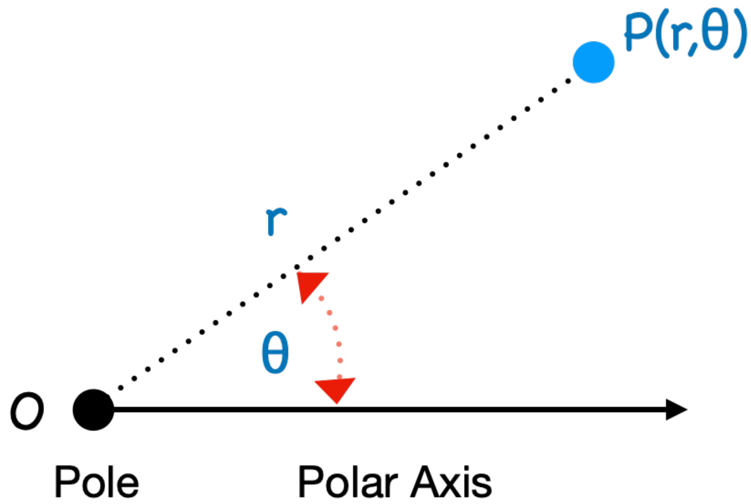
Polar Coordinates

The system was introduced by Newton to more easily describe curves in the rectangular coordinate system and consequently perform Calculus with those curves. It involves some concepts in Trigonometry and is based on two variables r (radius) and θ (central angle). That is, we can describe a point P in relation to a pole (origin) and the polar axis.



There are two points in this plane O and P. The point O is called the **Pole** and the ray is known as the **Polar Axis**. We define the distance from O to P as the radius and label that value r. That is, $r = |OP|$ and θ is the central angle found by rotating the polar axis either clockwise or counter clockwise. $\theta > 0$ for CCW rotation and $\theta < 0$ for CW rotation.

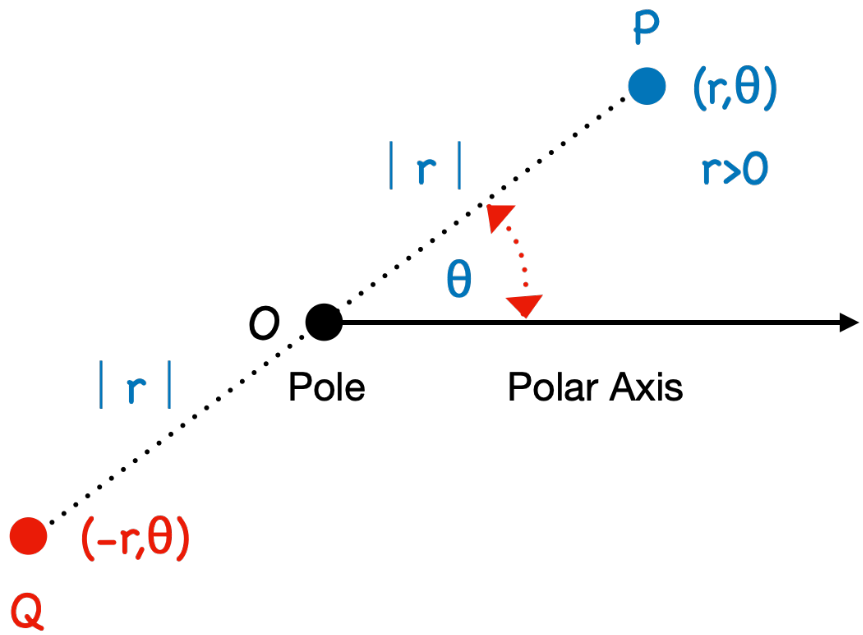
We are now able to define points (r, θ) in a plane called the **Polar Coordinate System**.

Facts

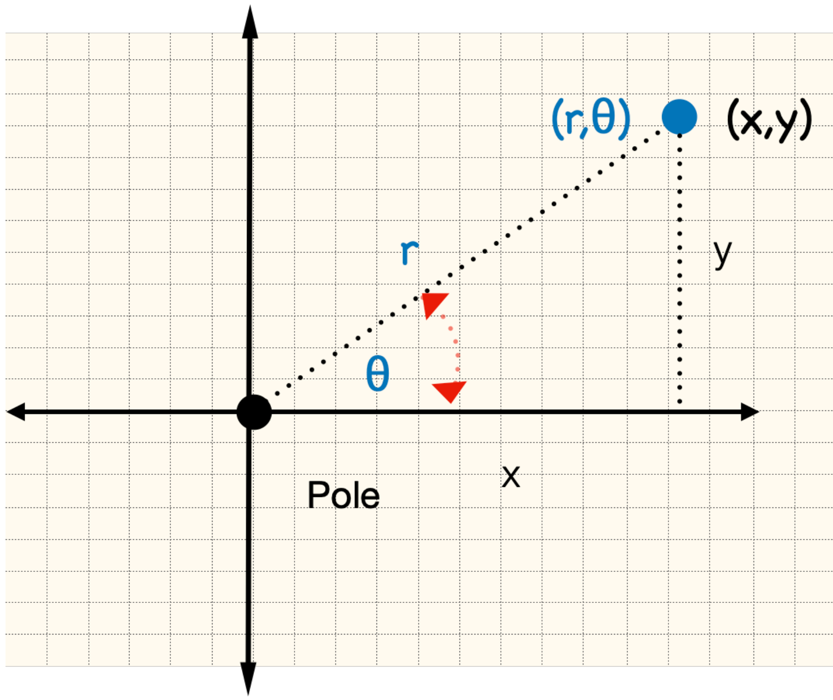
- If $P = O$, then we can say that $r = 0$ and the location (Origin/Pole) $O = P(0, \theta)$ for any angle θ.

- $(r, \theta) = (r, \theta + 2n\pi)$ for $n \epsilon Z$ which means these points are not well defined.

- We define a negative radius value $(r < 0)$ as being a location that is across the Pole. That is, the points (r, θ) and $(-r, \theta)$ lie across the Pole along a line that is the same distance $|r|$ in opposite sides from the Pole.



- Every point (x, y)on the Cartesian Coordinate System can be represented on the Polar Coordinate System as we superimpose both systems on top of each other.

