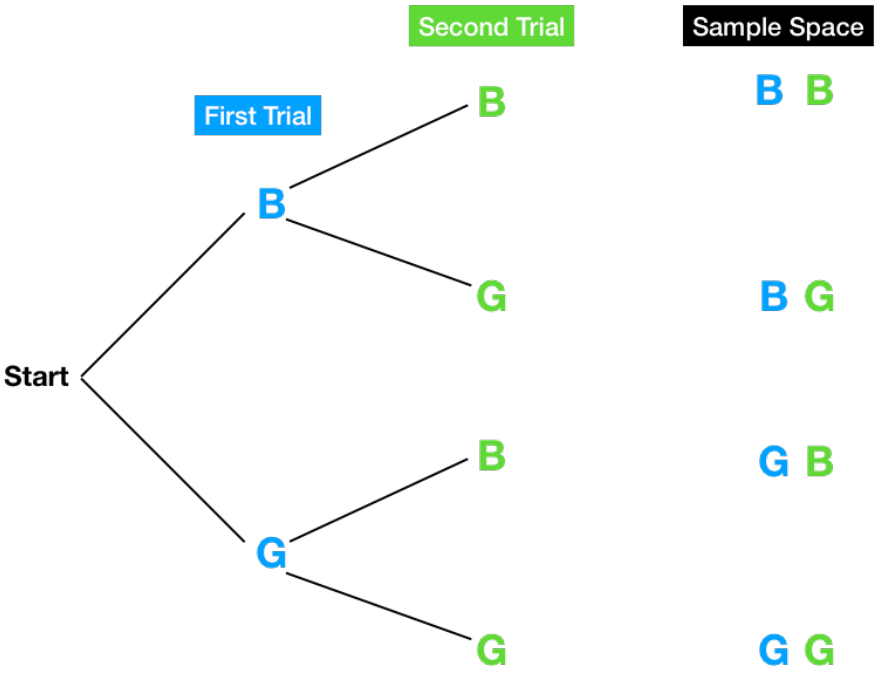
Binomial Probability Distribution

We use this distribution when dealing with a special type of experiment that involves counting success out of a fixed number of trials. Each trial is independent with a probability of success for a single trial.

Consider the experiment of having two children. The tree diagram is as follows.



This process assumes we have one child at a time and each trial is independent of one another. We let $x = \#\ of\ boys$ when having two children be our discrete random variable. We can create the following probability distribution by following our tree diagram.

x	P(x)
0	0.25
1	0.5
2	0.25

Sum 1

What if we want to compute probabilities for couples that want to have **4 children or more**? We can generalize this process by using the Binomial Probability Formula.

$$P(x) = nC_x p^x (1-p)^{n-x}$$

However, we must define each variable in our formula (n,x,p) and describe how this formula works. This formula is based on the concept of independent trials.

n is the number of trials.

x is the number of **successes** out of **n trials**.

p is the probability of a success for a **single trial**.

$q = 1 - p$ is the complement of p.

The key to using the Binomial Probability Formulas is to consider a **single trial**.

Having Four Children
A couple plans on having 4 children, what's the probability of having:

1. No boys?
2. One boy?
3. Two boys?
4. Three boys?
5. Four boys?

In this case, the number of trials is 4, that is $\boldsymbol{n = 4}$

Let x be the number of boys (successes) so that our questions can be posed in terms of the following values of x.

No boys
$x = 0$

One boy
$x = 1$

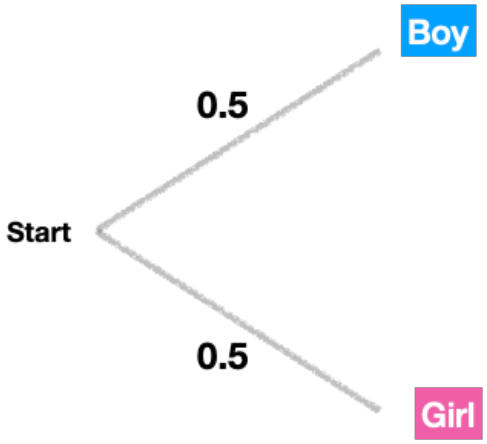
Two boys
$x = 2$

Three boys
$x = 3$

Four boys
$x = 4$

We need to know the value of p for this experiment. In order to determine this value we need to consider a **single trial** for having children.

Single Trial



Since we are counting successes as having boys, the probability of a success for a single trial is $p = \frac{1}{2} = 0.5$

The complement $q = 1 - p$ or $q = 1 - 0.5$ thus $q = 0.5$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

