East Los Angeles College
Department of Mathematics
Math 261
Test 2

40 ✓
4P ✓

Use the definition of derivative to differentiate the following.

1. $f(x) = \sqrt{2x+5}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2(x+h)+5} - \sqrt{2x+5}}{h}$$

✓

$$= \lim_{h \to \infty} \left(\frac{\sqrt{} - \sqrt{}}{h} \right) \frac{\sqrt{2x+2h+5} + \sqrt{2x+5}}{\sqrt{2x+2h+5} + \sqrt{2x+5}}$$

$$= \lim_{h \to 0} \frac{2}{\sqrt{2x+2h+5}\ \sqrt{2x+5}}$$

✓

$$= \frac{2}{2\sqrt{2x+5}}$$

$$= \boxed{\frac{1}{\sqrt{2x+5}}}$$ ✓

3 ✓

Let $s(t)$ represent a position function along a horizontal line in meters and t represent time t in seconds.

$$s(t) = -16t^2 + 64t + 96$$

Determine the following.

2. The initial position of the particle.

$S(0) = 96$ m ✓

3. The velocity function.

$v(t) = -32t + 64$ ✓ ✓

4. The initial velocity of the particle.

$v(0) = 64$ m/sec ✓

5. The initial speed of the particle.

$|v(0)| = |64| = 64$ m/sec ✓

6. The time t when the particle stops.

$t = 2$ sec ; $v(t) = 0$ ✓

7. The time t for which the particle is moving to the right.

$0 \leq t < 2$; $v(t) > 0$ ✓ ✓

8. The time t for which the particle is moving to the left.

$t > 2$; $v(t) < 0$ ✓ ✓

9. The speed of the particle at 5 seconds.

$|v(5)| = 96$ m/sec ✓ ✓

10. The total distance traveled in the first 5 seconds.

Total Distance $= S(2) - S(0) + S(2) - S(5)$

$= 64 + 144$

$= 208$ m ✓

13

Determine the equation of the tangent line for the following curves.

11. $f(x) = (x+1)(x^2-4)$ at $x = 1$

$m = y' = 3x^2 + 2x - 4 \Big|_{x=1}$

✓

; $m = 1$; $y - y_1 = m(x - x_1)$

✓ $y - -6 = 1(x-1)$

$y + 6 = x - 1$

$y = x - 7$

✓

12. $f(x) = x\sin(x)$ at $= 0$

$m = y' = x\cos x + \sin x \Big|_{x=0}$

✓

$m = 0$; $y - 0 = 0(x-0)$

✓ $y = 0$

✓

6 ✓

Determine the values of x for which the following functions have horizontal tangents.

