$

$f(-2) = -3$

$f(3) = -8$

$f(2) = -19$ min

$f(-1) = 8$ max

8

Determine the intervals of increasing/decreasing, local max and local min, intervals of concavity, inflection points (if any).

3. $f(x) = \frac{x}{x^2+1}$

$f'(x) = \frac{1-x^2}{(x^2+1)^2}$ ✓ ; CV $x = 1$ ✓; $x = -1$ ✓

f is decreasing on $(-\infty, -1) \cup (1, \infty)$ ✓ ✓

f is increasing on $(-1, 1)$ ✓

Rel max at $x = 1$; rel max $= \frac{1}{2}$ ✓

Rel min at $x = -1$; rel min $= -\frac{1}{2}$ ✓

$f''(x) = \frac{-2x(3-x^2)}{(1+x^2)^3}$ ✓

f is CU on $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$ ✓ ✓

f is CD on $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$ ✓ ✓

inflection points $(-\sqrt{3}, f(-\sqrt{3}))$ ✓

$(0, f(0))$ ✓

$(\sqrt{3}, f(\sqrt{3}))$ ✓

16 ✓

4. $f(\theta) = \sin(\theta) + \cos(\theta)$ *over* $0 \le \theta \le 2\pi$

$f'(\theta) = \cos(\theta) - \sin(\theta)$ ✓ ; $\theta = \frac{\pi}{4}$, $\theta = \frac{5\pi}{4}$ ✓ ✓

f is decreasing on $(\frac{\pi}{4}, \frac{5\pi}{4})$ ✓

f is increasing on $[0, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, 2\pi]$ ✓ ✓

f has a rel max $f(\frac{\pi}{4}) = \sqrt{2}$ ✓

f has a rel min $f(\frac{5\pi}{4}) = -\sqrt{2}$ ✓

$f''(\theta) = -\sin(\theta) - \cos(\theta)$ ✓

f is concave up $(\frac{3\pi}{4}, \frac{7\pi}{4})$ ✓

f is concave down $[0, \frac{3\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi]$ ✓ ✓

f has inflection points

$(\frac{3\pi}{4}, f(\frac{3\pi}{4}))$ ✓

$(\frac{7\pi}{4}, f(\frac{7\pi}{4}))$ ✓

14 ✓

Evaluate the following limits at infinity.

5. $\lim_{x \to \infty} \frac{1-x^2}{x^3-x+1}$

0

6. $\lim_{x \to -\infty} \left(x + \sqrt{x^2 + 2x}\right)$

-1

7. $\lim_{x \to -\infty} \frac{1+x^6}{x^4+1}$ ∞

8. $\lim_{x \to \infty} (x^3 - x^5)$ $-\infty$

12

Using the volume of a sphere formula $V = \frac{4}{3}\pi r^3$.

9. Find the average rate of change of V with respect to the radius when it goes from 5 inch to 8 inches.

$$\bar{v} = \frac{v(8) - v(5)}{8-5} \quad ; \quad \bar{v} = 172\pi$$

≈ 540.3 inch/time

✓ ✓

10. What is the related rates equation for the volume of a sphere.

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

✓ ✓

11. If the radius of sphere is decreasing at a rate of 5 inches per sec, what is the rate of change for the volume of a sphere when the radius is 12 inches?

$$\frac{dv}{dt} = 4\pi\, 12^2 \cdot 5 \quad \text{or} \quad \frac{dv}{dt} = -2880\pi$$

$\approx -9{,}047.7$ in^3/sec

✓ ✓

6 ✓

12. A particle moves along a curve the curve at $y = 2\sin\left(\frac{\pi x}{2}\right)$. As the particle passes through the point $\left(\frac{1}{3}, 1\right)$ its x-coordinate is increasing at a rate of $\sqrt{10}$ cm/sec. How fast is the distance from the particle to the origin changing at this instant?

$$d = \sqrt{x^2 + 4\sin^2\left(\frac{\pi x}{2}\right)}$$

or $d^2 = x^2 + 4\sin^2\left(\frac{\pi x}{2}\right)$

$$d \frac{dd}{dt} = x\frac{dx}{dt} + 2\pi \sin\left(\frac{\pi}{2}x\right)\cos\left(\frac{\pi}{2}x\right)\frac{dx}{dt}$$

at $\left(\frac{1}{3}, 1\right)$ $\quad \frac{dx}{dt} = \sqrt{10}$ $\quad$ ∴ $d = \frac{\sqrt{10}}{3}$

$$\frac{dd}{dt} = 1 + \frac{3\pi\sqrt{3}}{2}$$

≈ 9.1620

8

Determine the linearization function at the indicated points for the following functions.