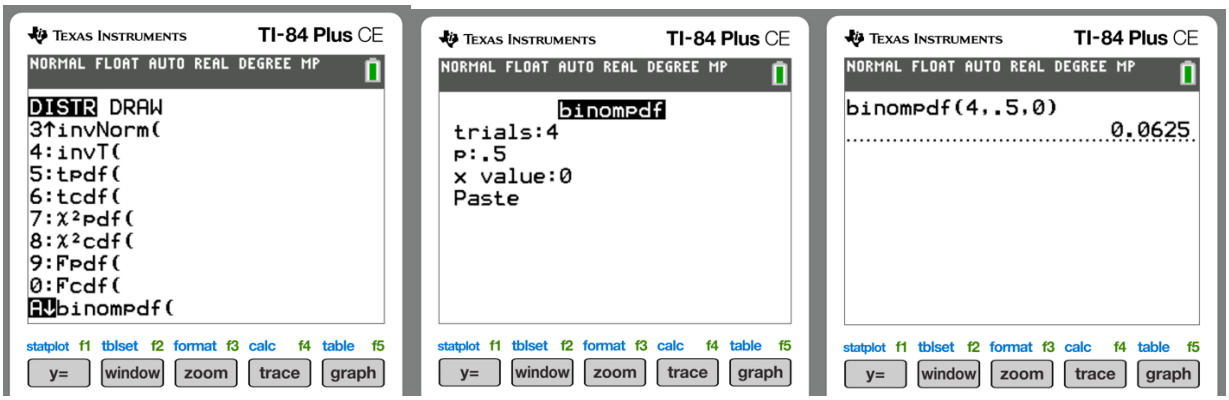
We now have all the information (n, x, p) we need to answer a Binomial Probability Distribution question.

1. **No boys**

$n = 4$
$x = 0$
$p = 0.5$

TI-83 or TI-84
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(4,0.5,0)



$$p(0) \approx 0.063$$

2. **One boy**

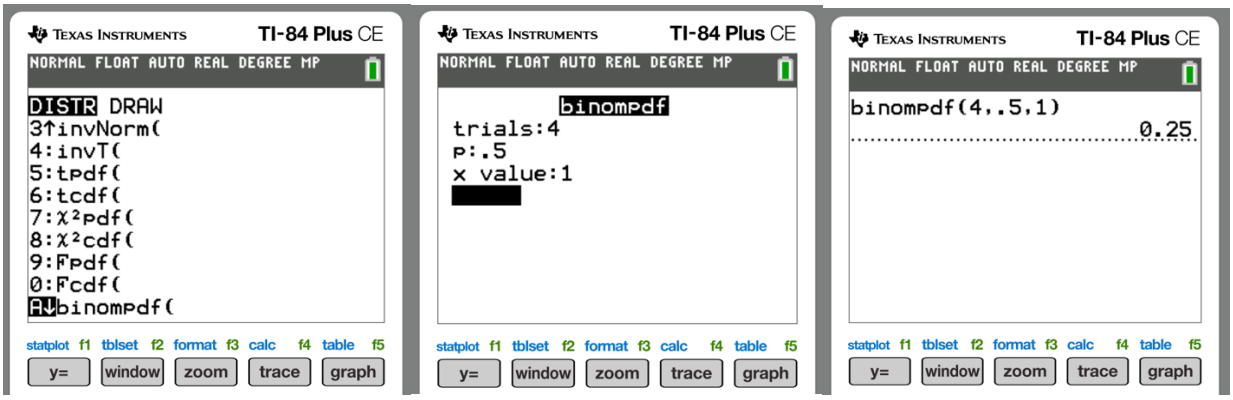
$n = 4$
$x = 1$
$p = 0.5$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84 Plus

1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(4,0.5,1)



$$\boldsymbol{p(1) = 0.25}$$

3. **Two boys**

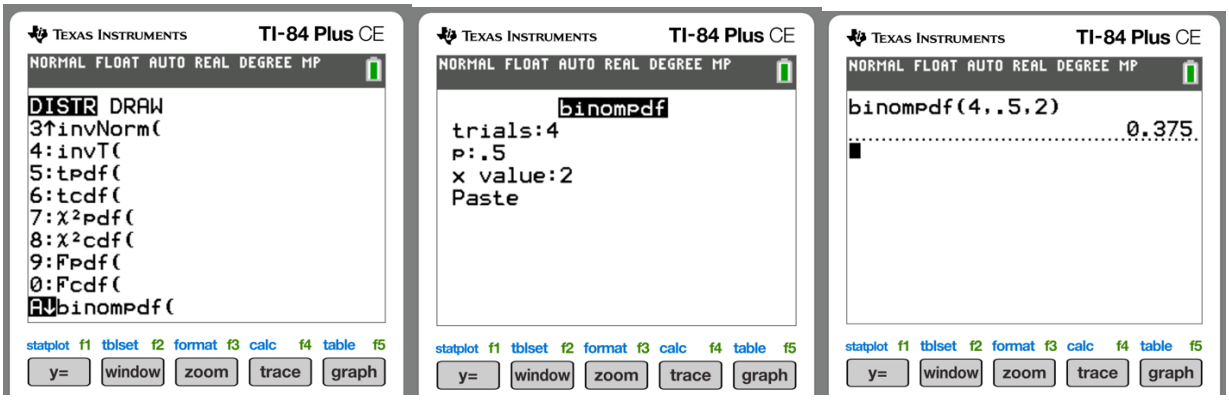
$n = 4$
$x = 2$
$p = 0.5$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84 Plus

