Exponential Integrals

(1) $\int e^{2\pi x}\,dx \quad = \quad \int e^u \frac{du}{2\pi}$

$u = 2\pi x$

$\frac{du}{dx} = \frac{d}{dx}(2\pi x) = 2\pi$

$\frac{du}{dx} = \frac{2\pi}{1}$; $dx = \frac{du}{2\pi}$

$\frac{1}{2\pi}\int e^u\,du$

$\frac{1}{2\pi} e^u + C$

$\boxed{\frac{1}{2\pi} e^{2\pi x} + C}$

(3) $\int e^{-\pi x}\,dx \quad = \quad \int e^u \frac{du}{-\pi}$

$u = -\pi x$

$\frac{du}{dx} = \frac{d}{dx}(-\pi x) = -\pi$

$\frac{du}{dx} = \frac{-\pi}{1}$; $dx = \frac{du}{-\pi}$

$-\frac{1}{\pi}\int e^u\,du$

$-\frac{1}{\pi} e^u + C$

$\boxed{-\frac{1}{\pi} e^{-\pi x} + C}$

(5) $\int (e^{\pi x} + 1)\,dx \quad = \quad \frac{1}{\pi} e^{\pi x} + C_1 + x + C_2$

$= \int e^{\pi x}\,dx + \int dx$ $\quad x + C$

$\boxed{\frac{1}{\pi} e^{\pi x} + x + C}$

u-Sub

ie, $\int e^{\pi x}\,dx = \int e^u \frac{du}{\pi}$

$u = \pi x$ $\quad dx = \frac{du}{\pi}$

$\frac{du}{dx} = \pi$

$= \frac{1}{\pi}\int e^u\,du = \frac{1}{\pi} e^u + C$

$= \frac{1}{\pi} e^{\pi x} + C$

(7) $\int x e^{x^2} dx = \int \cancel{x} e^u \frac{du}{2\cancel{x}} = \int e^u \frac{du}{2}$

$u = x^2$

$= \frac{1}{2} \int e^u \, du$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$= \frac{1}{2} e^u + C$

$= \boxed{\frac{1}{2} e^{x^2} + C}$

(9) $\int \cos(x) \, e^{\sin(x)} dx = \int \cancel{\cos(x)} \, e^u \frac{du}{\cancel{\cos(x)}}$

$u = \sin(x)$

$\frac{du}{dx} = \cos(x)$

$= \int e^u \, du$

so, $dx = \frac{du}{\cos(x)}$

$= e^u + C$

$= \boxed{e^{\sin(x)} + C}$

(11) $\int x e^{-x^2} dx = \int \cancel{x} e^u \frac{du}{-2\cancel{x}}$

$u = -x^2$

$= -\frac{1}{2} \int e^u \, du$

$\frac{du}{dx} = -\frac{2x}{1}$

$= -\frac{1}{2} e^u + C$

so, $dx = \frac{du}{-2x}$

$= \boxed{-\frac{1}{2} e^{-x^2} + C}$

(13) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int \frac{e^u}{\cancel{\sqrt{x}}}\, 2\cancel{\sqrt{x}}\, du$

$u = \sqrt{x}$

$$= \int 2e^u\,du$$

$\dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$

$$= 2\int e^u\,du$$

$dx = 2\sqrt{x}\,du$

$$= 2e^u + C$$

$$= \boxed{2e^{\sqrt{x}} + C}$$

(15) $\displaystyle\int e^{7x-5}\,dx = \int e^u \frac{du}{7}$

$u = 7x - 5$

$$= \frac{1}{7}\int e^u\,du$$

$\dfrac{du}{dx} = 7$; $dx = \dfrac{du}{7}$

$$= \frac{1}{7}e^u + C$$

$$= \boxed{\frac{1}{7}e^{7x-5} + C}$$

(17) $\displaystyle\int \frac{e^x}{(1+2e^x+e^{2x})}\,dx = \int \frac{\cancel{e^x}}{(1+2u+u^2)} \frac{du}{\cancel{e^x}}$

let $u = e^x$

$\dfrac{du}{dx} = e^x$; $dx = \dfrac{du}{e^x}$

$$= \int \frac{1}{1+2u+u^2}\,du$$

$$= \int \frac{1}{(1+u)^2}\,du = \int (1+u)^{-2}\,du$$

use another substitution !