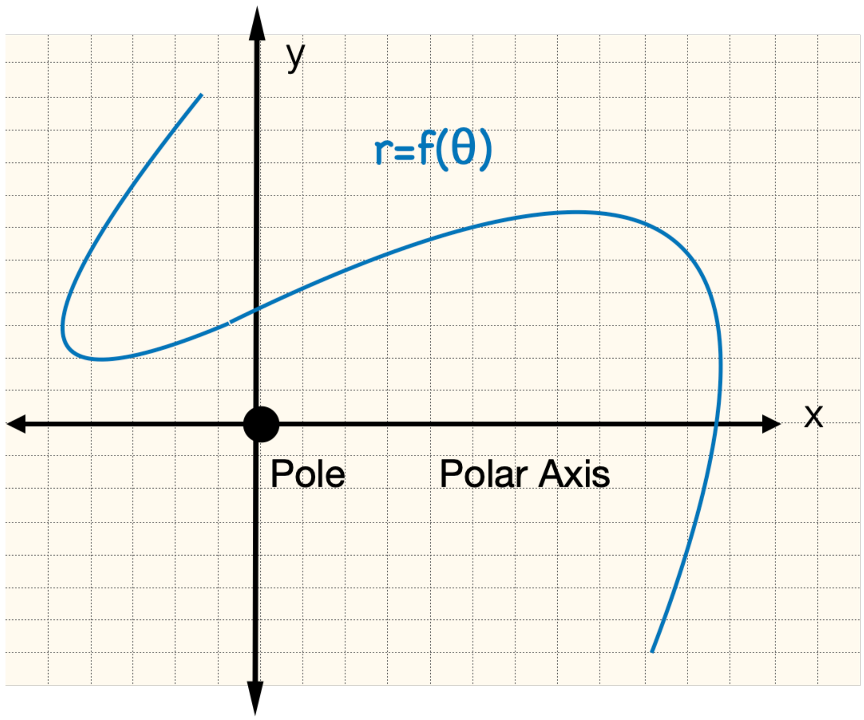


We are now able to convert from the Rectangular Coordinate System to the Polar Coordinate System and vice versa by considering the following equations.

Polar to Rectangular	**Rectangular to Polar**
$x = rcos(\theta)$	$x^2 + y^2 = r^2$
$y = rsin(\theta)$	$tan(\theta) = \frac{y}{x}, x \neq 0$

Polar Curves

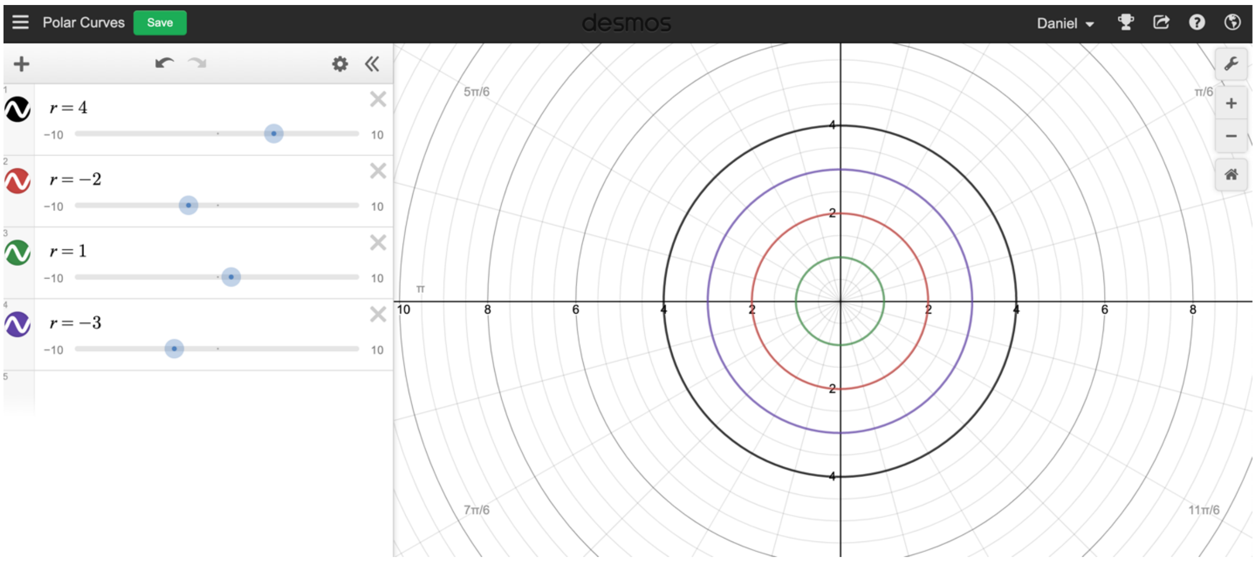
$r = f(\theta)$



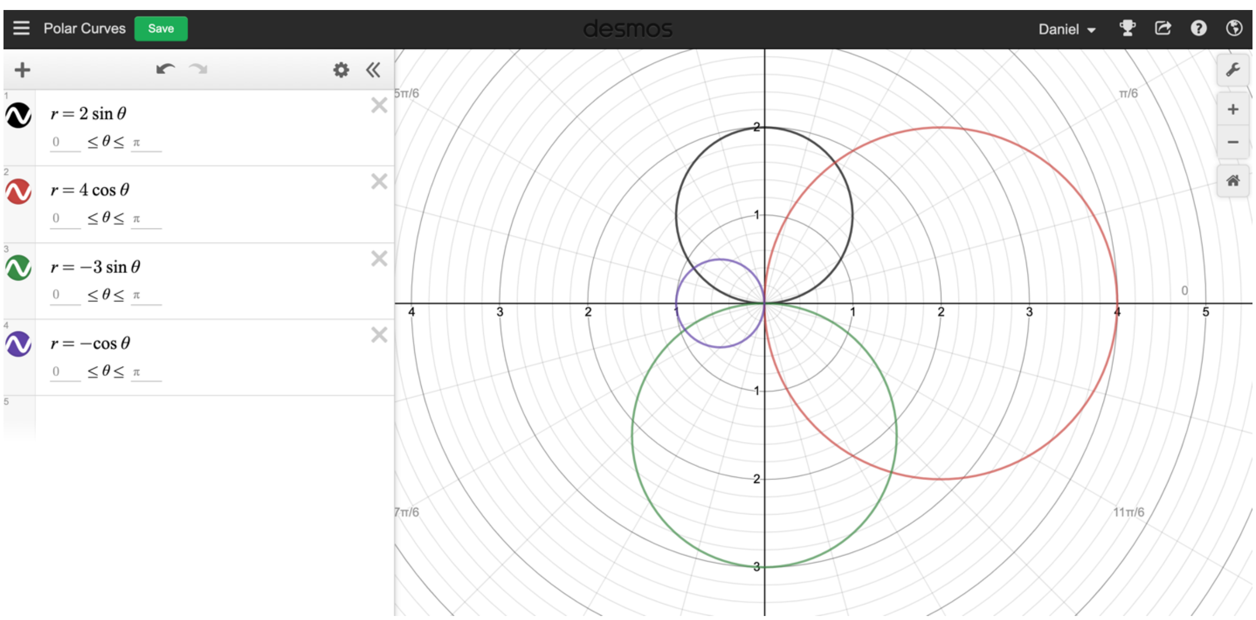
Interesting Polar Curves
(Some you may recognize)