1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(4,0.5,2)



$$p(2) = 0.375$$

4. **Three boys**

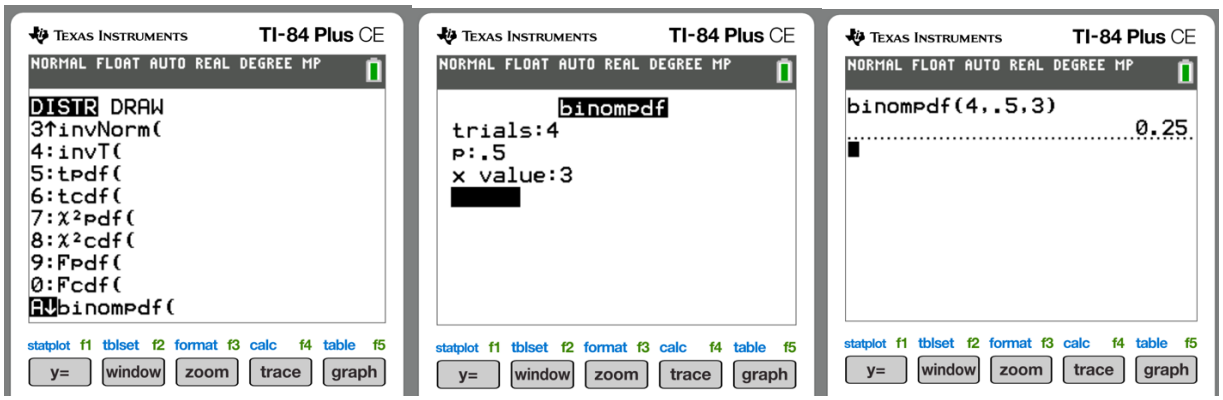
$n = 4$
$x = 3$
$p = 0.5$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84 Plus

1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(4,0.5,3)



$$p(3) = 0.25$$

5. **Four boys**

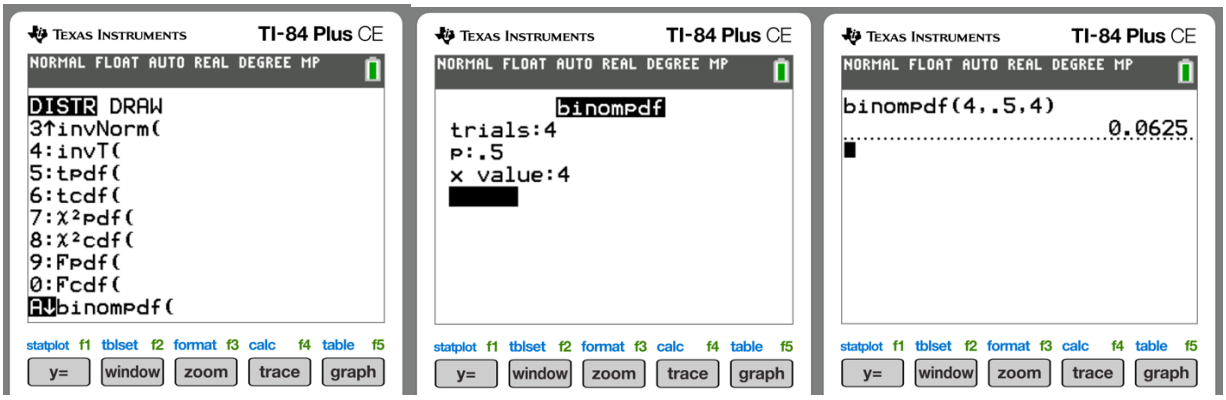
$n = 4$
$x = 4$
$p = 0.5$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84 Plus

1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(4,0.5,4)

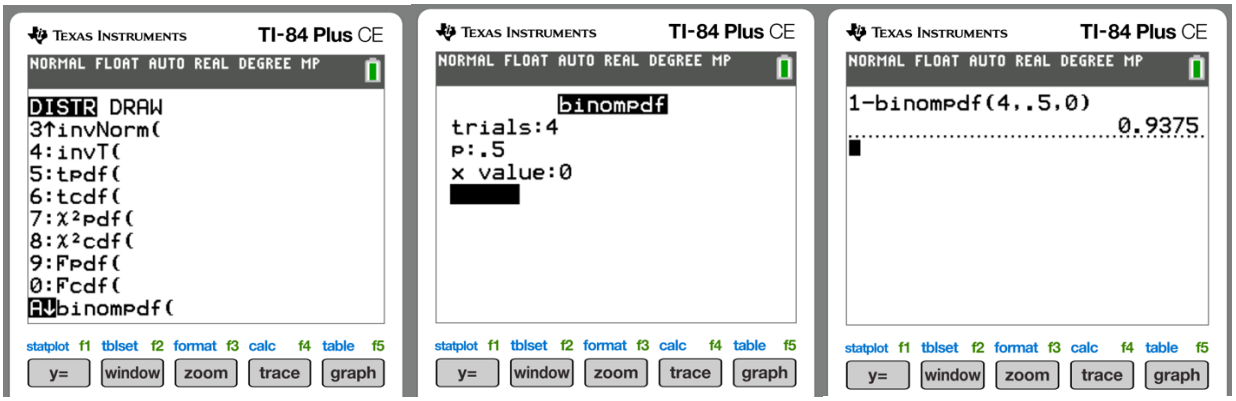


$$p(4) \approx 0.063$$

And of course my favorite questions.

