# Derivatives of logarithmic Functions

(1) $f(x) = \ln(\pi x)$

$$f'(x) = \frac{d}{dx}\left[\ln(\pi x)\right]$$

$$= \frac{1}{\pi x} \cancel{\frac{d}{dx}(\pi x)} \to \pi$$

ie, $f'(x) = \frac{\cancel{\pi}}{\cancel{\pi} x}$ ; $\boxed{f'(x) = \frac{1}{x}}$

(3) $f(x) = \ln(x^2)$

$$f'(x) = \frac{d}{dx}\left[\ln(x^2)\right]$$

$$= \frac{1}{x^2} \cancel{\frac{d}{dx}(x^2)} \to 2x$$

$$= \frac{2x}{x^2} ;$$

$$\boxed{f'(x) = \frac{2}{x}}$$

(3) $f(x) = \ln[\sin(x)]$

$$f'(x) = \frac{d}{dx}\left[\ln(\sin(x))\right]$$

$$= \frac{1}{\sin(x)} \cancel{\frac{d}{dx}(\sin(x))} \to \cos(x)$$

$f'(x) = \frac{\cos(x)}{\sin(x)}$ ; $\boxed{f'(x) = \cot(x)}$

(7) $f(x) = \ln(x^3)$

$$f'(x) = \frac{d}{dx}\left[\ln(x^3)\right]$$

$$= \frac{1}{x^3} \cdot \underbrace{\frac{d}{dx}(x^3)}_{3x^2}$$

$$= \frac{3x^2}{x^3}$$

$$\boxed{f'(x) = \frac{3}{x}}$$

(9) $f(x) = \ln(4x^5)$

$$f'(x) = \frac{d}{dx}\left[\ln(4x^5)\right]$$

$$= \frac{1}{4x^5} \cdot \underbrace{\frac{d}{dx}(4x^5)}_{20x^4}$$

$$= \frac{20x^4}{4x^5}$$

$$\boxed{f'(x) = \frac{5}{x}}$$

(11) $f(x) = x\ln(2x)$

$$f'(x) = \frac{d}{dx}\left[x\ln(2x)\right]$$

Product Rule

$$f'(x) = \frac{d}{dx}\left[x \ln(2x)\right]$$

$$= x \overbrace{\frac{d}{dx}\left[\ln(2x)\right]}^{\frac{1}{2x}\overbrace{\frac{d}{dx}(2x)}^{2}} + \ln(2x) \overbrace{\frac{d}{dx}(x)}^{1}$$

$$f'(x) = \cancel{x} \cdot \frac{\cancel{2}}{\cancel{2x}} + \ln(2x)$$

$$\boxed{f'(x) = 1 + \ln(2x)}$$

(13) $f(x) = x^2 \ln(x)$

$$f'(x) = \frac{d}{dx}\left[x^2 \ln(x)\right]$$

•Product Rule!

$$f'(x) = x^2 \overbrace{\frac{d}{dx}\left[\ln(x)\right]}^{\frac{1}{x}} + \ln(x) \overbrace{\frac{d}{dx}(x^2)}^{2x}$$

$$f'(x) = \frac{x^2}{x} + 2x \ln(x)$$

$$\boxed{f'(x) = x + 2x \ln(x)}$$

(15) $f(x) = x^3 \ln(5x)$

$$f'(x) = \frac{d}{dx}\left(x^3 \ln(x)\right)$$

product Rule!

$$f'(x) = x^3 \overbrace{\frac{d}{dx}(\ln(x))}^{\frac{1}{x}} + \ln(x) \overbrace{\frac{d}{dx}(x^3)}^{3x^2}$$

$$f'(x) = \frac{x^3}{x} + 3x^2 \ln(x)$$

$$\boxed{f'(x) = x^2 + 3x^2 \ln(x)}$$

17) $f(x) = \sqrt{\ln(x)}$

$f(x) = [\ln(x)]^{1/2}$

$f'(x) = \frac{d}{dx}[\ln(x)]^{1/2}$

$f'(x) = \frac{1}{2}[\ln(x)]^{\frac{1}{2}-1} \frac{d}{dx}(\ln(x))$ $\quad \frac{1}{x}$

