East Los Angeles College
Department of Mathematics

Math 261

Test 2 (class)

42 ✓

solutions

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Let $s(t) = 2t^3 - 7t^2 + 4t + 1$ be a position function measured in meters where t is measured in seconds and $t \geq 0$

2. Determine the average velocity over the interval [1,2]

$$\text{avg} = \frac{S(2) - S(1)}{2-1} \quad ; \quad \overline{\text{avg}} = -3 \text{ m/sec}$$

✓ ✓

3. Determine the initial position.

$S(0) = 1$ m ✓

4. Determine the velocity function.

$v(t) = 6t^2 - 14t + 4$ ✓ ✓ ✓

5. Determine the initial velocity.

$v(0) = 4$ m/sec ✓

6. Determine the velocity at t=3 seconds.

✓

$v(3) = 16$ m/sec

7. Determine the direction of travel at t=3 seconds.

$v(3) > 0$; Right or up

8

✓ ✓

8. Determine the speed at t=3 seconds.

speed $= |v(3)| = |16| = 16$ m/sec ✓ ✓

9. At what time t does the particle stop? ✓

$v(t) = 0$; $6t^2 - 14t + 4 = 0$

$t = \frac{1}{3}$ sec , $t = 2$ sec ✓ ✓

10. For what time interval t is the particle moving to the right?

$v(t) > 0$

$[0, \frac{1}{3}) \cup (2, \infty)$ ✓ ✓

11. For what time interval t is the particle moving to the left?

$v(t) < 0$ $(\frac{1}{3}, 2)$ ✓

12. Determine the acceleration function.

$a(t) = \frac{d}{dt}[v(t)] = 12t - 14$ ✓ ✓

13. What is the acceleration at t=3 seconds?

$a(3) = 22$ m/sec^2 ✓

14. Is the particle speeding up or slowing down at t=3 seconds?

$v(3) > 0$

same sign

$a(3) > 0$ ✓

particle is speeding up

12 ✓

15. Determine the equation of the line tangent to the curve at the indicated point.

$$y = \sec(x) - 2\cos(x) \text{ at the point } \left(\frac{\pi}{3}, 1\right)$$

$x \quad y$

$y - 1 = 3\sqrt{3}\left(x - \frac{\pi}{3}\right)$

$y = 3\sqrt{3}x - \sqrt{3}\pi + 1$

✓ ✓

as $m_{tan} = y'\Big|_{x=\frac{\pi}{3}} = \sec(x)\tan(x) + 2\sin(x)$

✓

$m_{tan} = 3\sqrt{3}$ ✓

16. Determine the points of horizontal tangents for the following curve

$$y = x^3 - x^2 - x + 1$$

$y' = 3x^2 - 2x - 1$; $\frac{dy}{dx} = 0$

✓ ✓ ✓ ✓

$3x^2 - 2x - 1 = 0$; $x = -1/3$, $x = 1$

$\left(-\frac{1}{3}, \frac{32}{27}\right)$; $(1, 0)$

8 ✓

Differentiate the following using Chain Rule

17. $y = (x+1)^2(x+3)^3$

$$\frac{dy}{dx} = \frac{d}{dx}\left[(x+1)^2(x+3)^3\right]$$

$$= (x+1)^2 \frac{d}{dx}\left[(x+3)^3\right] + (x+3)^3 \frac{d}{dx}\left[(x+1)^2\right]$$

$$\frac{dy}{dx} = 3(x+1)^2(x+3)^2 + 2(x+1)(x+3)^3$$

18. $y = tan\left(\frac{x}{x-1}\right)$

8

$$\frac{dy}{dx} = \frac{d}{dx}\left[\tan\left(\frac{x}{x-1}\right)\right]$$

$$= \sec^2\left(\frac{x}{x-1}\right)\frac{d}{dx}\left(\frac{x}{x-1}\right)$$

$$\frac{dy}{dx} = -\sec^2\left(\frac{x}{x-1}\right)\cdot\frac{1}{(x-1)^2}$$

Determine the first and second derivatives for the following functions.