6. At least one boy?

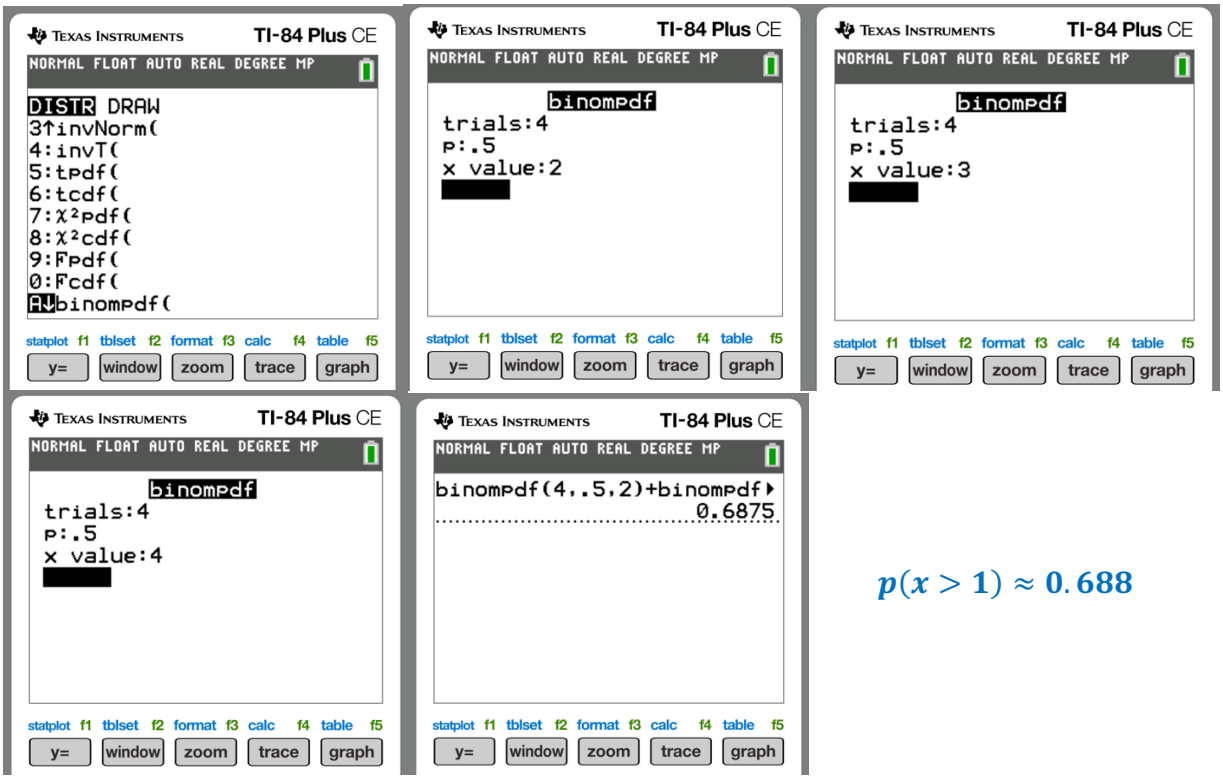
$$p(x \geq 1) = p(1) + p(2) + p(3) + p(4) = 1 - p(0)$$