13. $f(x) = \sin(x)\ at\ a = \frac{\pi}{6}$

$L(x) = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) + \frac{1}{2}$

✓ ✓

14. Approximate $\sin(0.48)$ using the linearization function.

$\sin(0.48) \approx \frac{\sqrt{3}}{2}\left(0.48 - \frac{\pi}{6}\right) + \frac{1}{2}$

≈ 0.4622 ✓ ✓

Verify the function satisfies the hypothesis of the Mean Value Theorem on the given interval and find the value of c such that the conclusion satisfies the mean value theorem.

15. $f(x) = \frac{1}{x}\ over\ [1,3]$; $f'(x) = -\frac{1}{x^2}$ ✓

MVT

$-\frac{1}{c^2} = \frac{\frac{1}{3} - \frac{1}{1}}{3 - 1}$

6 ✓

$c^2 = 3$; $c = \pm\sqrt{3}$

$c = \sqrt{3}$ ✓

Math 261 Test 3

(1) $f(x) = x^{1/3} - x^{-2/3}$

$f'(x) = \frac{d}{dx}(x^{1/3} - x^{-2/3})$

$= \frac{d}{dx}(x^{1/3}) - \frac{d}{dx}(x^{-2/3})$

$= \frac{1}{3}x^{\frac{1}{3}-1} - (-\frac{2}{3})x^{-2/3-1}$

$= \frac{1}{3}x^{\frac{1}{3}-\frac{3}{3}} + \frac{2}{3}x^{-\frac{2}{3}-\frac{3}{3}}$

$= \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-5/3}$

$= \frac{1}{3}x^{-5/3}(x+2)$

$= \frac{1}{3}\frac{(x+2)}{x^{5/3}}$

$f'(x) = \frac{x+2}{3x^{5/3}}$; $f'(x) = 0$; $x+2=0$

$-2 \; -2$

$x = -2$

$f'(x)$ DNE ; $3x^{5/3} = 0$

$x^{5/3} = 0$ or $x = 0$

CV

(2) $f(x) = 2x^3 - 3x^2 - 12x + 1$ $[-2, 3]$

$f'(x) = \frac{d}{dx}(2x^3 - 3x^2 - 12x + 1)$

$= 2\frac{d}{dx}(x^3) - 3\frac{d}{dx}(x^2) - 12\frac{d}{dx}(x) + \frac{d}{dx}(1)$

$= 2 \cdot 3x^2 - 3 \cdot 2x - 12 \cdot 1 + 0$

$= 6x^2 - 6x - 12$

$f'(x) = 6x^2 - 6x - 12$

CV $f'(x) = 0$;

$6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$6(x-2)(x+1) = 0$

$(x-2)(x+1) = 0$

$[-2, 3]$

$x - 2 = 0$	$x + 1 = 0$
$+2 \quad +2$	$-1 \quad -1$
$x = 2$	$x = -1$

$f(x) = 2x^3 - 3x^2 - 12x + 1$

$f(-2) = 2(-2)^3$

$-2 \leq x \leq 3$

$f(-2) = 2(-2)^3 - 3(-2)^2 - 12(-2) + 1$

$f(-2) = -3$

$f(3) = 2 \cdot 3^3 - 3 \cdot 3^2 - 12 \cdot 3 + 1$

$f(3) = -8$

$f(2) = 2 \cdot 2^3 - 3 \cdot 2^2 - 12 \cdot 2 + 1$

$f(2) = -19$ min

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$f(-1) = 8$ max

(3) $f(x) = \dfrac{x}{x^2+1}$

$$f'(x) = \frac{d}{dx}\left(\frac{x}{x^2+1}\right)$$

$$= \frac{(x^2+1)\cancel{\frac{d}{dx}(x)}^{\,1} - x\,\cancel{\frac{d}{dx}(x^2+1)}^{\,2x}}{(x^2+1)^2}$$

$$= \frac{(x^2+1) - x \cdot 2x}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2}$$

$f'(x) = \dfrac{1-x^2}{(x^2+1)^2}$; cv $f'(x) = 0$

$\dfrac{1-x^2}{(x^2+1)^2} = 0$

$1 - x^2 = 0$; $x^2 = 1$

$x = \pm 1$

<u>note</u> $f'(x)$ DNE

$(x^2+1)^2 = 0$ or

$x^2+1 = 0$; never happens!