Circles

$r = constant$

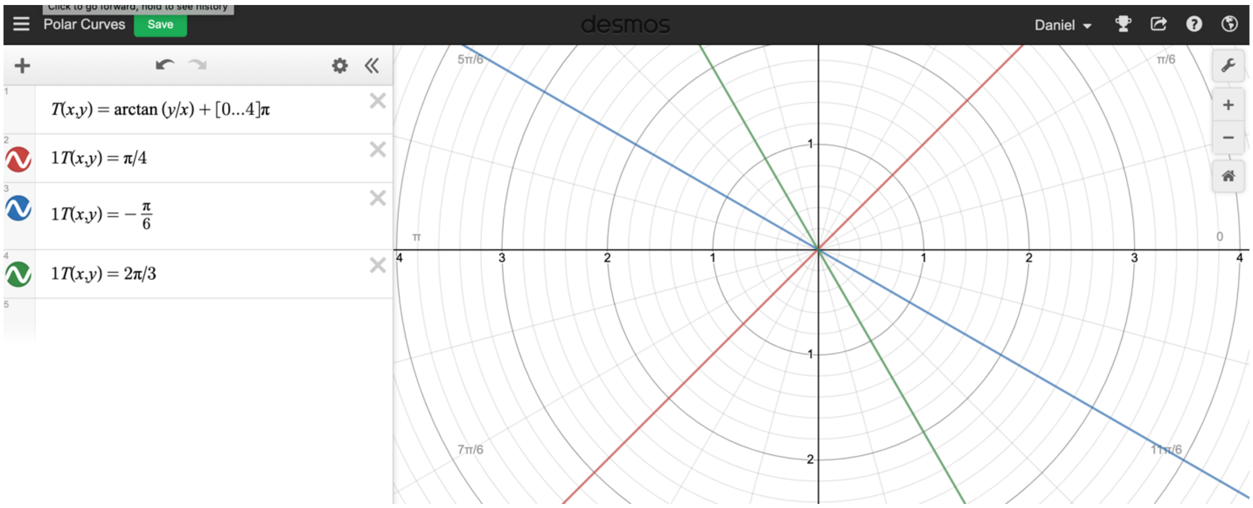


$r = acos(\theta)$ and r=$asin(\theta)$

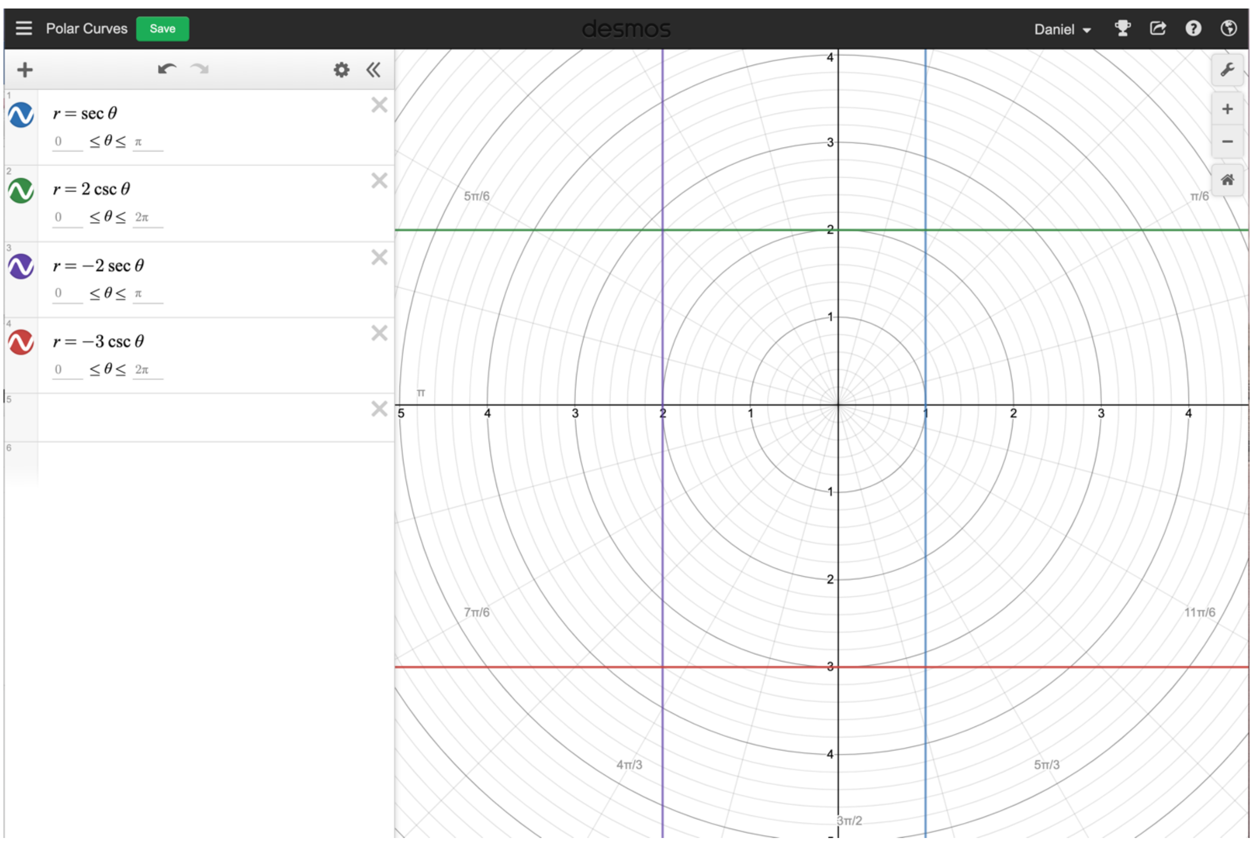


Lines

$\theta = constant$



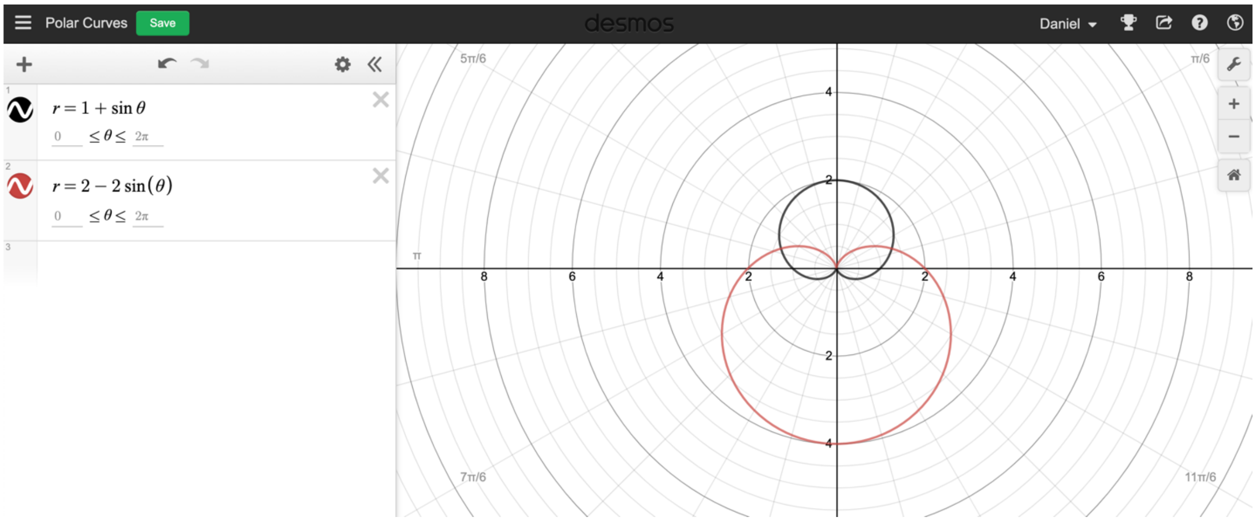
$r = asec(\theta)$ or $r = acsc(\theta)$