$$p(x \geq 1) \approx 0.938$$

7. More than one boy?

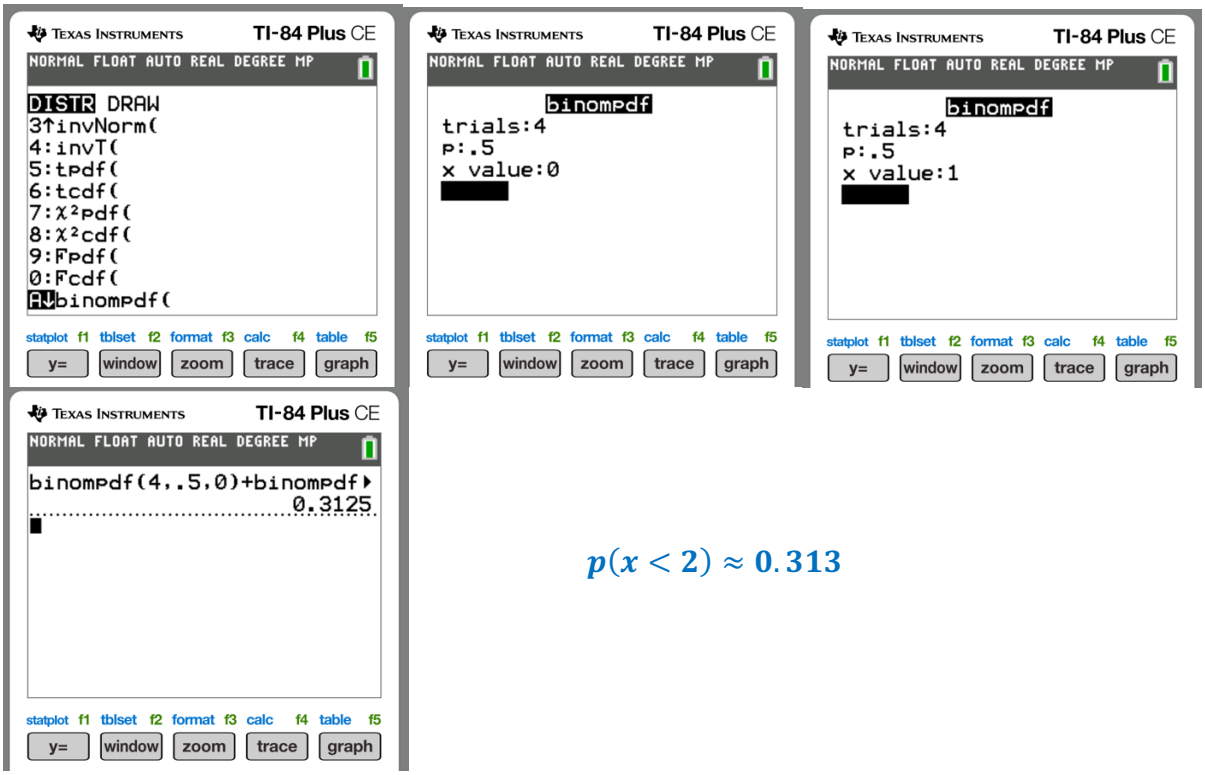
$$p(x > 1) = p(2) + p(3) + p(4)$$



$$p(x > 1) \approx 0.688$$

8. Less than two boys?

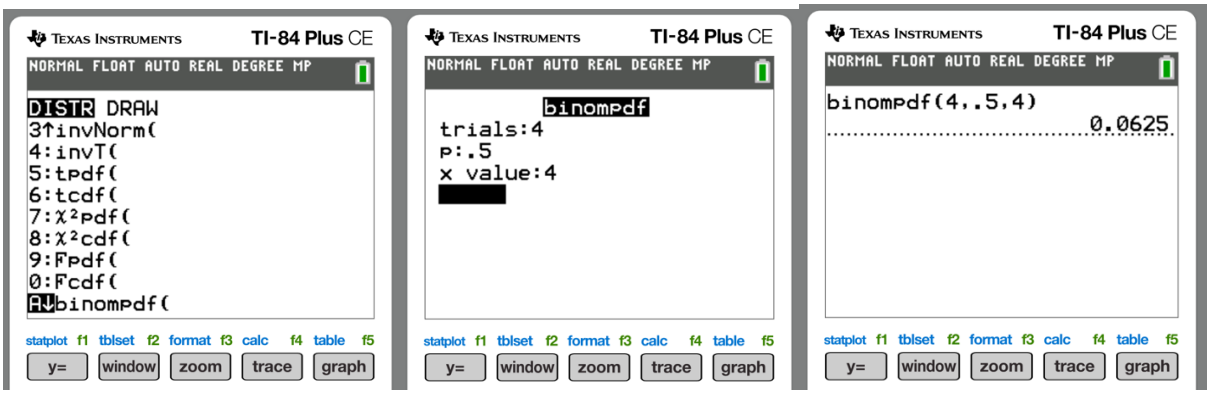
$$p(x < 2) = p(0) + p(1)$$



$$p(x < 2) \approx 0.313$$

9. More than three boys?

$$p(x > 3) = p(4)$$



$$p(x > 3) \approx 0.063$$

10. Between one and three boys?

$$p(1 \leq x \leq 3) = p(1) + p(2) + p(3)$$

$$p(1 \leq x \leq 3) = 0.875$$

Fact- Just because we have two outcomes, that does not mean that they are equally likely.

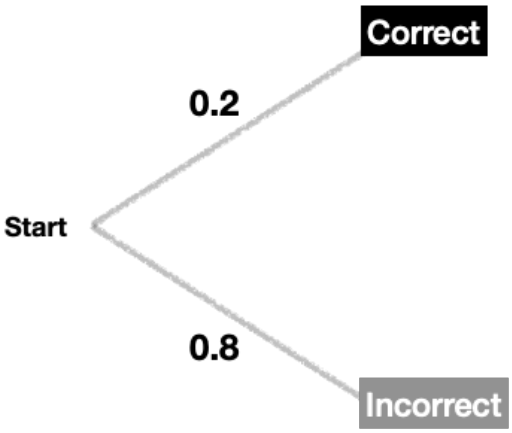
$$p \neq q$$

12 Question Multiple Choice Quiz

Every question has 5-possible answers (a-b-c-d-e). If you guess on every question, what's the probability of guess correct on:

11. No questions?
12. One question?
13. Two questions?
14. Three questions?
15. Four questions?

Single Trial



x = number of correct guesses

$n = 12$

$p = \frac{1}{5}$ or $p = 0.2$

$q = 1 - p$ or $q = 0.8$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

11. **No questions**?

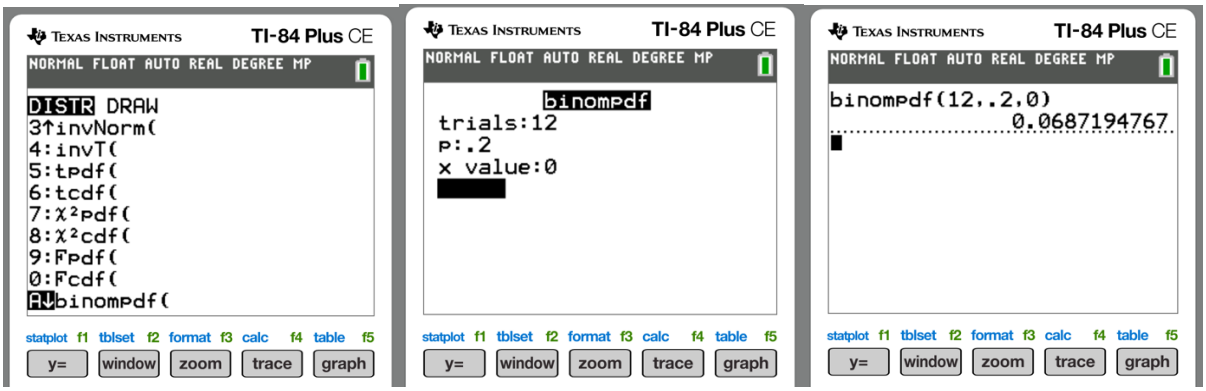
$n = 12$
$x = 0$
$p = 0.2$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84

1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(12,0.2,0)



$$p(0) \approx 0.069$$

12. **One questions**?

