East Los Angeles College
Department of Mathematics
Math 261
Test 1

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Show your work for credit.

Evaluate the following limits by using algebra.

1. $\lim_{x \to -5} \frac{x^2+7x+10}{x^2+4x-5}$

$\lim_{x \to -5} \frac{x+2}{x-1}$ $\frac{1}{2}$

2. $\lim_{x \to -1} \frac{x^3+1}{x^2-1}$

$\lim_{x \to -1} \frac{x^2-x+1}{x-1}$ $-\frac{3}{2}$

3. $\lim_{x \to 16} \frac{4-\sqrt{x}}{x-16}$

$\lim_{x \to 16} \frac{-1}{4+\sqrt{x}}$ $-\frac{1}{8}$

4. $\lim_{x \to 4^+} \frac{|x-4|}{x-4}$

$\lim_{x \to 4^+}$ 1 1

12

5. $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{|x|}\right)$

$\lim_{x \to 0^+} 0 = 0$

6. $\lim_{x \to -5} \left(\frac{\frac{1}{5}+\frac{1}{x}}{5+x}\right)$

$\lim_{x \to -5} \frac{1}{5x}$ $-\frac{1}{25}$

7. $\lim_{x \to -\pi/4} \tan(x)$

$\lim_{x \to -\frac{\pi}{4}} \tan(x) = \tan\left(-\frac{\pi}{4}\right)$

-1

8. $\lim_{x \to \pi^-} csc(x)$

$\lim_{x \to \pi^-} \csc(x) = \infty$

12

9. If $10 \le f(x) \le x^2 + 2x + 2$ for all x, then determine $\lim_{x \to 2} f(x)$

$\lim_{x \to 2} 10 = \lim_{x \to 2} x^2 + 2x + 2 = 10$

by Squeeze Theorem, $\lim_{x \to 2} f(x) = 10$

4✓

10. Determine whether the function is discontinuous or continuous at $x = 0, x = 1, x = 2$.

$$f(x) = \begin{cases} x - 4 & for\ x \le 1 \\ x^2 + x - 5 & for\ 1 < x \le 2 \\ 2 - x & for\ x > 2 \end{cases}$$

$\lim_{x \to 0} f(x) = f(0) = -5$

| f is continuous at 0 |

$\lim_{x \to 1} f(x) = f(1)$

| f is continuous at 1 |

$\lim_{x \to 2} f(x) \ne f(2)$

DNE

| f is discontinuous at 2

6✓

i.e., f is continuous at $x=0$, $x=1$
discontinuous at $x=2$

✓

11. Determine the intervals of continuity for the following functions.

$$f(x) = \sin(x)\sqrt{x^2 + 4x - 5}$$

4✓

$(-\infty, -5] \cup [1, \infty)$

14✓

12. Determine the intervals of continuity for the following functions.

$$f(x) = \frac{\sqrt{x}}{x^2-16}$$

$[0, 4) \cup (4, \infty)$

4 ✓

13. Show there is a root in the following interval $(0,1)$ for $f(x) = \sqrt[3]{x} + x - 1$

$f(0) = -1$ and $f(1) = 1$

↓ neg ↓ positive

by IVT there is a $c \in (0,1)$ such that $f(c) = 0$.

Thus there is a root for f.

4 ✓

8 ✓

Math 261 Test 1

(1) $$\lim_{x \to -5} \frac{x^2+7x+10}{x^2+4x-5}$$

$$\lim_{x \to -5} \frac{(x+2)\cancel{(x+5)}}{\cancel{(x+5)}(x-1)}$$

$$\lim_{x \to -5} \frac{x+2}{x-1}$$

$$\frac{-5+2}{-5-1} \qquad \frac{-3}{-6} \qquad \frac{1}{2}$$

(2) $$\lim_{x \to -1} \frac{x^3+1}{x^2-1}$$

$$\lim_{x \to -1} \frac{\cancel{(x+1)}(x^2 - x+1)}{\cancel{(x+1)}(x-1)}$$

$$\lim_{x \to -1} \frac{x^2-x+1}{x-1}$$

$$\frac{(-1)^2-(-1)+1}{-1-1}$$

$$\frac{1+1+1}{-2} \qquad -\frac{3}{2}$$

(3) $\lim_{x \to 16} \frac{(4-\sqrt{x})}{(x-16)} \cdot \frac{(4+\sqrt{x})}{(4+\sqrt{x})}$

$\lim_{x \to 16} \frac{16-x}{(x-16)(4+\sqrt{x})}$

$\lim_{x \to 16} \frac{-1\ \cancel{(x-16)}}{\cancel{(x-16)}(4+\sqrt{x})}$

$\lim_{x \to 16} \frac{-1}{4+\sqrt{x}}$

$\frac{-1}{4+\sqrt{16}}$

$\frac{-1}{4+4}$ $\quad -\frac{1}{16}$

(4) $\lim_{x \to 4^+} \frac{|x-4|}{x-4}$

$$|x-4| = \begin{cases} x-4 & ; \ x-4 \geq 0 \\ & \quad x \geq 4 \\ -(x-4) & ; \ x-4 < 0 \\ & \quad x < 4 \end{cases}$$

$\lim_{x \to 4^+} \frac{x-4}{x-4} = \lim_{x \to 4^+} 1 = 1$

(5) $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{|x|} \right)$

$$|x| = \begin{cases} x & ; \quad x \geq 0 \quad \text{(Right)} \\ -x & ; \quad x < 0 \quad \text{(left)} \end{cases}$$

$$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{x} \right) = \lim_{x \to 0^+} 0 = \boxed{0}$$

