Derivatives of Exponentials

(1) $f(x) = x^2 e^x$

$f'(x) = \frac{d}{dx}(x^2 e^x)$

$= x^2 \frac{d}{dx}(e^x) + e^x \frac{d}{dx}(x^2)$ ($\frac{d}{dx}(e^x) = e^x$, $\frac{d}{dx}(x^2) = 2x$)

$f'(x) = x^2 e^x + 2x e^x$

(3) $f(x) = \sqrt{x}\, e^x$

$f'(x) = \frac{d}{dx}(\sqrt{x}\, e^x)$

$= \sqrt{x} \frac{d}{dx}(e^x) + e^x \frac{d}{dx}(\sqrt{x})$ ($\frac{d}{dx}(e^x) = e^x$, $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$)

$f'(x) = \sqrt{x}\, e^x + \frac{1}{2\sqrt{x}} e^x$

(5) $f(x) = x^3 e^{-x}$

$f'(x) = \frac{d}{dx}(x^3 e^{-x})$

$= x^3 \frac{d}{dx}(e^{-x}) + e^{-x} \frac{d}{dx}(x^3)$ ($\frac{d}{dx}(x^3) = 3x^2$)

$e^{-x} \frac{d}{dx}(-x)$ ($\frac{d}{dx}(-x) = -1$)

$f'(x) = -x^3 e^{-x} + 3x^2 e^{-x}$

(7) $f(x) = \sqrt{x}\, e^{-4x}$

$$f'(x) = \frac{d}{dx}\left(\sqrt{x} \cdot e^{-4x}\right)$$

$$= \sqrt{x}\,\frac{d}{dx}\left(e^{-4x}\right) + e^{-4x}\,\frac{d}{dx}\left(\sqrt{x}\right)$$

$$e^{-4x}\,\frac{d}{dx}(-4x) \qquad \frac{1}{2\sqrt{x}}$$

$$-4$$

$$f'(x) = -4\sqrt{x}\, e^{-4x} + \frac{1}{2\sqrt{x}}\, e^{-4x}$$

(9) $f(x) = \dfrac{e^{2x}}{x}$

$$f'(x) = \frac{d}{dx}\left(\frac{e^{2x}}{x}\right) \qquad e^{2x}\,\frac{d}{dx}(2x) \quad (2)$$

$$= \frac{x\,\frac{d}{dx}(e^{2x}) - e^{2x}\,\frac{d}{dx}(x)}{x^2} \quad (1)$$

$$f'(x) = \frac{2x\,e^{2x} - e^{2x}}{x^2}$$

(11) $f(x) = \dfrac{e^{2x}}{x-1}$

$$f'(x) = \frac{d}{dx}\left(\frac{e^{2x}}{x-1}\right)$$

$$f'(x) = \frac{(x-1)\frac{d}{dx}(e^{2x}) - e^{2x}\frac{d}{dx}(x-1)}{(x-1)^2}$$

(e^{2x} → $\frac{d}{dx}(2x)$ → 2; $\frac{d}{dx}(x-1)$ → 1)

$$f'(x) = \frac{2(x-1)e^{2x} - e^{2x}}{(x-1)^2}$$

(13) $f(x) = e^{x^3}$

$$f'(x) = \frac{d}{dx}(e^{x^3})$$

$$= e^{x^3}\frac{d}{dx}(x^3) \quad (\frac{d}{dx}(x^3) \to 3x^2)$$

$$f'(x) = 3x^2 e^{x^3}$$

(15) $f(x) = x^2 e^{x^3}$

$$f'(x) = \frac{d}{dx}(x^2 e^{x^3})$$

$$= x^2\frac{d}{dx}(e^{x^3}) + e^{x^3}\frac{d}{dx}(2x)$$

($\frac{d}{dx}(e^{x^3}) \to e^{x^3}\frac{d}{dx}(x^3) \to e^{x^3}\cdot 3x^2$; $\frac{d}{dx}(2x) \to 2$)