Cardiod (Heart)

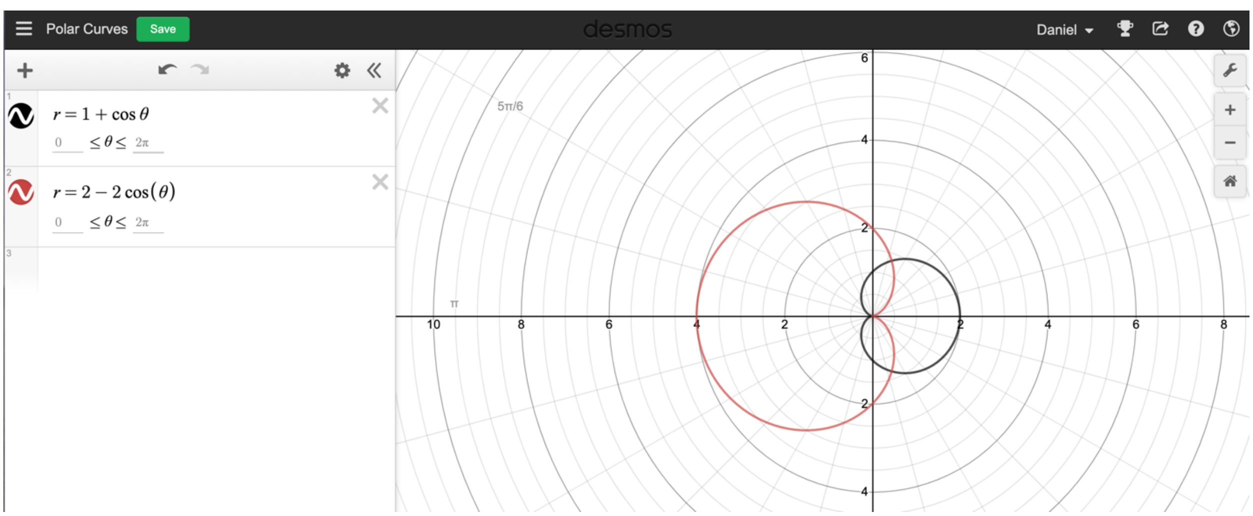
Vertical

$r = a \pm a\sin(\theta))$



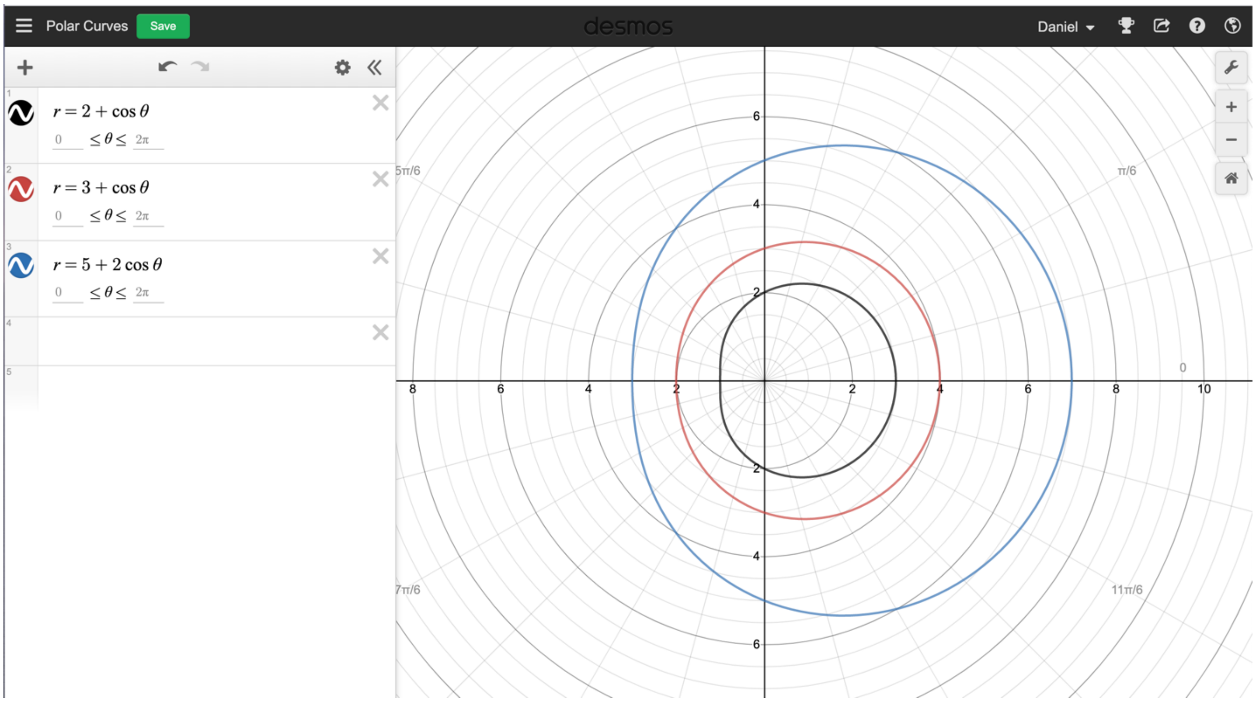
Horizontal

$r = a \pm a\cos(\theta)$

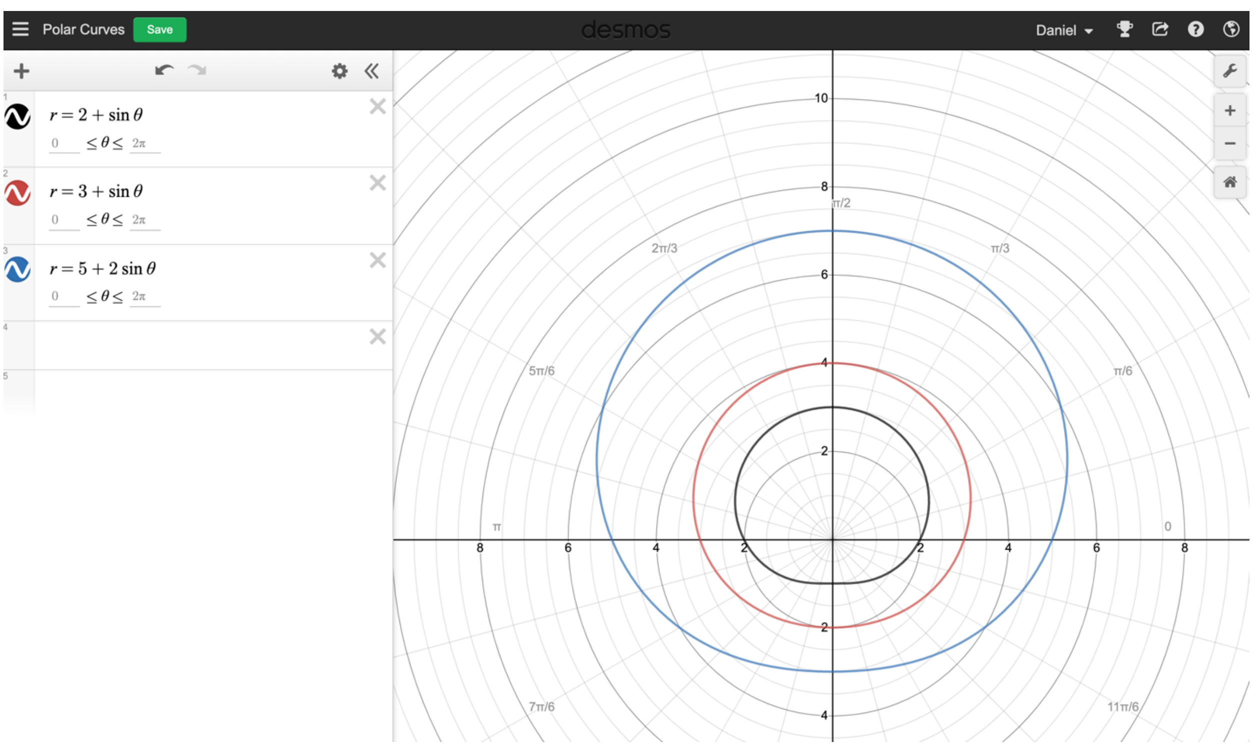


Convex Limacon