13. $f(x) = x - 2cos(x)$ over the interval $0 \leq x < 2\pi$

$y' = 1 + 2\sin x$; $y' = 0$ or

✓

$1 + 2\sin x = 0$

i.e, $\sin(x) = -\frac{1}{2}$

✓

$\boxed{x_1 = \frac{7\pi}{6}}$; $\boxed{x_2 = \frac{4\pi}{6}}$

✓ ✓

14. $f(x) = \frac{3x^2}{x^2+1}$

$y' = \frac{6x}{(x^2+1)^2}$; $y' = 0$

✓

$\frac{6x}{(x^2+1)^2} = 0$ ✓

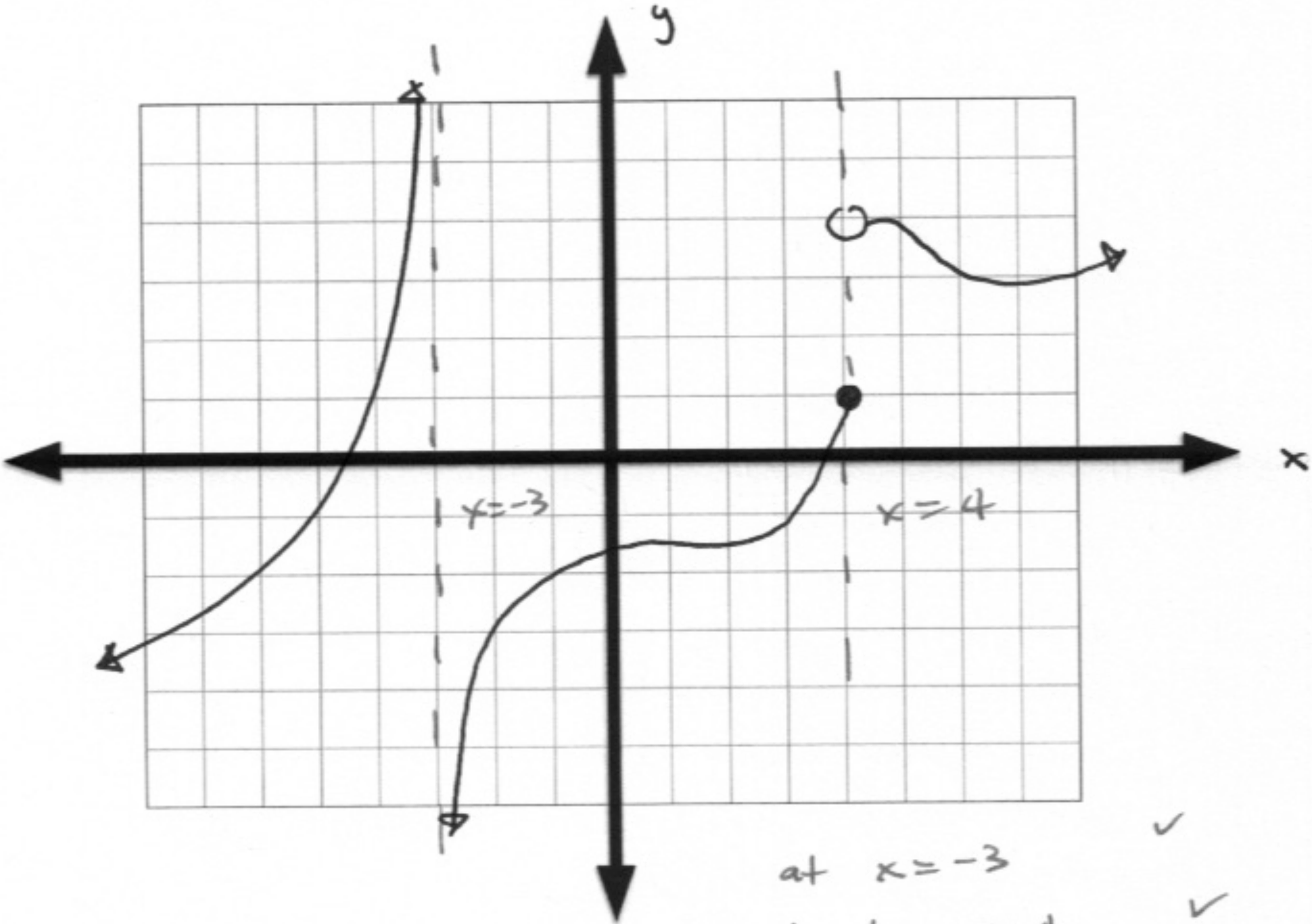
i.e, $6x = 0$ ✓

$\boxed{x = 0}$

✓

8 ✓

15. Where is the function not differentiable?



at $x=-3$

and at $x=4$

Differentiate the following functions.

16. $f(x) = 8tan(\sqrt{x})$; $y' = \dfrac{4\sec^2(\sqrt{x})}{\sqrt{x}}$

4

17. $f(x) = 2x + 3(x-1)^4$

$y' = 2 + 12(x-1)^3$

✓

✓

18. $f(x) = 2x\sqrt{x+5}$; $y' = \dfrac{3x+10}{\sqrt{x+5}}$ ✓ ✓

✓

19. $f(x) = x^3 sec(\pi x)$; $y' = \pi x^3 \sec(\pi x)\tan(\pi x) + 3x^2 \sec(\pi x)$ ✓