let $t = 1 + u$

$\frac{dt}{du} = 1$ or $dt = du$

$$= \int t^{-2}\,dt$$

$$= \frac{t^{-3}}{-3} + C$$

$$= -\frac{1}{3t^3} + C$$

$$-\frac{1}{3(1+u)^3} + C$$

$$\boxed{-\frac{1}{3(1+e^x)^3} + C}$$

(19) $\int \frac{e^{2x} - e^{4x}}{e^x}\,dx = \int \left(\frac{e^{2x}}{e^x} - \frac{e^{4x}}{e^x}\right)dx$

$$= \int (e^x - e^{3x})\,dx = \int e^x\,dx - \int e^{3x}\,dx$$

$e^x + C$

u-sub

<u>note</u> $\int e^{3x}\,dx$

$u = 3x$; $\frac{du}{dx} = 3$;

$dx = \frac{du}{3}$

$$\int e^u \frac{du}{3} = \frac{1}{3}\int e^u\,du$$

$\frac{1}{3}e^u + C$

$\frac{1}{3}e^{3x} + C$

$e^x + c_1 - \left(\frac{1}{3}e^{3x} + c_2\right)$

$e^x + c_1 - \frac{1}{3}e^{3x} - c_2$

$\boxed{e^x - \frac{1}{3}e^{3x} + C}$

21) $\int e^x(e^{2x}+1)^3\,dx = \int \cancel{e^x}((e^x)^2+1)^3 \frac{du}{\cancel{e^x}}$

let $u = e^x$ $\quad = \int [(e^x)^2+1]^3\,du$

$\frac{du}{dx} = e^x$;

$\quad = \int [u^2+1]^3\,du$

$dx = \frac{du}{e^x}$

use algebra !

$(u^2+1)^3 = (u^2+1)(u^2+1)(u^2+1)$

$= (u^4+2u^2+1)(u^2+1)$

$= u^6 + 2u^4 + u^2$

$\quad u^4 + 2u^2 + 1$

$= u^6 + 3u^4 + 3u^2 + 1$

$\int (u^6 + 3u^4 + 3u^2 + 1)\,du$

properties of integrals

$\int u^6\,du + 3\int u^4\,du + 3\int u^2\,du + \int du$

$\frac{1}{7}u^7 + 3 \cdot \frac{u^5}{5} + 3\frac{u^3}{3} + u + C$

$$\frac{1}{7}u^7 + \frac{3}{5}u^5 + u^3 + u + C$$

$u = e^x$

$$\frac{1}{7}(e^x)^7 + \frac{3}{5}(e^x)^5 + (e^x)^3 + e^x + C$$

$$\boxed{\frac{1}{7}e^{7x} + \frac{3}{5}e^{5x} + e^{3x} + e^x + C}$$

(23) $\int e^x \sqrt{e^x + 1}\, dx = \int \cancel{e^x} \sqrt{u+1}\, \frac{du}{\cancel{e^x}}$

let $u = e^x$

$= \int \sqrt{u+1}\, du$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

use another substitution!

let $t = \sqrt{u+1}$

so that $t^2 = u + 1$

use implicit diff to obtain $\frac{dt}{du}$

$$\frac{d}{du}(t^2) = \frac{d}{du}(u+1)$$

$$2t\frac{dt}{du} = 1 \quad \text{or} \quad \frac{dt}{du} = \frac{1}{2t}$$

$\frac{dt}{du} = \frac{1}{2t}$; $du = 2t\ dt$

so $\int \sqrt{u+1}\ du \underset{t\text{-sub}}{=} \int t\, 2t\ dt$

$= \int 2t^2\, dt = 2\int t^2\, dt$

$= 2\,\frac{t^3}{3} + C$

$= \frac{2}{3}\, t^3 + C$

but, $t = \sqrt{u+1}$

$= \frac{2}{3}\left(\sqrt{u+1}\right)^3 + C$

$= \frac{2}{3}\left[(u+1)^{1/2}\right]^3 + C$

$= \frac{2}{3}\,(u+1)^{3/2} + C$

but, $u = e^x$

$\boxed{\frac{2}{3}\,(e^x+1)^{3/2} + C}$

or $\boxed{\frac{2}{3}\,(e^x+1)\sqrt{e^x+1} + C}$