(6) $\lim_{x \to -5} \left(\frac{\frac{1}{5} + \frac{1}{x}}{5+x} \right)$

$$\lim_{x \to -5} \frac{\frac{x+5}{5x}}{5+x}$$

$$\lim_{x \to -5} \frac{x+5}{5x} \div 5+x$$

$$\lim_{x \to -5} \frac{\cancel{x+5}}{5x} \cdot \frac{1}{\cancel{5+x}}$$

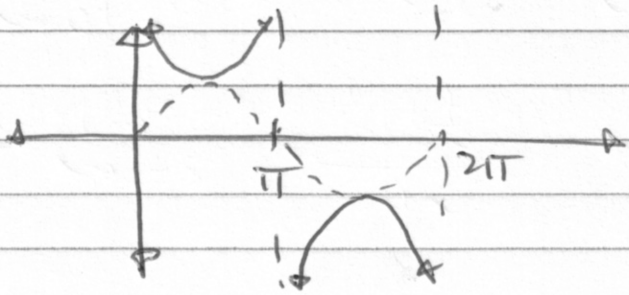
$$\lim_{x \to -5} \frac{1}{5x}$$

$$\frac{1}{5(-5)} \quad \boxed{-\frac{1}{25}}$$

(7) $\lim_{x \to -\frac{\pi}{4}} \tan(x)$

$\tan\left(-\frac{\pi}{4}\right)$ -1

(8) $\lim_{x \to \pi^-} \csc(x)$ note $\csc(x) = \frac{1}{\sin(x)}$



$\lim_{x \to \pi^-} \csc(x)$

$= \boxed{\infty}$

(9) $\lim_{x \to 2} 10 = 10$

$\lim_{x \to 2} x^2 + 2x + 2 = 2^2 + 2 \cdot 2 + 2$

$= 4 + 4 + 2$

$= 10$

∴ by the Squeeze Theorem

$\lim_{x \to 2} f(x) = \boxed{10}$

(10) $\lim_{x \to 1^-} f(x) = 1 - 4$

$= -3$

$\lim_{x \to 1^+} f(x) = 1^2 + 1 - 5$

$= 2 - 5$

$= -3$

$f(1) = 1 - 4 = -3$

ie, $\lim_{x \to 1} f(x) = f(1)$, f is continuous at $x = 1$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} x^2 + x - 5 = 0^2 + 0 - 5$

$= -5$

$f(0) = 0^2 + 0 - 5$

$= -5$

ie, $\lim_{x \to 0} f(x) = f(0)$; f is continuous at $x = 0$.

$\lim_{x \to 2^-} f(x) = 2^2 + 2 - 5$

$= 4 + 2 - 5$

$= 6 - 5$

$= 1$

$\lim_{x \to 2^+} 2 - x = 2 - 2 = 0$

$\lim_{x \to 2} f(x) = DNE$; f is discontinuous at $x = 2$.

(11)

$f(x) = \sin(x)\sqrt{x^2+4x-5}$

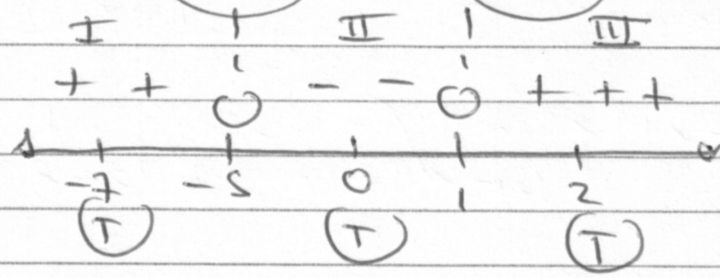
4
12

$x^2+4x-5 \geq 0$

$(x+5)(x-1) \geq 0$

<u>note</u> $x+5=0$ $\quad$ $x-1=0$

$x=-5$ $\quad$ $x=1$

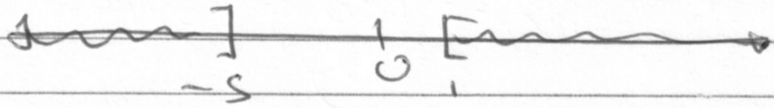


0 2 $\quad$ 0 2
$(x+5)(x-1)$
-7 $\quad$ -7

-2 (-8)

5 (-1)

7 · 1



$\boxed{(-\infty, -5] \cup [1, \infty)}$

(12) $f(x) = \dfrac{\sqrt{x}}{x^2-16}$; $x \geq 0$

$x^2-16=0$

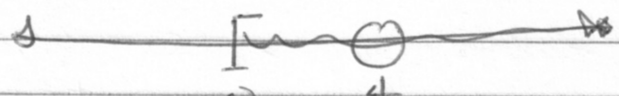
$x^2=16$

$x=\pm\sqrt{16}$

$x=\pm 4$

$x \geq 0$

ie, Domain $= \{x \mid x \neq \pm 4\}$



ie, $\boxed{[0,4) \cup (4,\infty)}$

(13) $f(x) = \sqrt[3]{x} + x - 1$ over $[0, 1]$

$f(0) = \sqrt[3]{0} + 0 - 1$

$= -1$

by IMVT

$f(1) = \sqrt[3]{1} + 1 - 1$

there is a $c \in (0, 1)$

$= 1 + 1 - 1$

Such that $f(c) = 0$

$= \cancel{0}\,1$

ie, f has a root