$f'(x) = \frac{1}{2}[\ln(x)]^{-1/2} \cdot \frac{1}{x}$

$$\boxed{f'(x) = \frac{1}{2x\sqrt{\ln(x)}}}$$

19) $f(x) = \ln^2(x)$

$f(x) = [\ln(x)]^2$

$f'(x) = \frac{d}{dx}\left\{[\ln(x)]^2\right\}$

$f'(x) = 2[\ln(x)]^{2-1} \frac{d}{dx}(\ln(x))$ $\quad \frac{1}{x}$

$f'(x) = 2\ln(x) \cdot \frac{1}{x}$

$$\boxed{f'(x) = \frac{2\ln(x)}{x}}$$

(21) $f(x) = 4 \ln^3(x)$

$f(x) = 4 [\ln(x)]^3$

$f'(x) = \frac{d}{dx}\left[4[\ln(x)]^3\right]$

$= 4 \frac{d}{dx}[\ln(x)]^3$

$= 4 \cdot 3[\ln(x)]^{3-1} \frac{d}{dx}(\ln(x))$ $\frac{1}{x}$

$= 12 \ln^3(x) \cdot \frac{1}{x}$

$$\boxed{f'(x) = \frac{12 \ln^3(x)}{x}}$$

(23) $f(x) = \frac{\ln(x)}{x}$

$f'(x) = \frac{d}{dx}\left[\frac{\ln(x)}{x}\right]$ $\frac{1}{x}$

$= \frac{x \frac{d}{dx}(\ln(x)) - \ln(x) \frac{d}{dx}(x)}{x^2}$ $1$

$= \frac{x \cdot \frac{1}{x} - \ln(x) \cdot 1}{x^2}$

$$\boxed{f'(x) = \frac{1 - \ln(x)}{x^2}}$$

(25) $f(x) = \dfrac{\ln(3x)}{x^4}$

$$f'(x) = \frac{d}{dx}\left[\frac{\ln(3x)}{x^4}\right] \quad \frac{1}{3x}\,\cancel{\frac{d}{dx}(3x)}^{\,3}$$

$$= \frac{x^4 \cancel{\frac{d}{dx}(\ln(3x))} - \ln(3x)\cancel{\frac{d}{dx}(x^4)}^{\,4x^3}}{(x^4)^2}$$

$$f'(x) = \frac{x^4 \cdot \frac{3}{3x} - 4x^3 \ln(3x)}{x^8}$$

$$\boxed{f'(x) = \frac{x^3 - 4x^3 \ln(3x)}{x^8}}$$

(27) $f(x) = \ln(\sqrt{x})$

$$f'(x) = \frac{d}{dx}\left[\ln(\sqrt{x})\right]$$

$$= \frac{1}{\sqrt{x}}\,\cancel{\frac{d}{dx}(\sqrt{x})}\ \frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$\boxed{f'(x) = \frac{1}{2x}}$$

(29) $f(x) = \dfrac{\ln(4x)}{\sqrt{x}}$

$$f'(x) = \frac{d}{dx}\left[\frac{\ln(4x)}{\sqrt{x}}\right]$$

Quotient Rule

$$f'(x) = \frac{\sqrt{x}\,\overbrace{\frac{d}{dx}(\ln(4x))}^{\frac{1}{4x}\,\cdot\,4} - \ln(4x)\,\overbrace{\frac{d}{dx}(\sqrt{x})}^{\frac{1}{2\sqrt{x}}}}{(\sqrt{x})^2}$$

$$f'(x) = \frac{\sqrt{x} \cdot \dfrac{4}{4x} - \dfrac{\ln(4x)}{2\sqrt{x}}}{x}$$

$$f'(x) = \left(\frac{\sqrt{x}}{x} - \frac{\ln(4x)}{2\sqrt{x}}\right) \div x$$

$$f'(x) = \left(\frac{\sqrt{x}}{x} - \frac{\ln(4x)}{2\sqrt{x}}\right) \cdot \frac{1}{x}$$