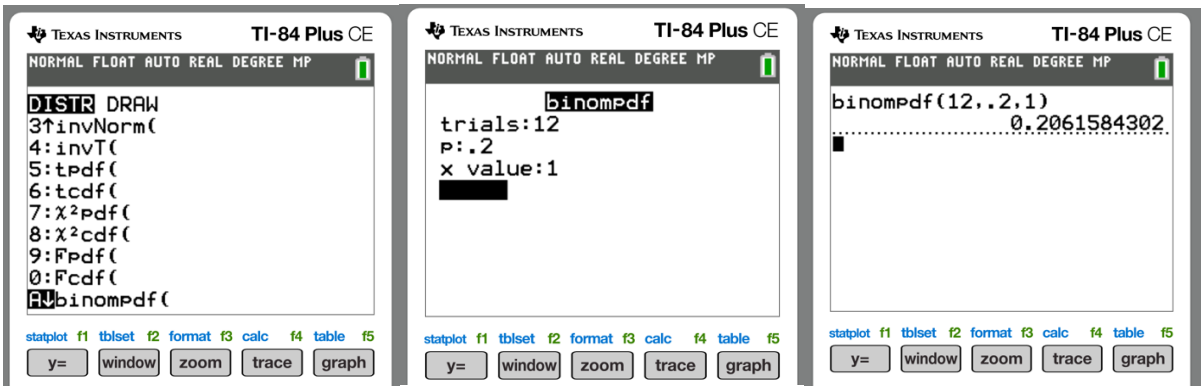
$n = 12$
$x = 1$
$p = 0.2$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84 Plus

1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(12,0.2,1)



$$p(1) \approx 0.206$$

13. **Two questions**?

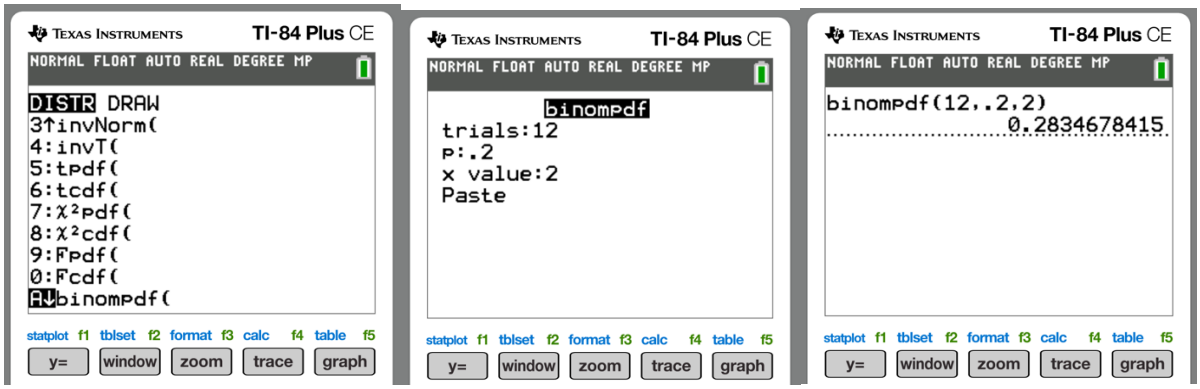
$n = 12$
$x = 2$
$p = 0.2$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84 Plus

1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(12,0.2,2)



$$p(2) \approx 0.283$$

14. **Three questions**?

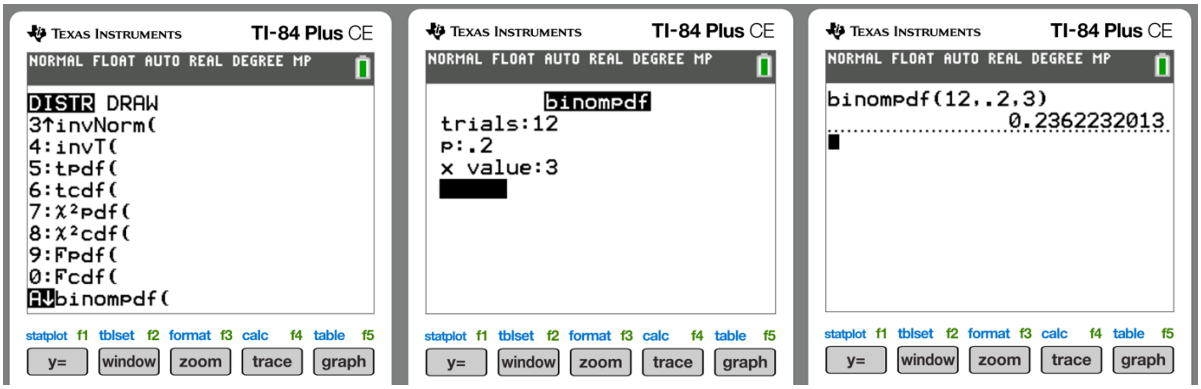
$n = 12$
$x = 3$
$p = 0.2$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84 Plus

1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(12,0.2,3)



$$p(3) \approx 0.236$$

15. **Four questions**?