$r = b + a\cos(\theta)$ where $b \geq 2a$

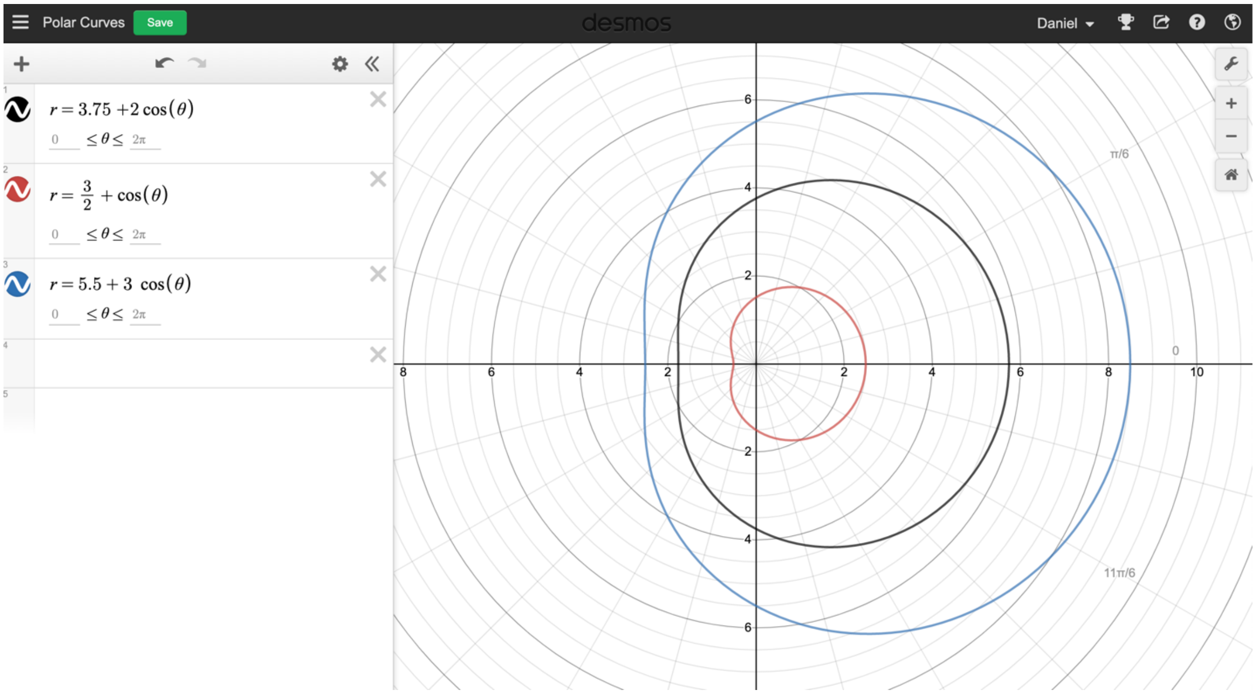


$r = b + a\cos(\theta)$ where $b \geq 2a$

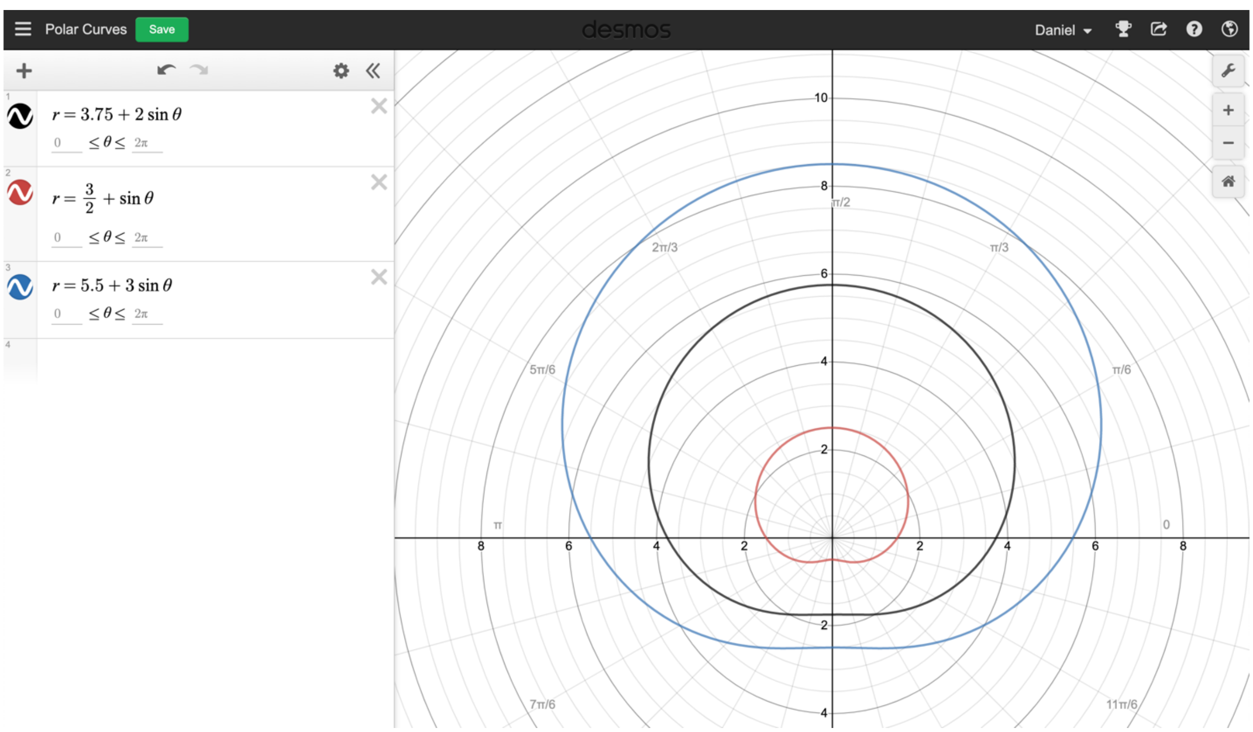


Dimpled Limacon

$r = b + a\cos(\theta)$ where $2a < b < a$