6 ✓

① $f(x) = \sqrt{2x+5}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2(x+h)+5} - \sqrt{2x+5}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2x+2h+5} - \sqrt{2x+5}}{h}$$

note Difference Quotient

$$\frac{\left(\sqrt{2x+2h+5} - \sqrt{2x+5}\right)}{h} \cdot \frac{\left(\sqrt{2x+2h+5} + \sqrt{2x+5}\right)}{\left(\sqrt{2x+2h+5} + \sqrt{2x+5}\right)}$$

$$\frac{2x+2h+5 - (2x+5)}{h\left(\sqrt{2x+2h+5} + \sqrt{2x+5}\right)}$$

$$\frac{2x+2h+5 - 2x - 5}{h\left(\sqrt{2x+2h+5} + \sqrt{2x+5}\right)}$$

$$\lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}\left(\sqrt{2x+2h+5} + \sqrt{2x+5}\right)}$$

$$\lim_{h\to 0} \frac{2}{\sqrt{2x+2\underset{0}{h}+5} + \sqrt{2x+5}}$$

$$\frac{2}{\sqrt{2x+5} + \sqrt{2x+5}}$$

$$\frac{\cancel{2}}{\cancel{2}\sqrt{2x+5}}$$

$$\boxed{f'(x) = \frac{1}{\sqrt{2x+5}}}$$

(2) $s(t) = -16t^2 + 64t + 96$

$s(0) = -16 \cdot 0^2 + 64 \cdot 0 + 96$

$\boxed{s(0) = 96 \text{ m}}$

(3) $v(t) = s'(t) = \frac{d}{dt}(-16t^2 + 64t + 96)$

$= -16 \frac{d}{dt}(t^2) + 64 \frac{d}{dt}(t) + \frac{d}{dt}(96)$

$= -16 \cdot 2t + 64 + 0$

$\boxed{v(t) = -32t + 64}$

(4) $v(0) = -32 \cdot 0 + 64$

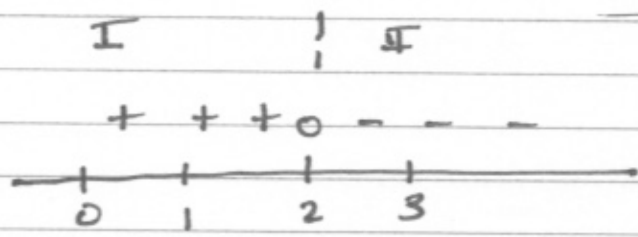
$\boxed{v(0) = 64 \text{ m/sec}}$

(5) $\boxed{|v(0)| = |64| = 64 \text{ m/sec}}$

(6) $v(t) = 0$; $-32t + 64 = 0$

$-32t = -64$; $\boxed{t = 2 \text{ sec}}$

(7)



$v(t) = -32t + 64$

(8) Left for $t > 2$

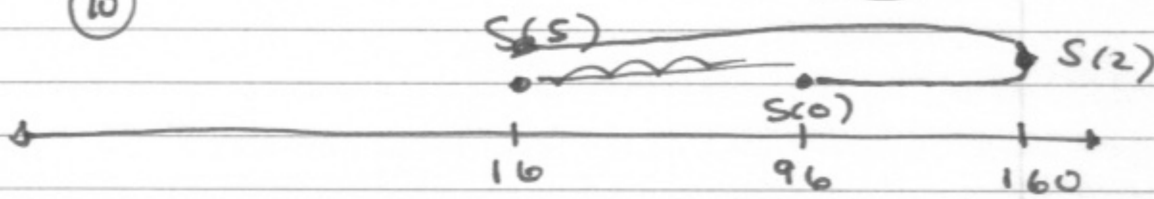
(7) Right for $0 \le t < 2$

(9) $|v(5)| = |-32 \cdot 5 + 64|$

$= |-160 + 64|$

$= |-96| = 96$

(10)