19. $y = sin(x^3)$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin(x^3) \right]$$

$$= \cos(x^3) \frac{d}{dx}(x^3)$$

$$\frac{dy}{dx} = 3x^2 \cos(x^3)$$

✓ ✓ ✓

20. $y = cos\left(\frac{1}{\sqrt{x}}\right)$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\cos\left(\frac{1}{\sqrt{x}}\right) \right]$$

$$= -\sin\left(\frac{1}{\sqrt{x}}\right) \frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)$$

$$\frac{dy}{dx} = \frac{1}{2x\sqrt{x}} \sin\left(\frac{1}{\sqrt{x}}\right).$$

✓ ✓ ✓

6

✓

math 261 Test 2

(2) $S(t) = 2t^3 - 7t^2 + 4t + 1$;

$$\underset{[1,2]}{\overline{avg}} = \frac{S(2) - S(1)}{2-1}$$

$$= \frac{(2\cdot 2^3 - 7\cdot 2^2 + 4\cdot 2 + 1) - (2\cdot 1^3 - 7\cdot 1^2 + 4\cdot 1 + 1)}{1}$$

$$= (2\cdot 8 - 7\cdot 4 + 8 + 1) - (2 - 7 + 4 + 1)$$

$$= (16 - 28 + 8 + 1) - 0$$

$$= \boxed{-3 \ m/sec}$$

(3) $S(0) = 2\cdot 0^3 - 7\cdot 0^2 + 4\cdot 0 + 1$

$$\boxed{S(0) = 1} m$$

(4) $V(t) = S'(t) = \frac{d}{dt}(2t^3 - 7t^2 + 4t + 1)$

$$= 2\frac{d}{dt}(t^3) - 7\frac{d}{dt}(t^2) + 4\frac{d}{dt}(t) + \frac{d}{dt}(1)$$

$$= 2\cdot 3t^2 - 7\cdot 2t + 4$$

$$\boxed{V(t) = 6t^2 - 14t + 4}$$

(5) $V(0) = 6\cdot 0^2 - 14\cdot 0 + 4$

$$\boxed{V(0) = 4 \ m/Sec}$$

(6) $v(3) = 6\cdot 3^2 - 14\cdot 3 + 4$

$v(3) = 6\cdot 9 - 42 + 4$

$v(3) = 54 - 42 + 4$

$\boxed{v(3) = 16 \text{ m/sec}}$

(7) $v(3) > 0$; Right or UP

(8) Speed $= |v(3)| = |16| = \boxed{16 \text{ m/sec}}$

(9) $v(t) = 0$; $6t^2 - 14t + 4 = 0$

or $3t^2 - 7t + 2 = 0$

$(3t - 1)(t - 2) = 0$

$3t - 1 = 0$	$t - 2 = 0$
$3t = 1$	$+2 \quad +2$
$\frac{\cancel{3}t}{\cancel{3}} = \frac{1}{3}$	$\boxed{t = 2 \text{ sec}}$
$\boxed{t = 1/3 \text{ sec}}$	

(10) Sign analysis, on $v(t)$

	I		II		III
$v(t)$	$+ + +$	0	$- -$	0	$+ + +$
t	0, $\frac{1}{4}$	$1/3$	1	2	3
	(T)		(T)		(T)

(Right) $\boxed{[0, 1/3) \cup (2, \infty)}$

(11) left $(\frac{1}{3}, 2)$

(12) $v(t) = 6t^2 - 14t + 4$

$$a(t) = \frac{d}{dt}[v(t)]$$

$$= \frac{d}{dt}\left[6t^2 - 14t + 4\right]$$

$$= 6\frac{d}{dt}\cancel{(t^2)}^{2t} - 14\frac{d}{dt}\cancel{(t)}^{1} + \frac{d}{dt}\cancel{(4)}^{0}$$

$$= 6 \cdot 2t - 14$$

$$\boxed{a(t) = 12t - 14}$$

(13) $a(3) = 12 \cdot 3 - 14$

$= 36 - 14$; $\boxed{a(3) = 22 \text{ m/sec}^2}$

(14) $v(3) = 16$ m/sec $\quad a(3) = 22$ m/sec^2

(+) $\quad$ (+)

Both are the same sign!

$\boxed{\text{Speeding up}}$

(15) $y - y_1 = m_{tan}(x - x_1)$; $(\frac{\pi}{3}, 1)$ [x y]

m_{tan}

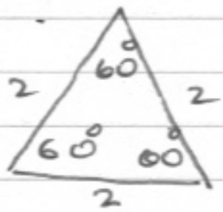
$$m_{tan} = \frac{dy}{dx} = \frac{d}{dx}\left(\sec(x) - 2\cos(x)\right)$$

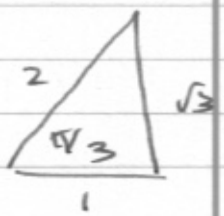
$$= \frac{d}{dx}[\sec(x)] - 2\frac{d}{dx}[\cos(x)]$$

$$m_{tan} = \sec(x)\tan(x) - 2(-\sin(x))$$

$$m_{tan} = \sec(x)\tan(x) + 2\sin(x) \Big|_{x=\frac{\pi}{3}}$$

$$m_{tan} = \sec\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{3}\right) + 2\sin\left(\frac{\pi}{3}\right)$$