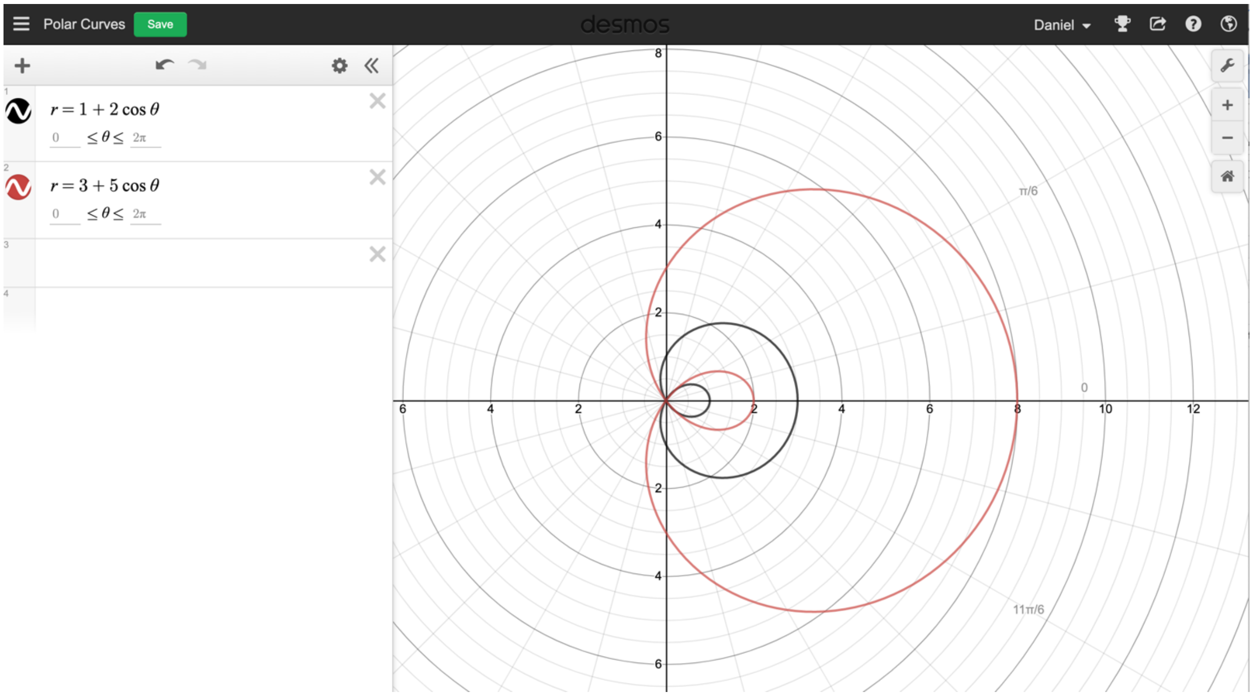


$r = b + a\sin(\theta)$ where $2a < b < a$

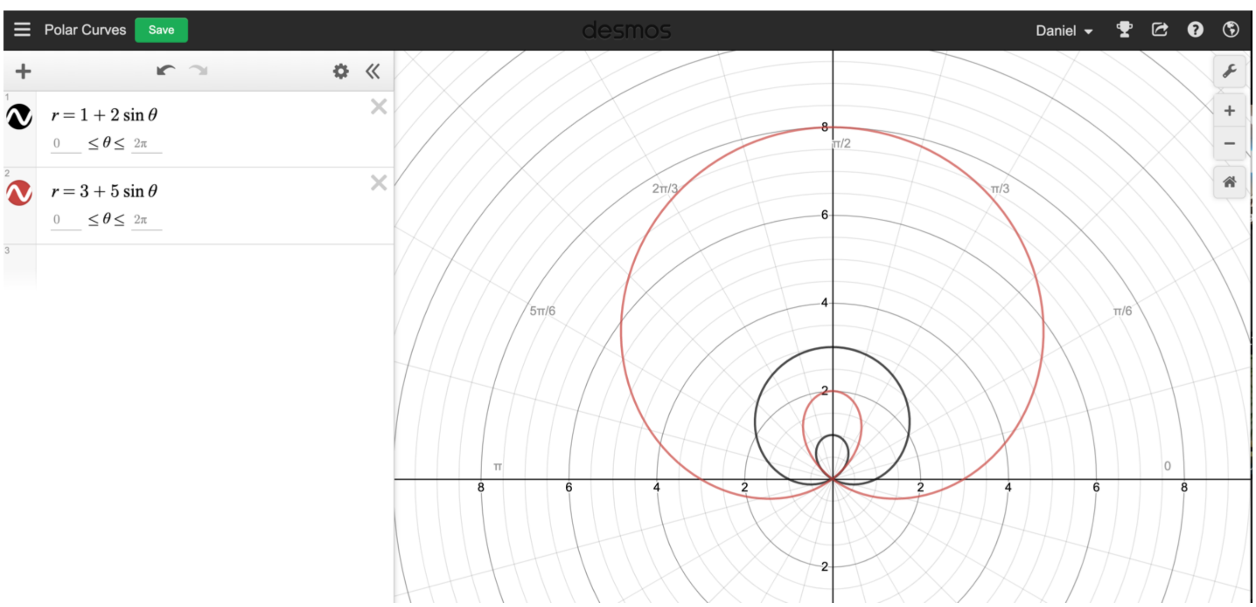


Inner Loop Limacon

$r = b + a\cos(\theta)$ where $b < a$

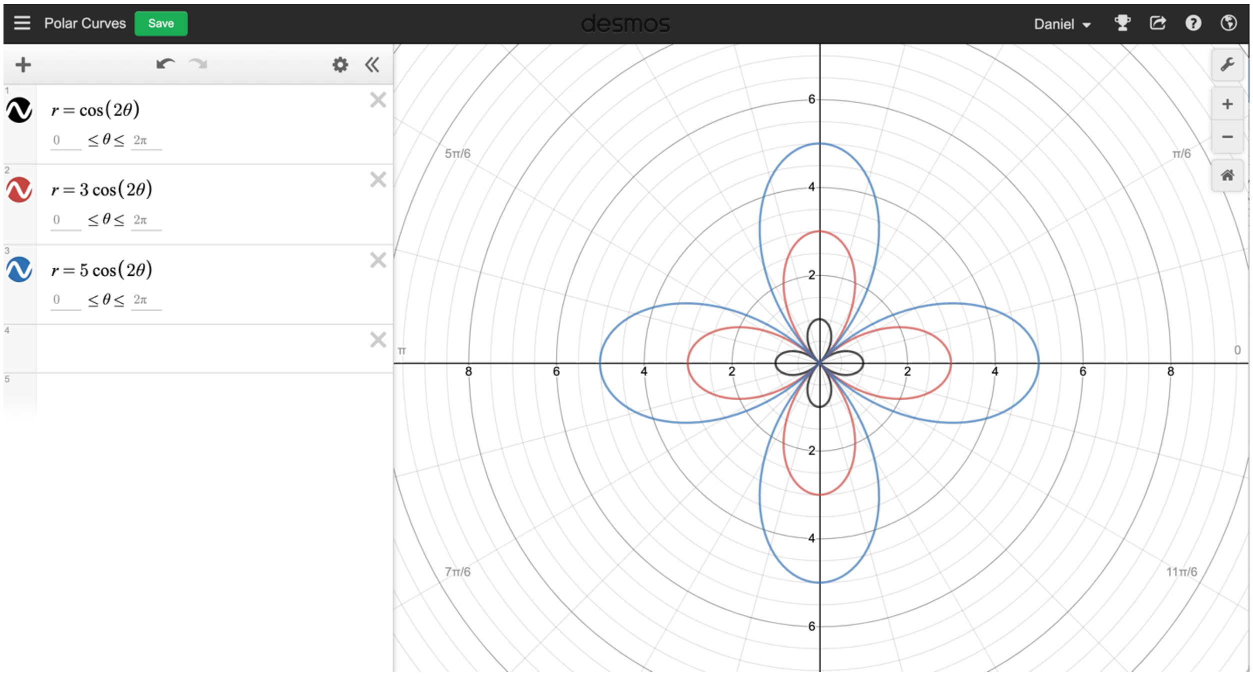