note $S(2) = -16 \cdot 2^2 + 64 \cdot 2 + 96$

$= -16 \cdot 4 + 128 + 96$

$= -64 + 128 + 96$

$= 64 + 96$

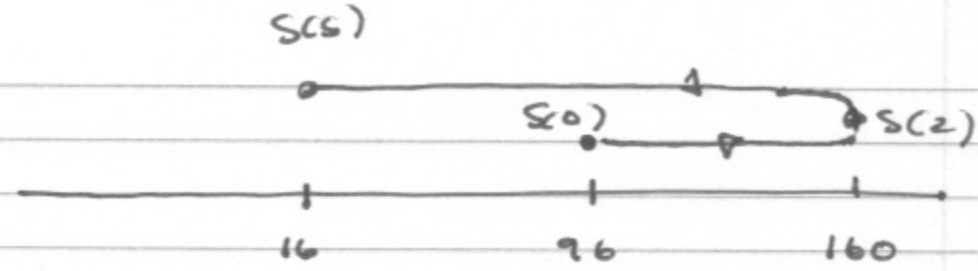
$S(2) = 160$

note $S(5) = -16 \cdot 5^2 + 64 \cdot 5 + 96$

$= -16 \cdot 25 + 320 + 96$

$= -400 + 320 + 96$

$S(5) = 16$



Distance $= 160 - 96 + 160 - 16$

$= 64 + 144$

(10)

$\boxed{\text{Distance} = 208}$

(11) $y = (x+1)(x^2-4)$

$y' = \frac{d}{dx}\left[(x+1)(x^2-4)\right]$

$= (x+1)\frac{d}{dx}(x^2-4) + (x^2-4)\frac{d}{dx}(x+1)$

$= (x+1)\cdot 2x + (x^2-4)\cdot 1$

$= 2x^2 + 2x + x^2 - 4$

$y' = 3x^2 + 2x - 4$; $m = 3x^2 + 2x - 4 \Big|_{x=1}$

$m = 3\cdot 1^2 + 2\cdot 1 - 4$

$m = 1$

$y - y_1 = m(x - x_1)$

note $y = (x+1)(x^2-4)$

$x=1$; $y = 2\cdot(-3)$

$x = 1 \;:\; y = -6$

$$y - -6 = 1\,(x - 1)$$

$$y + 6 = x - 1$$
$$-6 \qquad -6$$

$$\boxed{y = x - 7}$$

(12) $y = x \sin x$

$$y' = \frac{d}{dx}(x \sin x)$$

$$= x \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(x)$$

$$y' = x \cos x + \sin x$$

$$m = x \cos x + \sin x \Big|_{x = 0}$$

$$m = 0$$

$y - y_1 = m\,(x - x_1)$; $\quad x = 0$
$y = 0 \sin 0$
$y = 0$

$$y - 0 = 0\,(x - 0)$$

$$\boxed{y = 0}$$

(13) $y = x - 2\cos x$

$$y' = \frac{d}{dx}(x - 2\cos x)$$

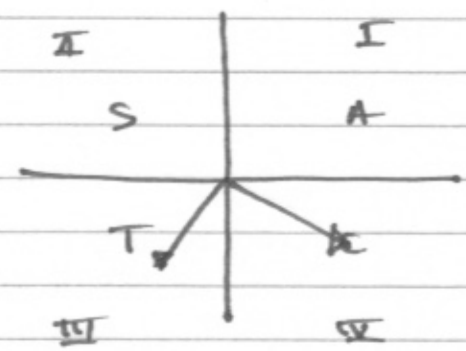
$$= \frac{d}{dx}(x) - 2\frac{d}{dx}(\cos x)$$

$$= 1 - 2(-\sin x)$$

$$y' = 1 + 2\sin x$$

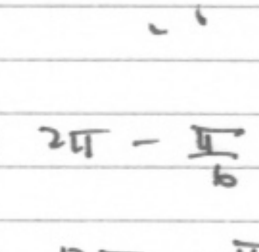
$y' = 0$; $\quad 1 + 2\sin x = 0$

$2\sin x = -1$; $\quad \sin x = -\frac{1}{2}$



Ref

$\sin x = \frac{1}{2}$



$x = \frac{\pi}{6}$

$$x_1 = \pi + \frac{\pi}{6}$$

$$= \frac{6\pi}{6} + \frac{\pi}{6}$$

$$x_1 = \frac{7\pi}{6}$$

$$x_2 = 2\pi - \frac{\pi}{6}$$

$$x_2 = \frac{12\pi}{6} - \frac{\pi}{6}$$

$$x_2 = \frac{4\pi}{6}$$

Since $y = x - 2\cos(x)$

at

$x_1 = \frac{7\pi}{6}$; $\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$$\boxed{\left(\frac{7\pi}{6}, \frac{7\pi}{6} - \frac{\sqrt{3}}{2}\right)}$$

