East Los Angeles College

Department of Mathematics

Math 261

Test 1

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Evaluate the following limits

1. $\lim_{x\to 3^-} \frac{4}{x-3}$ $-\infty$

2. $\lim_{x\to \frac{\pi}{2}^+} \tan(x)$ $-\infty$

3. $\lim_{x\to \frac{\pi}{2}^-} \sec(x)$ ∞

4. $\lim_{x\to 0}(5x^3 - 7x + 1)$ 1

5. $\lim_{x\to 1}\left(\frac{4x+1}{x^2-3x+2}\right)$ DNE

6. $\lim_{x\to -2}\left(\frac{x^2+8x+12}{x^2-x-6}\right)$ $-\frac{4}{5}$

7. $\lim_{x\to 4}\left(\frac{x-4}{|x-4|}\right)$ DNE

8. $\lim_{x\to 0^+}(|x| + 2x)$ 0

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Let $f(x) = \begin{cases} x^2 - 4 & for\ x \geq 1 \\ 3x - 6 & 0 \leq x < 1 \\ \frac{1}{x} & x < 0 \end{cases}$

Answer the following questions.

9. $\lim_{x \to 1} f(x)$ -3

10. $f(1)$ -3

11. Is the function continuous at $x = 1$, explain why or why not?

f is continuous at 1 ; $\lim_{x \to 1} f(x) = f(1)$

12. $\lim_{x \to 0} f(x)$ DNE

13. $f(0)$ -6

14. Is the function continuous at $x = 0$, explain why or why not?

f is not continuous at 0 ; $\lim_{x \to 0} f(x) \neq f(0)$

15. $\lim_{x \to -5} f(x)$ $-\frac{1}{5}$

16. $f(-5)$ $-\frac{1}{5}$

17. Is the function continuous at -5, explain why or why not?

f is continuous at -5 ; $\lim_{x \to -5} f(x) = f(-5)$

18. $\lim_{x \to 2} f(x)$ 0

19. $f(2)$ 0

20. Is the function continuous at $x = 2$, explain why or why not?

f is continuous at 2 ; $\lim_{x \to 2} f(x) = f(2)$

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21. Show that $x^3 = \sqrt{x} + 5$ has a solution in the interval $(1,4)$.
Hint: Use the Intermediate Value Theorem

$x^3 = \sqrt{x} + 5$; let $f(x) = x^3 - \sqrt{x} - 5$

$f(1) = 1^3 - \sqrt{1} - 5$; $f(1) = -5$ negative

$f(4) = 4^3 - \sqrt{4} - 5$; $f(4) = 64 - 2 - 5$;

$f(4) = 57$ positive

so, by the IVT, there is a $r \in (1,4)$ such that $f(r) = 0$.

ie, there is a root to the function f

ie, $f(r) = r^3 - \sqrt{r} - 5$ and $f(r) = 0$

so $r^3 - \sqrt{r} - 5 = 0$ for $r \in (1,4)$ #

✓ ✓ ✓

Determine the points of discontinuity for the following functions.

22. $f(x) = \frac{\cos(x)}{x^2-4}$

$x^2 - 4 = 0$; $x = \pm 2$ (VA)

f is discontinuous at $x = \pm 2$

✓ ✓ ✓

6 ✓

Determine the interval of continuity for the following functions.

23. $f(x) = \frac{\sqrt{x}}{x^2-9}$ f is continuous where it is defined.

ie, $x \geq 0$ and $x^2 - 9 \neq 0$

$x^2 \neq 9$; $x \neq \pm 3$

f is continuous for $x \geq 0$ and $x \neq 3$

$\{x \mid x \geq 0, x \neq 3\}$ ✓

$[0, 3) \cup (3, \infty)$

✓ ✓

24. Determine the value for c that makes the function continuous.

$$f(x) = \begin{cases} x^2 - c & for\ x > 2 \\ 4x + 2c & for\ x \leq 2 \end{cases}$$

$\lim_{x \to 2} f(x) = f(2)$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