$r = b + a\sin(\theta)$ where $b < a$

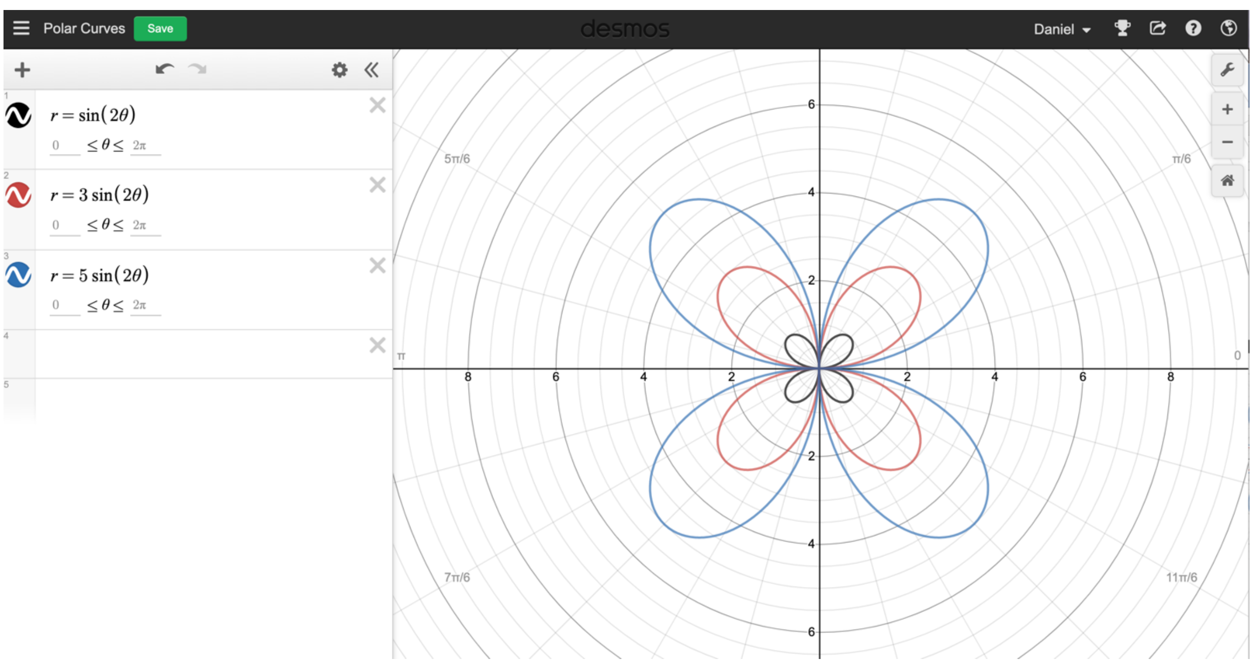


4-leaf Rose

$r = a\cos(2\theta)$

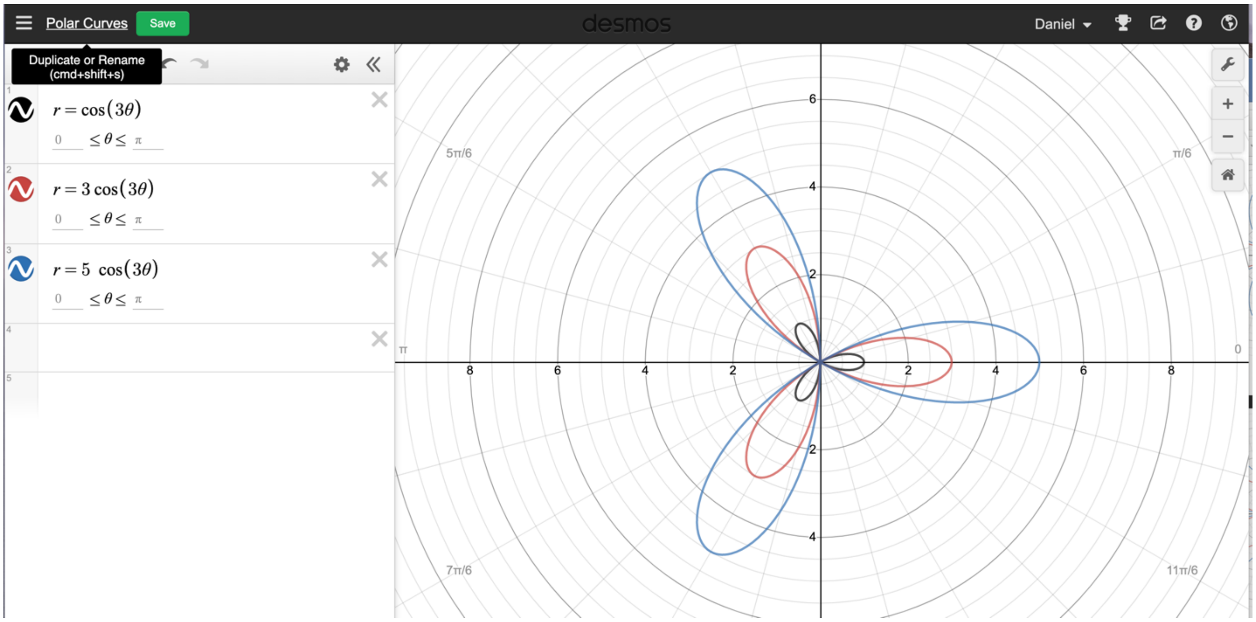


$r = a\sin(2\theta)$

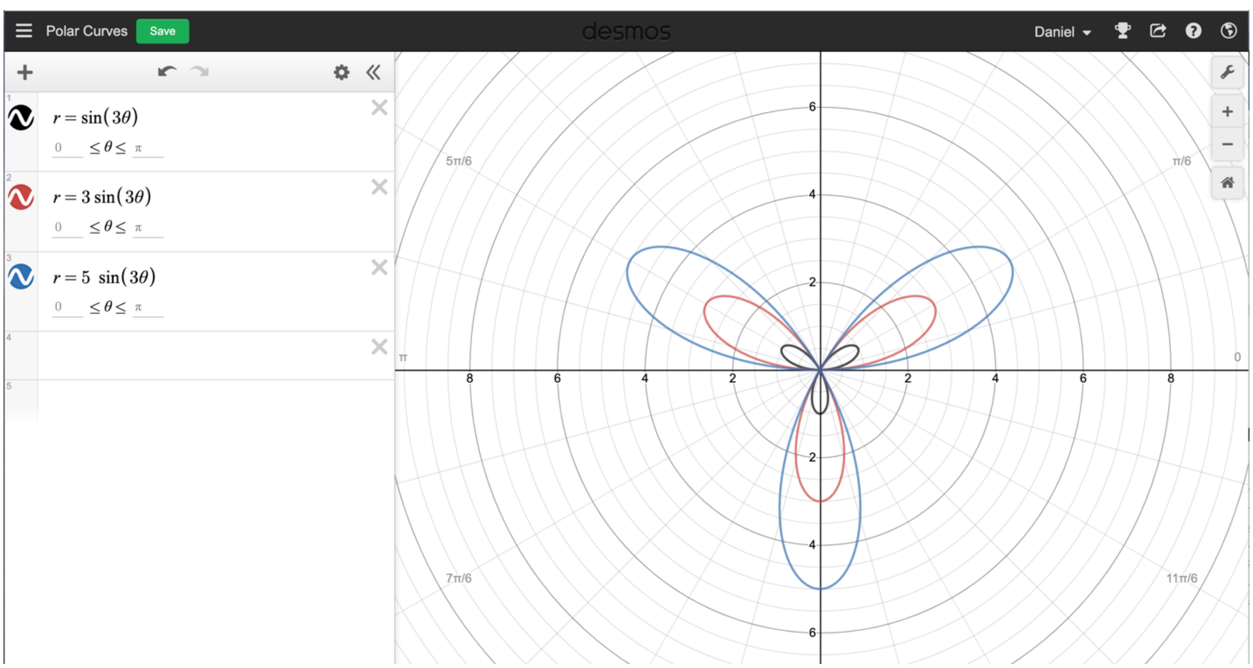


3-leaf Rose