at $x_2 = \frac{11\pi}{6}$

$$\boxed{\left(\frac{11\pi}{6}, \frac{11\pi}{6} + \frac{\sqrt{3}}{2}\right)}$$

(14) $y = \dfrac{3x^2}{x^2+1}$

$$y' = \frac{(x^2+1)\overset{6x}{\cancel{\frac{d}{dx}(3x^2)}} - 3x^2 \overset{2x}{\cancel{\frac{d}{dx}(x^2+1)}}}{(x^2+1)^2}$$

$$y' = \frac{(x^2+1)\cdot 6x - 3x^2 \cdot 2x}{(x^2+1)^2}$$

$$y' = \frac{6x^3 + 6x - 6x^3}{(x^2+1)^2}$$

$$\boxed{y' = \frac{6x}{(x^2+1)^2}}$$

HT ; $y' = 0$ or $6x = 0$

$x = 0$

$y = \frac{3x^2}{x^2+1}$; $x = 0$

$y = \frac{3 \cdot 0^2}{0^2+1}$

$\boxed{(0,0)}$ $y = \frac{0}{1}$ ∴ $y = 0$

(16) $y = 8 \tan(\sqrt{x})$

$y = 8 \tan(x^{1/2})$

$y' = \frac{d}{dx}\left[8 \tan(x^{1/2}) \right]$

$= 8 \frac{d}{dx}\left[\tan(x^{1/2}) \right]$

$= 8 \sec^2(x^{1/2}) \cancel{\frac{d}{dx}(x^{1/2})} \cdot \frac{1}{2\sqrt{x}}$

$$y' = 8\,\sec^2(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

$$\boxed{y' = \frac{4\,\sec^2(\sqrt{x})}{\sqrt{x}}}$$

(17) $y = 2x + 3(x-1)^4$

$$y' = \frac{d}{dx}\left[2x + 3(x-1)^4\right]$$

$$= \frac{d}{dx}(2x) + \frac{d}{dx}\left[3(x-1)^4\right]$$

$$= 2\frac{d}{dx}(x) + 3\frac{d}{dx}\left[(x-1)^4\right]$$

$$= 2\cdot 1 + 3\cdot 4(x-1)^{4-1}\cancelto{1}{\frac{d}{dx}(x-1)}$$

$$= 2 + 12(x-1)^3 \cdot 1$$

$$\boxed{y' = 2 + 12(x-1)^3}$$

(18) $y = 2x\sqrt{x+5}$

$$y' = \frac{d}{dx}\left[2x(x+5)^{1/2}\right]$$

$$= 2x \frac{d}{dx}\left[(x+5)^{1/2}\right] + (x+5)^{1/2}\frac{d}{dx}[2x]$$

$$= 2x \tfrac{1}{2}(x+5)^{\frac{1}{2}-1} \frac{d}{dx}(x+5)$$

$$+ (x+5)^{1/2} \cdot 2$$

$$y' = \frac{x}{\sqrt{x+5}} \cdot 1 + \sqrt{x+5} \cdot 2$$

$$y' = 2\sqrt{x+5} + \frac{x}{\sqrt{x+5}}$$

$$y' = \frac{2\sqrt{x+5}\sqrt{x+5}}{\sqrt{x+5}} + \frac{x}{\sqrt{x+5}}$$

$$y' = \frac{2(x+5) + x}{\sqrt{x+5}} \quad ; \quad \boxed{y' = \frac{3x+10}{\sqrt{x+5}}}$$

(19) $y = x^3 \sec(\pi x)$

$$y' = \frac{d}{dx}\left[x^3 \sec(\pi x)\right]$$

$$= x^3 \frac{d}{dx}\left[\sec(\pi x)\right] + \sec(\pi x)\frac{d}{dx}(x^3)$$

$$= x^3 \sec(\pi x)\tan(\pi x)\cancel{\frac{d}{dx}(\pi x)}\ \cdot \pi$$

$$+ 3x^2 \sec(\pi x)$$

$$\boxed{y' = \pi x^3 \sec(\pi x)\tan(\pi x) + 3x^2 \sec(\pi x)}$$