$2^2 - c = 4 \cdot 2 + 2c$

$4 - c = 8 + 2c$ ✓

$-2c \qquad -2c$

✓

$4 - 3c = 8$

$-4 \qquad -4$

$-3c = 4$

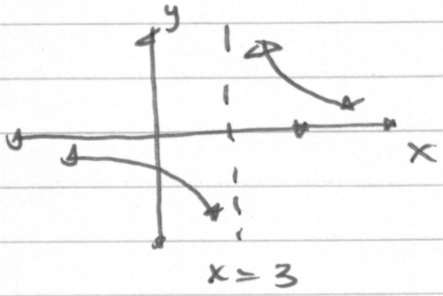
$\frac{-3c}{-3} = \frac{4}{-3}$ ✓

$c = -\frac{4}{3}$

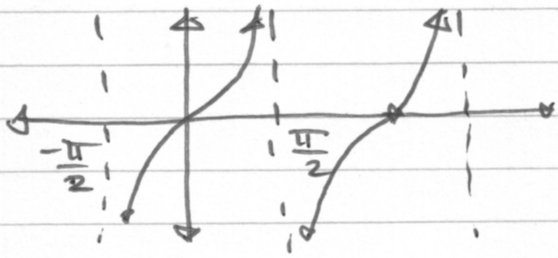
6 ✓

Math 261 Test 1

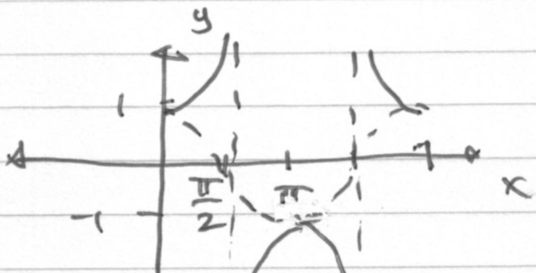
(1) $\lim\limits_{x \to 3^-} \dfrac{4}{x-3} = -\infty$



(2) $\lim\limits_{x \to \frac{\pi}{2}^+} \tan(x) = -\infty$



(3) $\lim\limits_{x \to \frac{\pi}{2}^-} \sec(x) = \infty$



(4) $\lim\limits_{x \to 0} (5x^3 - 7x + 1) = 5 \cdot 0^3 - 7 \cdot 0 + 1$

$= 1$

(5) $\lim_{x \to 1} \left(\frac{4x+1}{x^2-3x+2} \right)$ (VA) $x = 1$, $x = 2$

factor

$(x-2)(x-1)$

let $x = 0.999$; $y = \frac{4x+1}{(x-2)(x-1)}$

small

as $x \to 1^-$; $y \to -\infty$

let $x = 1.0001$; $y = \frac{4x+1}{(x-2)(x-1)}$

small

as $x \to 1^+$; $y \to \infty$

$\therefore$, $\lim_{x \to 1} \frac{4x+1}{(x-2)(x-1)} = \text{DNE}$

(6) $\lim_{x \to -2} \left(\frac{x^2+8x+12}{x^2-x-6} \right)$

$\lim_{x \to -2} \frac{(x+6)\cancel{(x+2)}}{(x-3)\cancel{(x+2)}}$

$\lim_{x \to -2} \frac{x+6}{x-3} = \frac{-2+6}{-2-3}$

$= \frac{4}{-5}$

$= -\frac{4}{5}$

(7)

$$\lim_{x \to 4} \left(\frac{x-4}{|x-4|} \right)$$

$$\frac{x-4}{|x-4|} = \begin{cases} \frac{x-4}{x-4}, & \text{if } x-4 > 0 \\ \frac{x-4}{4-x}, & \text{if } x-4 < 0 \end{cases}$$

$$\frac{x-4}{|x-4|} = \begin{cases} 1, & x > 4 \text{ ; R of 4} \\ -1, & x < 4 \text{ ; L of 4} \end{cases}$$

$$\lim_{x \to 4^+} \left(\frac{x-4}{|x-4|} \right) = 1$$

$$\lim_{x \to 4^-} \left(\frac{x-4}{|x-4|} \right) = -1$$

$$\lim_{x \to 4} \left(\frac{x-4}{|x-4|} \right) = \text{DNE}$$

(8) $\lim_{x \to 0^+} (|x| + 2x)$

$$|x| + 2x = \begin{cases} x + 2x \text{ ; } & x \geq 0 \\ -x + 2x \text{ ; } & x < 0 \end{cases}$$

$$|x| + 2x = \begin{cases} 3x & ; \quad x \geq 0 \quad R \\ x & ; \quad x < 0 \quad L \end{cases}$$

$$\lim_{x \to 0^+} (|x| + 2x) = 3 \cdot 0$$

$$= 0$$

(9) $\lim_{x \to 1^+} f(x) = 1^2 - 4$

$$= 1 - 4$$

$$= -3$$

ie, $\lim_{x \to 1} f(x) = \boxed{-3}$

$\lim_{x \to 1^-} f(x) = 3 \cdot 1 - 6$

$= 3 - 6$

$= -3$

(11) f is continuous at 1 as $\lim_{x \to 1} f(x) = f(1)$

(10) $f(1) = 1^2 - 4$

$= -3$

(12) $\lim_{x \to 0^+} f(x) = 3 \cdot 0 - 6$

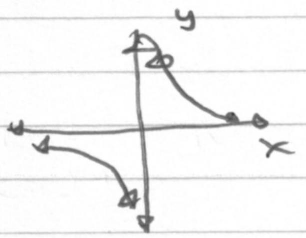
$= -6$

$\lim_{x \to 0} f(x) \neq$ DNE

$\lim_{x \to 0^-} f(x) = -\infty$

(13) $f(0) = 3 \cdot 0 - 6$

$= -6$



(14) f is discontinuous at 0 as $\lim_{x \to 0} f(x) \neq f(0)$

(15)

$\lim_{x \to -5} f(x) = -\frac{1}{5}$ by Direct Sub

(16) $f(-5) = -\frac{1}{5}$; (17) f is continuous at -5

(18) $\lim_{x \to 2} f(x) = 2^2 - 4$ by direct sub

$= 0$

(19) $f(2) = 0$ (20) f is continuous at 0