$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\tan\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{1}$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\sec\left(\frac{\pi}{3}\right) = 2$$

$$m_{tan} = 2\cdot\sqrt{3} + 2\,\frac{\sqrt{3}}{2}$$

$$\boxed{m_{tan} = 3\sqrt{3}}$$

$$y - \underset{1}{y_1} = m(x - \underset{\frac{\pi}{3}}{x_1})$$

$$y - 1 = 3\sqrt{3}\left(x - \frac{\pi}{3}\right)$$

$$y - 1 = 3\sqrt{3}x - \sqrt{3}\pi \quad ; \quad \boxed{y = 3\sqrt{3}x - \sqrt{3}\pi + 1}$$

(16) $y = x^3 - x^2 - x + 1 \quad ; \quad y' = 0$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^3 - x^2 - x + 1\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) - \frac{d}{dx}(x^2) - \frac{d}{dx}(x) + \frac{d}{dx}(1)$$

$$\frac{dy}{dx} = 3x^2 - 2x - 1 \quad ; \quad \frac{dy}{dx} = 0$$

$$3x^2 - 2x - 1 = 0$$

$$(3x + 1)(x - 1) = 0$$

$3x + 1 = 0$	$x - 1 = 0$
$-1 \quad -1$	$x = 1$
$3x = -1$	
$x = -1/3$	

(17) $y = (x+1)^2 (x+3)^3$

$$\frac{dy}{dx} = \frac{d}{dx}\left[(x+1)^2 (x+3)^3\right]$$

$$= (x+1)^2 \frac{d}{dx}\left[(x+3)^3\right] + (x+3)^3 \frac{d}{dx}\left[(x+1)^2\right]$$

$$= (x+1)^2 \cdot 3(x+3)^2 \cancel{\frac{d}{dx}\left[(x+3)\right]}^{\,1}$$

$$+ (x+3)^3 \cdot 2(x+1) \cancel{\frac{d}{dx}(x+1)}^{\,1}$$

$$\boxed{\frac{dy}{dx} = 3(x+1)^2(x+3)^2 + 2(x+1)(x+3)^3}$$

(18) $y = \tan\left(\frac{x}{x-1}\right)$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\tan\left(\frac{x}{x-1}\right)\right]$$

$$= \sec^2\left(\frac{x}{x-1}\right)\frac{d}{dx}\left(\frac{x}{x-1}\right)$$

$$= \sec^2\left(\frac{x}{x-1}\right)\left[\frac{(x-1)\overset{1}{\cancel{\frac{d}{dx}(x)}} - x\overset{1}{\cancel{\frac{d}{dx}(x-1)}}}{(x-1)^2}\right]$$

$$= \sec^2\left(\frac{x}{x-1}\right)\left[\frac{(x-1) - x}{(x-1)^2}\right]$$

$$= \sec^2\left(\frac{x}{x-1}\right)\left(\frac{x-1-x}{(x-1)^2}\right)$$

$$= \sec^2\left(\frac{x}{x-1}\right)\left(-\frac{1}{(x-1)^2}\right)$$

$$\boxed{\frac{dy}{dx} = -\sec^2\left(\frac{x}{x-1}\right)\cdot\frac{1}{(x-1)^2}}$$

(19) $y = \sin(x^3)$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin(x^3)\right]$$

$$= \cos(x^3)\ \cancel{\frac{d}{dx}(x^3)} \quad 3x^2$$

$$= \cos(x^3) \cdot 3x^2$$

$$\boxed{\frac{dy}{dx} = 3x^2 \cos(x^3)}$$

(20) $y = \cos\left(\frac{1}{\sqrt{x}}\right)$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\cos\left(\frac{1}{\sqrt{x}}\right)\right]$$

$$= -\sin\left(\frac{1}{\sqrt{x}}\right)\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)$$

$x^{-1/2}$

note $\frac{d}{dx}\left(x^{-1/2}\right)$

$$= -\frac{1}{2}x^{-1/2-1}$$

$$= -\frac{1}{2}x^{-3/2}$$

$$= -\frac{1}{2\sqrt{x^3}}$$

$$= -\frac{1}{2x\sqrt{x}}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{2x\sqrt{x}} \cdot \sin\left(\frac{1}{\sqrt{x}}\right)}$$