I | II | III

$- - -$ 0 $+ + +$ 0 $- - -$

$-2 \quad -1 \quad 0 \quad 1 \quad 2$ $\quad f'(x)$

(t) (t) (t)

f is decreasing on $(-\infty,-1) \cup (1,\infty)$

f is increasing on $(-1,1)$

f has a rel max at $x=1$

Rel max $= f(1) = \dfrac{1}{1^2+1} = \dfrac{1}{2}$

f has a rel min at $x=-1$

rel min $= f(-1) = \dfrac{-1}{(-1)^2+1} = -\dfrac{1}{2}$

(4) $f(\theta) = \sin(\theta) + \cos(\theta) \qquad 0 \le \theta \le 2\pi$

$$f'(\theta) = \frac{d}{d\theta}\left(\sin(\theta) + \cos(\theta)\right)$$

$$= \frac{d}{d\theta}(\sin(\theta)) + \frac{d}{d\theta}(\cos(\theta))$$

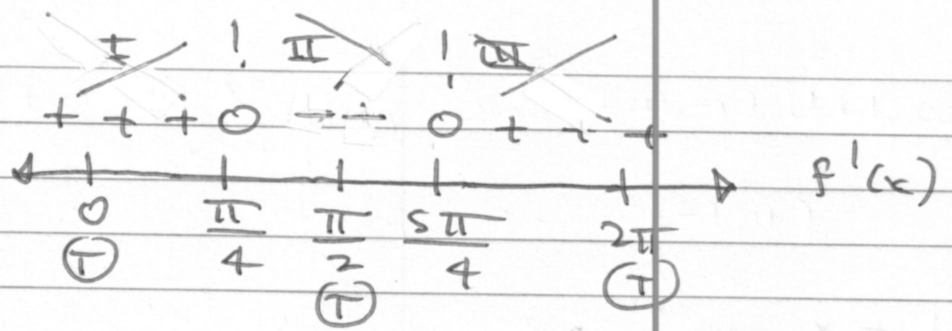
$$f'(\theta) = \cos(\theta) - \sin(\theta)$$

$f'(\theta) = 0$; $\cos(\theta) - \sin(\theta) = 0$

$\sin(\theta) = \cos(\theta)$; $\dfrac{\sin(\theta)}{\cos(\theta)} = 1$

$\tan(\theta) = 1$

$\theta = \dfrac{\pi}{4}$; $\theta = \dfrac{5\pi}{4}$



$$f'(x) = \cos(x) - \sin(x)$$

f is decreasing on $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

f is increasing on $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$

f has a relative max at $x = \frac{\pi}{4}$

relative max = $f\left(\frac{\pi}{4}\right) = \sqrt{2}$

f has a relative min at $x = \frac{5\pi}{4}$

relative min = $f\left(\frac{5\pi}{4}\right) = -\sqrt{2}$

concavity for (3) $f(x) = \frac{x}{x^2+1}$,

$$f'(x) = \frac{1-x^2}{(1+x^2)^2}$$

$$f''(x) = \frac{d}{dx}\left[\frac{1-x^2}{(1+x^2)^2}\right]$$

$$f''(x) = \frac{(1+x^2)^2 \overbrace{\frac{d}{dx}(1-x^2)}^{-2x} - (1-x^2)\overbrace{\frac{d}{dx}\left[(1+x^2)^2\right]}^{2(1+x^2)\cdot 2x}}{(1+x^2)^4}$$

$$= \frac{(1+x^2)^2(-2x) - (1-x^2)\cdot 2(1+x^2)\cdot 2x}{(1+x^2)^4}$$

$$= \frac{-2x(1+x^2)^2 - 4x(1-x^2)(1+x^2)}{(1+x^2)^4}$$

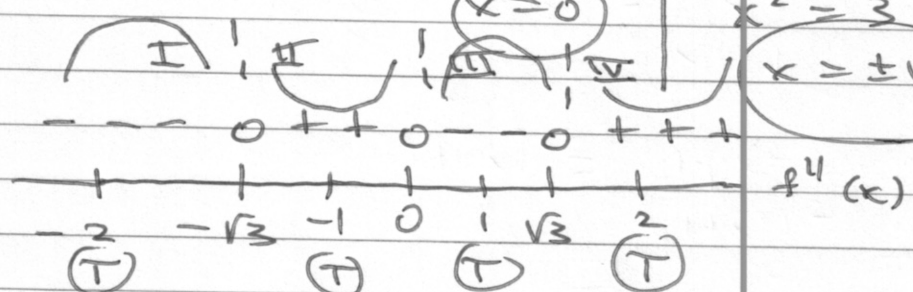
$$= \frac{-2x(1+x^2)\left[(1+x^2) + 2(1-x^2)\right]}{(1+x^2)^4}$$