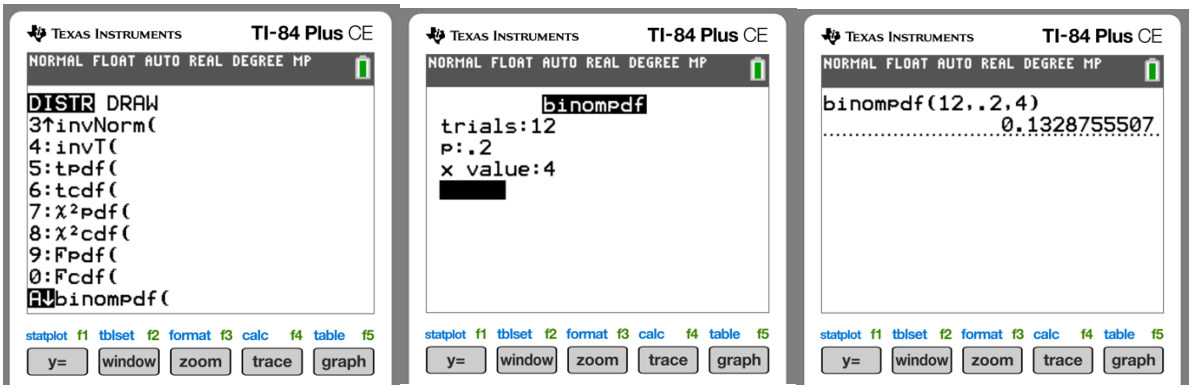
$n = 12$
$x = 4$
$p = 0.2$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84 Plus

1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(12,0.2,4)

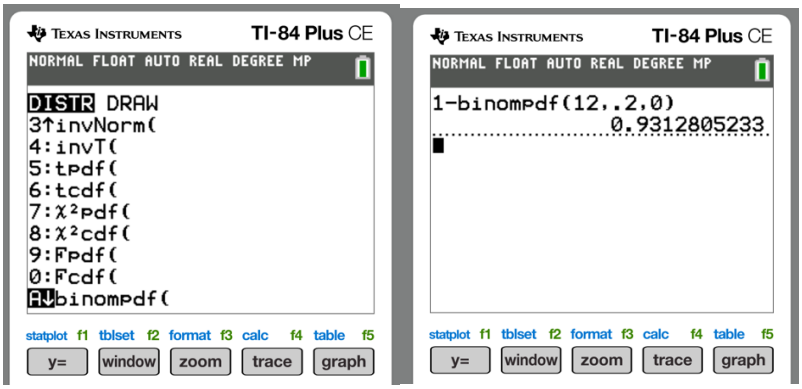


$$p(4) \approx 0.133$$

And, my favorite questions.

16. At least one correct?

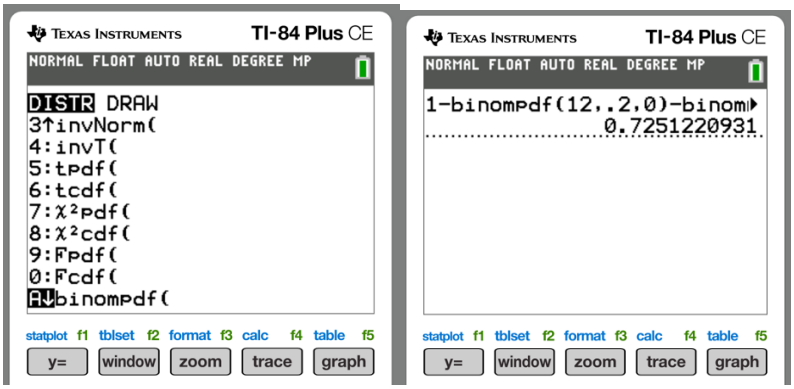
$$p(x \geq 1) = p(1) + p(2) + \cdots + p(11) + p(12) = 1 - p(0)$$



$$p(x \geq 1) \approx 0.931$$

17. At least two correct?

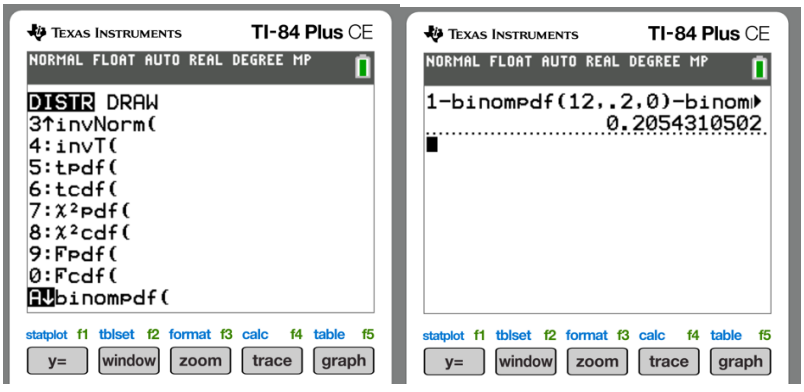
$$p(x \geq 2) = p(2) + p(3) + \cdots p(11) + p(12) = 1 - p(0) - p(1)$$