$$f'(x) = \frac{\sqrt{x}}{x^2} - \frac{\ln(4x)}{2x\sqrt{x}}$$

$$f'(x) = \frac{x^{1/2}}{x^2} - \frac{\ln(4x)}{2x\sqrt{x}}$$

$$f'(x) = x^{-3/2} - \frac{\ln(4x)}{2x\sqrt{x}}$$

$$f'(x) = \frac{1}{x^{3/2}} - \frac{\ln(4x)}{2x\sqrt{x}}$$

$$f'(x) = \frac{1}{x\sqrt{x}} - \frac{\ln(4x)}{2x\sqrt{x}}$$

$$f'(x) = \frac{2}{2x\sqrt{x}} - \frac{\ln(4x)}{2x\sqrt{x}}$$

$$\boxed{f'(x) = \frac{2 - \ln(4x)}{2x\sqrt{x}}}$$

(31) $f(x) = \sin\left[\ln\left(\frac{1}{x}\right)\right]$

$$f'(x) = \frac{d}{dx}\left[\sin\left(\ln\left(\frac{1}{x}\right)\right)\right]$$

$$= \cos\left(\ln\left(\frac{1}{x}\right)\right)\frac{d}{dx}\left(\ln\left(\frac{1}{x}\right)\right)$$

$$\frac{1}{1/x}\ \frac{d}{dx}\left(\frac{1}{x}\right)$$

$$-\frac{1}{x^2}$$

$$f'(x) = \cos\left(\ln\left(\frac{1}{x}\right)\right)\cdot x\left(-\frac{1}{x^2}\right)$$

$$\boxed{f'(x) = -\frac{\cos\left(\ln\left(1/x\right)\right)}{x}}$$

(33) $f(x) = \tan\left[\ln\left(\pi/x\right)\right]$

$f'(x) = \frac{d}{dx}\left[\tan\left[\ln\left(\frac{\pi}{x}\right)\right]\right]$

$= \sec^2\left[\ln\left(\frac{\pi}{x}\right)\right] \frac{d}{dx}\left[\ln\left(\frac{\pi}{x}\right)\right]$

$\frac{1}{\pi/x} \frac{d}{dx}\left[\frac{\pi}{x}\right]$

$-\frac{\pi}{x^2}$

$f'(x) = \sec^2\left[\ln\left(\frac{\pi}{x}\right)\right] \cdot \frac{x}{\pi} \cdot \frac{-\pi}{x^2}$

$f'(x) = \sec^2\left[\ln\left(\frac{\pi}{x}\right)\right] \cdot \left(-\frac{1}{x}\right)$

$f'(x) = -\frac{\sec^2\left(\ln\left(\pi/x\right)\right)}{x}$

(35) $f(x) = x^{\tan(x)}$

let $y = x^{\tan(x)}$ ; $\ln(y) = \ln\left(x^{\tan(x)}\right)$

$\ln(y) = \tan(x) \cdot \ln(x)$

$\frac{d}{dx}\left[\ln(y)\right] = \frac{d}{dx}\left[\tan(x) \cdot \ln(x)\right]$

product Rule!

$$\frac{d}{dx}[\ln(y)] = \frac{d}{dx}[\tan(x)\ln(x)]$$

$$\frac{1}{y}\cdot y' \qquad = \tan(x)\frac{d}{dx}(\ln(x)) \qquad \frac{1}{x}$$

$$+ \ln(x)\frac{d}{dx}(\tan(x))$$

$$\sec^2(x)$$

$$\frac{1}{y}\cdot y' = \frac{\tan(x)}{x} + \sec^2(x)\ln(x)$$

$$y' = y\left[\frac{\tan(x)}{x} + \sec^2(x)\ln(x)\right]$$

$$y' = x^{\tan(x)}\left[\frac{\tan(x)}{x} + \sec^2(x)\ln(x)\right]$$

37) $f(x) = (\sin(x))^x$

Let $y = (\sin(x))^x$ ; $\ln(y) = \ln(\sin(x))^x$

$$\ln(y) = x\cdot\ln(\sin(x))$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}(x\cdot\ln(\sin(x)))$$

Product Rule!