$$= -\frac{2x\left[1+x^2+2-2x^2\right]}{(1+x^2)^3}$$

$$f''(x) = -\frac{2x\left[3-x^2\right]}{(1+x^2)^3}$$

$f''(x) = 0$; $-2x(3-x^2) = 0$

$-2x = 0$	$3 - x^2 = 0$
$x = 0$	$x^2 = 3$
	$x = \pm\sqrt{3}$



f is concave up on $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

f is concave down on $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$

inflection points at $(-\sqrt{3}, f(-\sqrt{3}))$

$(0, f(0))$

$(\sqrt{3}, f(\sqrt{3}))$

concavity 4 $f(\theta) = \sin(\theta) + \cos(\theta)$ over $[0, 2\pi]$

$$f'(\theta) = \cos(\theta) - \sin(\theta)$$

$$f''(\theta) = \frac{d}{d\theta}\left[\cos(\theta) - \sin(\theta)\right]$$

$$= \frac{d}{d\theta}\left[\cos(\theta)\right] - \frac{d}{d\theta}\left[\sin(\theta)\right]$$

$$= -\sin(\theta) - \cos(\theta)$$

so, $f''(\theta) = 0$; $-\sin(\theta) - \cos(\theta) = 0$

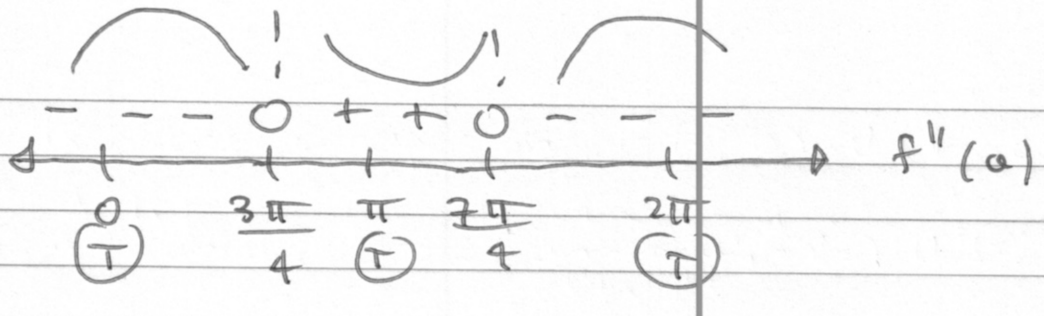
$$-\sin(\theta) = \cos(\theta)$$

$$\sin(\theta) = -\cos(\theta)$$

$$\frac{\sin(\theta)}{\cos(\theta)} = -1$$

$$\tan(\theta) = -1$$

$$\theta = \frac{3\pi}{4}; \quad \theta = \frac{7\pi}{4}$$



$f''(\theta) = -\sin(\theta) - \cos(\theta)$

f is concave up on $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

f is concave down on $\left[0, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right]$

f has inflection points at $\left(\frac{3\pi}{4}, f\left(\frac{3\pi}{4}\right)\right)$

$\left(\frac{7\pi}{4}, f\left(\frac{7\pi}{4}\right)\right)$

(5) $\lim_{x\to\infty} \frac{1-x^2}{x^3-x+1}$

$$\lim_{x\to\infty} \frac{\frac{1}{x^3}-\frac{x^2}{x^3}}{\frac{x^3}{x^3}-\frac{x}{x^3}+\frac{1}{x^3}} = \lim_{x\to\infty} \frac{\cancel{\frac{1}{x^3}}^{0}-\cancel{\frac{1}{x}}^{0}}{1-\cancel{\frac{1}{x^2}}^{0}+\cancel{\frac{1}{x^3}}^{0}} = \frac{0}{1} = \boxed{0}$$

(6) $\lim_{x\to-\infty} \left(x+\sqrt{x^2+2x}\right)$

$\downarrow$

$\infty - \infty$ indeterminant

use algebra!