$r = acos(3\theta)$



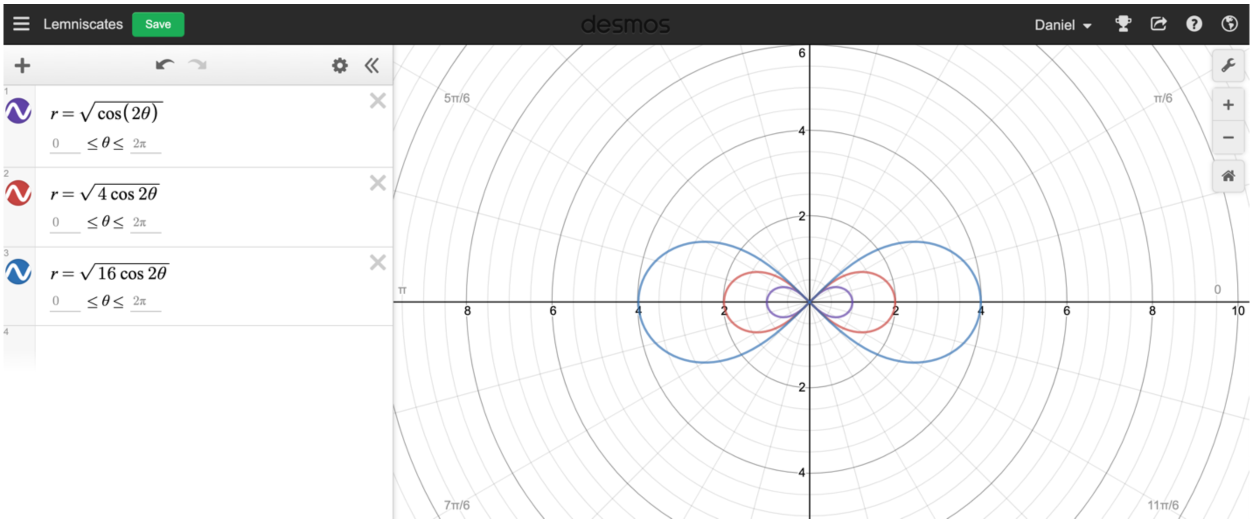
$r = asin(3\theta)$



Leminiscates

Figure-eight shaped curves

$r^2 = acos(2\theta)$ where $a > 0$ but $a < 0$



$r^2 = asin(2\theta)$ where $a > 0$ but $a < 0$