$$\frac{1}{y}\cdot y'$$

$$\frac{1}{y}\cdot y' = x\,\frac{d}{dx}[\ln(\sin(x))] + \ln[\sin(x)]\,\frac{d}{dx}(x)$$

$$\frac{1}{\sin(x)}\frac{d}{dx}(\sin(x)) \qquad 1$$

$$\cos(x)$$

$$\frac{y'}{y} = x\,\frac{\cos(x)}{\sin(x)} + \ln[\sin(x)]$$

$$\frac{y'}{y} = x\cot(x) + \ln[\sin(x)]$$

$$y' = y\left[x\cot(x) + \ln(\sin(x))\right]$$

$$\boxed{y' = \sin^{x}(x)\left[x\cot(x) + \ln(\sin(x))\right]}$$

(41) $f(x) = (3x+5)^4 (x-1)^3$

$$y = (3x+5)^4 (x-1)^3$$

$$\ln(y) = \ln\left[(3x+5)^4 (x-1)^3\right]$$

$$\ln(y) = \ln\left[(3x+5)^4\right] + \ln\left[(x-1)^3\right]$$

$$\ln(y) = 4\ln(3x+5) + 3\ln(x-1)$$

$$\frac{d}{dx}[\ln(y)] = \frac{d}{dx}\left[4\ln(3x+5) + 3\ln(x-1)\right]$$

$$\frac{1}{y}\cdot y' = 4\frac{d}{dx}[\ln(3x+5)] + 3\frac{d}{dx}[\ln(x-1)]$$

$$\frac{1}{3x+5}\frac{d}{dx}(3x+5) \qquad \frac{1}{x-1}\frac{d}{dx}(x-1)$$

$$3 \qquad 1$$

$$\frac{y'}{y} = \frac{12}{3x+5} + \frac{3}{x-1};$$

$$y' = y\left[\frac{12}{3x+5} + \frac{3}{x-1}\right]$$

$$y' = (3x+5)^4 (x-1)^3 \left[ \frac{12}{3x+5} + \frac{3}{x-1} \right]$$

(43) $$f(x) = \frac{\sqrt{x}\,(3x-2)^4}{(x+5)^3}$$

Let $$y = \frac{x^{1/2}\,(3x-2)^4}{(x+5)^3}$$

$$\ln(y) = \ln\left( \frac{x^{1/2}\,(3x-2)^4}{(x+5)^3} \right)$$

$$\ln(y) = \ln\left(x^{1/2}\,(3x-2)^4\right) - \ln\left((x+5)^3\right)$$

$$\ln(y) = \ln\left(x^{1/2}\right) + \ln\left((3x-2)^4\right) - \ln\left((x+5)^3\right)$$

$$\ln(y) = \frac{1}{2}\ln(x) + 4\ln(3x-2) - 3\ln(x+5)$$

$$\frac{d}{dx}\left[\ln(y)\right] = \frac{d}{dx}\left[ \frac{1}{2}\ln(x) + 4\ln(3x-2) - 3\ln(x+5) \right]$$

$$\frac{1}{x}$$

$$\underbrace{\frac{d}{dx}\left[\ln(y)\right]}_{\frac{1}{y}\cdot y'} = \frac{1}{2}\underbrace{\frac{d}{dx}\left[\ln(x)\right]}_{\frac{1}{x}} + 4\underbrace{\frac{d}{dx}\left[\ln(3x-2)\right]}_{\frac{1}{3x-2}\underbrace{\frac{d}{dx}(3x-2)}_{3}} - 3\underbrace{\frac{d}{dx}\left[\ln(x+5)\right]}_{\frac{1}{x+5}}$$

$$\frac{y'}{y} = \frac{1}{2x} + \frac{12}{3x-2} - \frac{3}{x+5}$$

$$y' = y\left[\frac{1}{2x} + \frac{12}{3x-2} - \frac{3}{x+5}\right]$$

$$\boxed{y' = \frac{\sqrt{x}\,(3x-2)^4}{(x+5)^3}\left[\frac{1}{2x} + \frac{12}{3x-2} - \frac{3}{x+5}\right]}$$