$$f'(x) = 3x^4 e^{x^3} + 2e^{x^3}$$

(17) $f(x) = x e^{-x^3}$

$$f'(x) = \frac{d}{dx}\left[x e^{-x^3}\right]$$

$$= x \frac{d}{dx}\left[e^{-x^3}\right] + e^{-x^3} \frac{d}{dx}(x)^1$$

$e^{-x^3} \frac{d}{dx}(-x^3)$

$-3x^2$

$$\boxed{f'(x) = -3x^3 e^{-x^3} + e^{-x^3}}$$

(19) $f(x) = \sin(e^{2\pi x})$

$$f'(x) = \frac{d}{dx}\left[\sin(e^{2\pi x})\right]$$

$e^{2\pi x} \frac{d}{dx}(2\pi x)$ (2π)

$$= \cos(e^{2\pi x}) \frac{d}{dx}(e^{2\pi x})$$

$$\boxed{f'(x) = 2\pi e^{2\pi x} \cos(e^{2\pi x})}$$

(21) $f(x) = 3e^{4x} - 5e^{-2x} + 2$

$$f'(x) = \frac{d}{dx}(3e^{4x} - 5e^{-2x} + 2)$$

$$= \frac{d}{dx}(3e^{4x}) - \frac{d}{dx}(5e^{-2x}) + \frac{d}{dx}(2)$$

$$= 3\frac{d}{dx}(e^{4x}) - 5\frac{d}{dx}(e^{-2x}) + \frac{d}{dx}(2)$$

(23) $f(x) = \sqrt{x - e^{2x}}$

$$f'(x) = \frac{d}{dx}\left[(x - e^{2x})^{1/2}\right]$$

$$= \frac{1}{2}(x - e^{2x})^{\frac{1}{2}-1} \frac{d}{dx}(x - e^{2x})$$

$$= \frac{1}{2}(x - e^{2x})^{-1/2} \left[\frac{d}{dx}(x) - \frac{d}{dx}(e^{2x})\right]$$

$\frac{d}{dx}(x) = 1$

$e^{2x}\frac{d}{dx}(2x)$, $\frac{d}{dx}(2x) = 2$

$$= \frac{1}{2}(x - e^{2x})^{-1/2}(1 - 2e^{2x})$$

$$\boxed{f'(x) = \frac{1 - 2e^{2x}}{2\sqrt{x - e^{2x}}}}$$

(25) $f(x) = \sin(e^{2\pi x})$

$$f'(x) = \frac{d}{dx}\left[\sin(e^{2\pi x})\right]$$

$$= \cos(e^{2\pi x}) \frac{d}{dx}(e^{2\pi x})$$

$e^{2\pi x}\frac{d}{dx}(2\pi x)$, $\frac{d}{dx}(2\pi x) = 2\pi$

$$\boxed{f'(x) = 2\pi e^{2\pi x} \cos(e^{2\pi x})}$$

(27) $f(x) = (1 - 4e^{-x})^3$

$f'(x) = \frac{d}{dx}\left[(1 - 4e^{-x})^3\right]$

$= 3(1 - 4e^{-x})^{3-1} \frac{d}{dx}(1 - 4e^{-x})$

$= 3(1 - 4e^{-x})^2 \left[\frac{d}{dx}(1) - \frac{d}{dx}(4e^{-x})\right]$ ($\frac{d}{dx}(1) \to 0$)

$-4 \frac{d}{dx}(e^{-x})$

$e^{-x} \frac{d}{dx}(-x)$ ($\frac{d}{dx}(-x) \to -1$)

$f'(x) = 3(1 - 4e^{-x})^2 \cdot 4e^{-x}$

$f'(x) = \frac{3(1 - 4e^{-x})^2}{4ex}$

(29) $f(x) = \sec(x)(x + 2e^{-x})$

$f'(x) = \frac{d}{dx}\left[\sec(x)(x + 2e^{-x})\right]$

product Rule !

