East Los Angeles College

Department of Mathematics

Math 261

Test 1

42

Let s(t) represent the position (meters) of an object moving along a straight line where time t is measured in seconds. Use your calculator.

$$s(t) = 2t^2 - 3$$

Determine the average velocity over each time period.

1. [4,5]

$$\frac{s(5) - s(4)}{5 - 4}$$

$$\frac{18}{1}$$

$$\boxed{18}$$

2. [4.5,5]

$$\frac{s(5) - s(4.5)}{5 - 4.5}$$

$$\frac{9.5}{0.5}$$

$$\boxed{19}$$

3. [4.9,5]

$$\frac{s(5) - s(4.9)}{5 - 4.9}$$

$$\frac{1.98}{0.1}$$

$$\boxed{19.8}$$

4. [4.99,5]

$$\frac{s(5) - s(4.99)}{5 - 4.99}$$

$$\frac{0.20}{0.01}$$

$$\boxed{19.98}$$

5. Determine the instantaneous velocity at time t=5 seconds.

$\approx \boxed{20}$

$\frac{9}{10}$

See solutions for details

Determine the following limits by using algebra. Show your work for credit.

6. $\lim_{x \to 3} \left(\frac{x^2-9}{x^2+2x-3} \right)$

$\lim_{x \to 3} \left(\frac{x-3}{x-1} \right)$

$\frac{0}{2}$

$\boxed{0}$

7. $\lim_{h \to 0} \left(\frac{\sqrt{9+h}-3}{h} \right)$

$\lim_{h \to 0} \left(\frac{1}{\sqrt{9+h}+3} \right)$

$\boxed{\frac{1}{6}}$

8. $\lim_{x \to 2} \left(\frac{x^2-4}{x^3-8} \right)$

$\lim_{x \to 2} \left(\frac{x+2}{x^2+2x+4} \right)$

$= \frac{1}{3}$

9. $\lim_{h \to 0} \left(\frac{(-5+h)^2-25}{h} \right)$

$\lim_{h \to 0} (h-10)$

$\boxed{-10}$

12

10. $\lim_{x\to 4}\left(\frac{|4-x|}{4-x}\right)$ DNE

$\lim_{x\to 4^-} 1 = 1$

$\lim_{x\to 4^+} -1 = -1$

11. If $2x \le f(x) \le x^4 - x^2 + 2$ for all x, then determine $\lim_{x\to 1} f(x)$

See Solutions.

$\lim_{x\to 1} f(x) = 2$ by Squeeze Therom

Let $f(x) = \begin{cases} \frac{1}{x^2} & for\ x \neq 0 \\ -2 & for\ x = 0 \end{cases}$

Answer the following questions.

12. $\lim_{x\to 0^-} f(x)$ $= \boxed{\infty}$

13. $\lim_{x\to 0^+} f(x)$ $= \boxed{\infty}$

14. $\lim_{x\to 0} f(x)$ $= \boxed{\infty}$

15. $f(0)$ $= \boxed{-2}$

11

16. Is the function continuous at $x = 0$? discontinuous

$$\text{Let } f(x) = \begin{cases} x^2 + 6 & for\ x \geq 2 \\ 3x + 4 & 0 < x < 2 \\ \frac{1}{x} & x < 0 \end{cases}$$

Answer the following questions.

17. $\lim_{x \to 2} f(x)$

$\boxed{10}$ ✓

18. $f(2)$

~~undefined~~

$\boxed{10}$ ✓

19. Is the function continuous at $x = 2$?

discontinuous ✓

20. $\lim_{x \to 0} f(x)$ $\boxed{DNE}$ ✓

21. $f(0)$ undefined ✓

22. Is the function continuous at $x = 0$?

Dis continuous ✓

23. $\lim_{x \to 5} f(x)$ $\boxed{31}$ ✓

24. $f(5)$ $\boxed{31}$ ✓

25. Is the function continuous at $x = 5$?

continuous ✓

9✓

	t1	t2	s(t1)	s(t2)	Delta t	Delta s	ds/dt
1)	4	5	29	47	1	18.00	18.00
2)	4.5	5	37.5	47	0.5	9.50	19.00
3)	4.9	5	45.02	47	0.1	1.98	19.80
4)	4.99	5	46.8002	47	0.01	0.20	19.98

(6) $\lim_{x \to 3} \left(\dfrac{x^2 - 9}{x^2 + 2x - 3} \right)$ $\quad \dfrac{0}{0}$ Indeterminant

$x \neq 3$

$$\frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}(x-1)}$$

$$\lim_{x \to 3} \left(\frac{x-3}{x-1} \right) = \frac{3-3}{3-1} = \frac{0}{2} = \boxed{0}$$