$$p(x \geq 2) \approx 0.725$$

18. More than three correct?

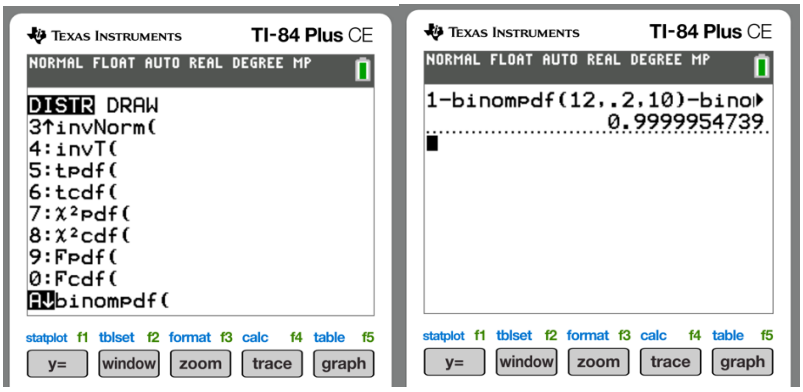
$$p(x > 3) = p(4) + p(5) + \cdots + p(11) + p(12) = 1 - p(0) - p(1) - p(2) - p(3)$$



$$p(x > 3) \approx 0.205$$

19. Less than ten correct?

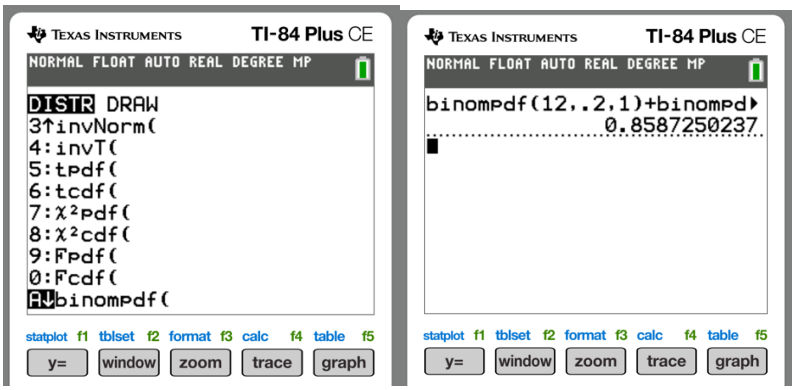
$$p(x < 10) = p(0) + p(1) + \cdots + p(8) + p(9) = 1 - p(10) - p(11) - p(12)$$



$$p(x < 10) \approx 1.000$$

20. Between one and four correct?

$$p(1 \le x \le 4) = p(1) + p(2) + p(3) + p(4)$$



$$p(1 \le x \le 4) \approx 0.859$$

This Binomial Porbability Distribution also has a way to compute its mean, variance, and standard deviation. In fact, these are the short cut formulas.

Mean $\mu = np$

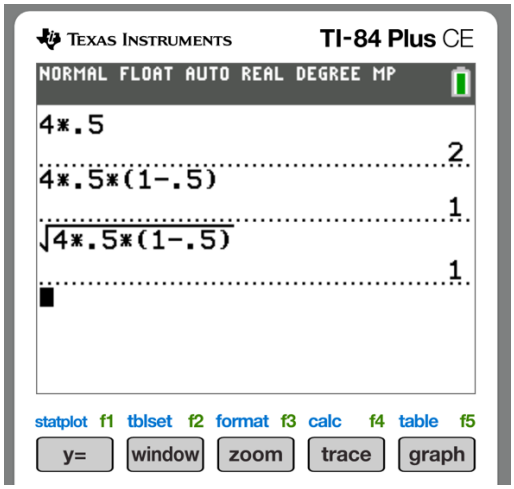
Variance $\sigma^2 = np(1-p)$

Standard Deviation $\sigma = \sqrt{np(1-p)}$

Determine the mean, variance, and standard deviation.

Four Children
Let x be the number of boys

x	p(x)
0	0.063
1	0.25
2	0.375
3	0.25
4	0.063
Sum	**1.001**



$$\mu = 2$$

$$\sigma^2 = 1$$

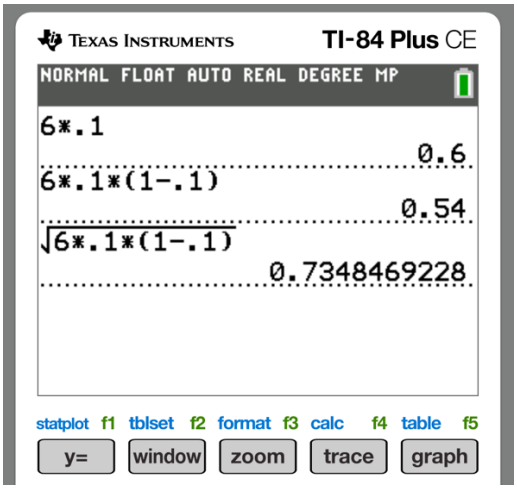
$$\sigma = 1$$

Left Handed (select 6 students at random)

Let x be the number of left handed students where the probability a person is left handed is 0.10 as 10% of the population is left handed.

x	p(x)
0	0.531441
1	0.354294
2	0.098415
3	0.01458
4	0.001215
5	0.000054
6	0.000001

Sum **1**



$$\mu = 0.6$$

$$\sigma^2 = 0.54$$

$$\sigma = 0.735$$