$\lim_{x\to-\infty} (x^2+2x) = \lim_{x\to-\infty} x(x+2) = \infty$

but, $\lim_{x\to-\infty} (x+\sqrt{x^2+2x}) = -\infty+\infty$ indeterminant

$$\left(x+\sqrt{x^2+2x}\right)\left(\frac{x-\sqrt{x^2+2x}}{x-\sqrt{x^2+2x}}\right)$$

$$\frac{x^2-(x^2+2x)}{x-\sqrt{x^2+2x}} = \frac{x^2-x^2-2x}{x-\sqrt{x^2+2x}}$$

$$= \frac{-2x}{x-\sqrt{x^2+2x}} \; ; \; \lim_{x\to-\infty} \frac{-2x}{x-\sqrt{x^2+2x}} \quad \frac{\infty}{\infty}$$

indeterminant

use algebra

Divide every term by x

$$\lim_{x\to-\infty} \frac{-2x/x}{\frac{x}{x}-\frac{\sqrt{x^2+2x}}{x}}$$

$$\lim_{x\to-\infty} \frac{-2}{1-\frac{\sqrt{x^2+2x}}{x}}$$

But $\sqrt{x^2} = x = |x|$

$$= \begin{cases} x \; ; & x \geq 0 \\ -x \; ; & x < 0 \end{cases}$$

$$\lim_{x\to-\infty} \frac{-2}{1-\frac{\sqrt{x^2+2x}}{-\sqrt{x^2}}}$$

ie, $\sqrt{x^2} = -x$ or $x = -\sqrt{x^2}$

$$\lim_{x\to-\infty} \frac{-2}{1+\sqrt{\frac{x^2}{x^2}+\frac{2x}{x^2}}} = \lim_{x\to-\infty} \frac{-2}{1+\sqrt{1+\frac{2}{x}}} = -1$$

7. $$\lim_{x\to-\infty} \frac{1+x^6}{x^4+1}$$

$$\lim_{x\to-\infty} \frac{\frac{1}{x^4}+\frac{x^6}{x^4}}{\frac{x^4}{x^4}+\frac{1}{x^4}} = \lim_{x\to-\infty} \frac{\frac{1}{x^4}+x^2}{1+\frac{1}{x^4}} = \infty$$

(8) $\lim_{x \to \infty} (x^2 - x^5)$ $\quad \infty - \infty$

indeterminant

$\lim_{x \to \infty} x^2 (1 - x^3) = \infty \cdot (-\infty) = -\infty$

(9) $V = \frac{4}{3}\pi r^3 \quad [5, 8]$

$\overline{V} = \frac{V(8) - V(5)}{8-5}$

$= \frac{\frac{4}{3}\pi 8^3 - \frac{4}{3}\pi 5^3}{3}$

$= \frac{\frac{4}{3}\pi [8^3 - 5^3]}{3} = \frac{4}{9}\pi [387]$

$= 172\pi$

(10) $\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$

$\frac{dV}{dt} = \frac{4}{3}\pi \frac{d}{dt}(r^3) \rightarrow 3r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3} r^2 \frac{dr}{dt}$

$\boxed{\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}}$

(11) $\frac{dr}{dt} = -5$; $\frac{dv}{dt} = ?$; $r = 12$

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt} \quad ; \quad \frac{dv}{dt} = 4\pi\, 12^2(-5)$$

$$\boxed{\frac{dv}{dt} = -2880\pi \ m^3/sec}$$

(12) $y = 2\sin\left(\frac{\pi x}{2}\right)$ $P\left(\frac{1}{3}, 1\right)$; $\frac{dx}{dt} = \sqrt{10}$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x-0)^2 + (y-0)^2}$$

$$d = \sqrt{x^2 + y^2} \qquad d = \sqrt{x^2 + 4\sin^2\left(\frac{\pi x}{2}\right)}$$

$$d^2 = x^2 + 4\sin^2\left(\frac{\pi x}{2}\right)$$

$$\frac{d}{dt}(d^2) = \frac{d}{dt}\left[x^2 + 4\sin^2\left(\frac{\pi x}{2}\right)\right]$$

$$2d\frac{dd}{dt} = 2x\frac{dx}{dt} + 8\sin\left(\frac{\pi x}{2}\right)\cdot\cos\left(\frac{\pi x}{2}\right)\cdot\frac{\pi}{2}\frac{dx}{dt}$$

$$d\frac{dd}{dt} = x\frac{dx}{dt} + 4\sin\left(\frac{\pi x}{2}\right)\cos\left(\frac{\pi x}{2}\right)\cdot\frac{\pi}{2}\frac{dx}{dt}$$

$$d\frac{dd}{dt} = x\frac{dx}{dt} + 2\pi\sin\left(\frac{\pi}{2}x\right)\cos\left(\frac{\pi}{2}x\right)\frac{dx}{dt}$$

at $P\left(\overset{x}{\frac{1}{3}}, \overset{y}{1}\right)$; $\frac{dx}{dt} = \sqrt{10}$; we need d ?