(7) $\lim_{h \to 0} \left(\dfrac{\sqrt{9+h} - 3}{h} \right)$ $\quad \dfrac{0}{0}$ indeterminant

$$\left(\frac{\sqrt{9+h} - 3}{h} \right) \cdot \frac{(\sqrt{9+h} + 3)}{(\sqrt{9+h} + 3)}$$

$$\frac{9 + h - 9}{h[\sqrt{9+h} + 3]} = \frac{\cancel{h}}{\cancel{h}[\sqrt{9+h} + 3]}$$

$$\frac{1}{\sqrt{9+h} + 3} \; ; \; \lim_{h \to 0} \left(\frac{1}{\sqrt{9+h} + 3} \right)$$

$$\frac{1}{\sqrt{9+0} + 3}$$

$$\frac{1}{3+3} \qquad \frac{1}{6}$$

(8) $\lim_{x \to 2} \left(\frac{x^2 - 4}{x^3 - 8} \right)$ $\frac{0}{0}$ indeterminant

$$\frac{x^2-4}{x^3-8} = \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}(x^2+2x+4)}$$

$$= \frac{x+2}{x^2+2x+4}$$

$$\lim_{x \to 2} \frac{x+2}{x^2+2x+4} = \frac{2+2}{2^2+2 \cdot 2+4}$$

$$= \frac{4}{4+4+4} = \frac{4}{12}$$

$$\frac{1}{3}$$

(9) $\lim_{h \to 0} \left(\frac{(-5+h)^2 - 25}{h} \right)$ $\frac{0}{0}$ indeterminant

$$\frac{(-5+h)^2-25}{h} = \frac{(h-5)^2-25}{h}$$

$$= \frac{h^2-10h+25-25}{h}$$

$$= \frac{h^2-10h}{h} = \frac{\cancel{h}(h-10)}{\cancel{h}}$$

$$= h-10$$

$$\lim_{h \to 0} (h-10) \qquad 0-10$$

$$-10$$

(10) $$\lim_{x \to 4} \left(\frac{|4-x|}{4-x} \right)$$

note $$\frac{|4-x|}{4-x} = \begin{cases} \frac{4-x}{4-x} & ; 4-x > 0 \\ \frac{-(4-x)}{4-x} & ; 4-x < 0 \end{cases}$$

$$\frac{|4-x|}{4-x} = \begin{cases} 1 & , x < 4 \\ -1 & , x > 4 \end{cases}$$

$$\lim_{x \to 4^-} \left(\frac{|4-x|}{4-x} \right) = 1$$

$$\lim_{x \to 4^+} \left(\frac{|4-x|}{4-x} \right) = -1$$

$$\lim_{x \to 4} \left(\frac{|4-x|}{4-x} \right) \quad \text{DNE}$$

(11) $2x \le f(x) \le x^4 - x^2 + 2$ all x

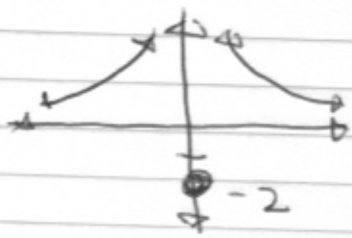
$$\lim_{x\to 1} 2x = 2\cdot 1 = 2$$

$$\lim_{x\to 1} x^4 - x^2 + 2 = 1^4 - 1^2 + 2$$
$$= 2$$

ie, $\lim_{x\to 1} f(x) = \boxed{2}$ By Squeeze Theorem

(12) $f(x) = \begin{cases} \frac{1}{x^2} & ; x \neq 0 \\ -2 & ; x = 0 \end{cases}$

$\lim_{x\to 0^-} f(x) = \boxed{\infty}$



(13) $\lim_{x\to 0^+} f(x) = \boxed{\infty}$

(14) $\lim_{x\to 0} f(x) = \boxed{\infty}$

$f(0) = \boxed{-2}$

(15) f is discontinuous at $x=0$

(16)

$$f(x) = \begin{cases} x^2 + 6 & ; \quad x \geq 2 \\ 3x + 4 & ; \quad 0 < x < 2 \\ \dfrac{1}{x} & ; \quad x < 0 \end{cases}$$

(17) $\lim\limits_{x \to 2} f(x) = 10$

as $\lim\limits_{x \to 2^-} f(x) = 3 \cdot 2 + 4$ $\quad$ $\lim\limits_{x \to 2^+} f(x) = 2^2 + 6$

$= 6 + 4$ $\quad$ $4 + 6$

$= 10$ $\quad$ 10

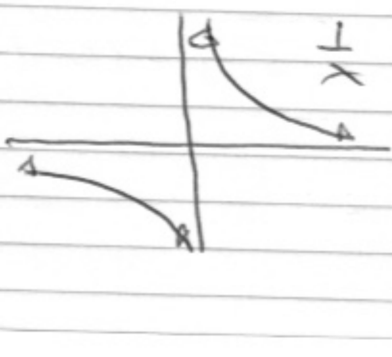
so, $\lim\limits_{x \to 2} f(x) = \boxed{10}$

but, (18) $f(2) =$ ~~undefined~~. $f(2) = 2^2 + 6$

$f(2) = \boxed{10}$

(19) so, f is ~~dis~~ continuous at $x = 2$

(20) $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x)$



$-\infty$ $\quad$ ∞

DNE

(21) $f(0) = 0^2+6$

$= 6$

$f(0) =$ undefined

(22) f is dis-continuous at $x=0$.

(23) $\lim_{x\to 5} f(x) = \lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x)$

5^2+6 $\quad$ 5^2+6

$25+6$ $\quad$ $25+6$

31 $\quad$ 31

$\lim_{x\to 5} f(x) = 31$

(24) $f(5) = 5^2+6$

$= 25+6$

$= 31$

(25) f is continuous at $x=5$.