$= \sec(x) \frac{d}{dx}(x + 2e^{-x}) + (x + 2e^{-x}) \frac{d}{dx}(\sec(x))$

$\frac{d}{dx}(x) + 2\frac{d}{dx}(e^{-x})$; $\sec(x)\tan(x)$

1 ; $e^{-x} \frac{d}{dx}(-x)$

-1

$$f'(x) = \sec(x)\left(1-2e^{-x}\right) + (x+2e^{-x})\sec(x)\tan(x)$$

$$f'(x) = \sec(x)\left[(1-2e^{-x}) + (x+2e^{-x})\tan(x)\right]$$

(31) $f(x) = x\sin(e^{2\pi x})$

$$f'(x) = \frac{d}{dx}\left[x\sin(e^{2\pi x})\right]$$

$$= x\frac{d}{dx}\left[\sin(e^{2\pi x})\right] + \sin(e^{2\pi x})\frac{d}{dx}(x)$$

$$\cos(e^{2\pi x})\frac{d}{dx}(e^{2\pi x})$$

$$e^{2\pi x}\frac{d}{dx}(2\pi x)$$

$$2\pi$$

$$f'(x) = 2\pi x e^{2\pi x}\cos(e^{2\pi x}) + \sin(e^{2\pi x})$$

(33) $f(x) = 5e^{\tan(2x)}$

$$f'(x) = \frac{d}{dx}\left[5e^{\tan(2x)}\right]$$

$$= 5\frac{d}{dx}\left[e^{\tan(2x)}\right]$$

$$e^{\tan(2x)}$$

$$\frac{d}{dx}\left[\tan(2x)\right]$$

$$\sec^2(2\pi x)\frac{d}{dx}(2\pi x)$$

$$2\pi$$

$$f'(x) = 5\, e^{2\pi x} \cdot \sec^2(2\pi x) \cdot 2\pi$$

$$\boxed{f'(x) = 10\pi\, e^{2\pi x} \sec^2(2\pi x)}$$

(35) $f(x) = 4e^{\sqrt{x}} - 7$

$$f'(x) = \frac{d}{dx}\left[4e^{\sqrt{x}} - 7\right]$$

$$= 4\frac{d}{dx}\left[e^{\sqrt{x}}\right] - \frac{d}{dx}(7)$$

$$e^{\sqrt{x}} \frac{d}{dx}(\sqrt{x})$$

$$\frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{4e^{\sqrt{x}}}{2\sqrt{x}} \quad ; \quad \boxed{f'(x) = \frac{2e^{\sqrt{x}}}{\sqrt{x}}}$$

(37) $f(x) = e^{-\sqrt{x} + 3\cos(x^2)} + 2$

$$f'(x) = \frac{d}{dx}\left[e^{-\sqrt{x} + 3\cos(x^2)} + 2\right]$$

$$= \frac{d}{dx}\left[e^{-\sqrt{x} + 3\cos(x^2)}\right] + \frac{d}{dx}[2]$$

$$= e^{-\sqrt{x} + 3\cos(x^2)} \cdot \frac{d}{dx}\left[-\sqrt{x} + 3\cos(x^2)\right]$$

$$f'(x) = e^{-\sqrt{x} + 3\cos(x^2)} \cdot \frac{d}{dx}\left[-\sqrt{x} + 3\cos(x^2)\right]$$

$$-\frac{d}{dx}(\sqrt{x}) + 3\frac{d}{dx}\left[\cos(x^2)\right]$$

$$\frac{1}{2\sqrt{x}} \qquad \sin(x^2)\,\frac{d}{dx}(x^2)$$

$$2x$$

$$f'(x) = e^{-\sqrt{x}+3\cos(x^2)}\left[-\frac{1}{2\sqrt{x}} + 3\sin(x^2)\cdot 2x\right]$$

$$f'(x) = e^{-\sqrt{x}+3\cos(x^2)}\left[-\frac{1}{2\sqrt{x}} + 6x\sin(x^2)\right]$$