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(\tfrac{1}{3} - 0)^2 + (1 - 0)^2}$$

$$d = \sqrt{\tfrac{1}{9} + 1} \quad ; \quad d = \sqrt{\tfrac{1}{9} + \tfrac{9}{9}}$$

$$d = \sqrt{\tfrac{10}{9}} \quad , \quad d = \frac{\sqrt{10}}{3}$$

$$d \frac{dd}{dt} = x \frac{dx}{dt} + 2\pi \sin\left(\tfrac{\pi}{2} x\right) \cos\left(\tfrac{\pi}{2} x\right) \frac{dx}{dt}$$

$$\frac{\sqrt{10}}{3} \frac{dd}{dt} = \frac{1}{3} \cdot \sqrt{10} + 2\pi \sin\left(\frac{\pi}{2} \cdot \frac{1}{3}\right) \cos\left(\frac{\pi}{2} \cdot \frac{1}{3}\right) \sqrt{10}$$

$$\frac{\sqrt{10}}{3} \frac{dd}{dt} = \frac{\sqrt{10}}{3} + 2\pi \underbrace{\cancel{\sin}\left(\frac{\pi}{6}\right)}_{\to \frac{1}{2}} \underbrace{\cancel{\cos}\left(\frac{\pi}{6}\right)}_{\to \sqrt{3}/2} \sqrt{10}$$

$$\frac{\sqrt{10}}{3} \frac{dd}{dt} = \frac{\sqrt{10}}{3} + 2\pi \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} \sqrt{10}$$

$$\frac{\sqrt{10}}{3} \frac{dd}{dt} = \frac{\sqrt{10}}{3} + \frac{\sqrt{30}\,\pi}{2}$$

$$\frac{dd}{dt} = \left(\frac{\sqrt{10}}{3} + \frac{\sqrt{30}\,\pi}{2}\right) \frac{3}{\sqrt{10}}$$

$$\frac{dd}{dt} = 1 + \frac{\sqrt{30}\,\pi \cdot 3}{2\sqrt{10}}$$

$$\boxed{\frac{dd}{dt} = 1 + \frac{3\pi\sqrt{3}}{2}}$$

(13) $f(x) = \sin(x)$; $a = \frac{\pi}{6}$

$y - y_1 = m(x - x_1)$

$y - f(a) = f'(a)(x-a)$

$y = f'(a)(x-a) + f(a)$

$f'(x) = \cos(x)$; $f'(a) = \cos(\frac{\pi}{6})$; $f(a)$

$f'(a) = \frac{\sqrt{3}}{2}$ $\quad = \sin(\frac{\pi}{6})$

$= \frac{1}{2}$

$y = \underbrace{\frac{\sqrt{3}}{2}(x - \frac{\pi}{6}) + \frac{1}{2}}_{L(x)}$

ie, $\boxed{L(x) = \frac{\sqrt{3}}{2}(x - \frac{\pi}{6}) + \frac{1}{2}}$

(14) $\sin(0.48) \approx \frac{\sqrt{3}}{2}(0.48 - \frac{\pi}{6}) + \frac{1}{2}$

≈ 0.4622

~~(14)~~ (15) $f(x) = \frac{1}{x}$ over $[1, 3]$

MVT $f'(c) = \frac{f(b) - f(a)}{b-a}$

$f'(x) = \frac{d}{dx}(\frac{1}{x}) = -\frac{1}{x^2}$; $-\frac{1}{c^2} = \frac{\frac{1}{b} - \frac{1}{a}}{b-a}$

$$-\frac{1}{c^2} = \frac{\frac{1}{3} - \frac{1}{1}}{3-1}$$

$$-\frac{1}{c^2} = \frac{\frac{1}{3} - 1}{2} \quad ; \quad -c^2 = \frac{2}{\frac{1}{3} - \frac{3}{3}}$$

$$-c^2 = \frac{2}{-2/3} \quad ; \quad -c^2 = 2 \div (-2/3)$$

$$-c^2 = 2 \cdot \left(-\frac{3}{2}\right) \quad ; \quad -c^2 = -3 \quad ; \quad c^2 = 3$$

$$c = \pm\sqrt{3} \quad ; \quad c = \sqrt{3}$$