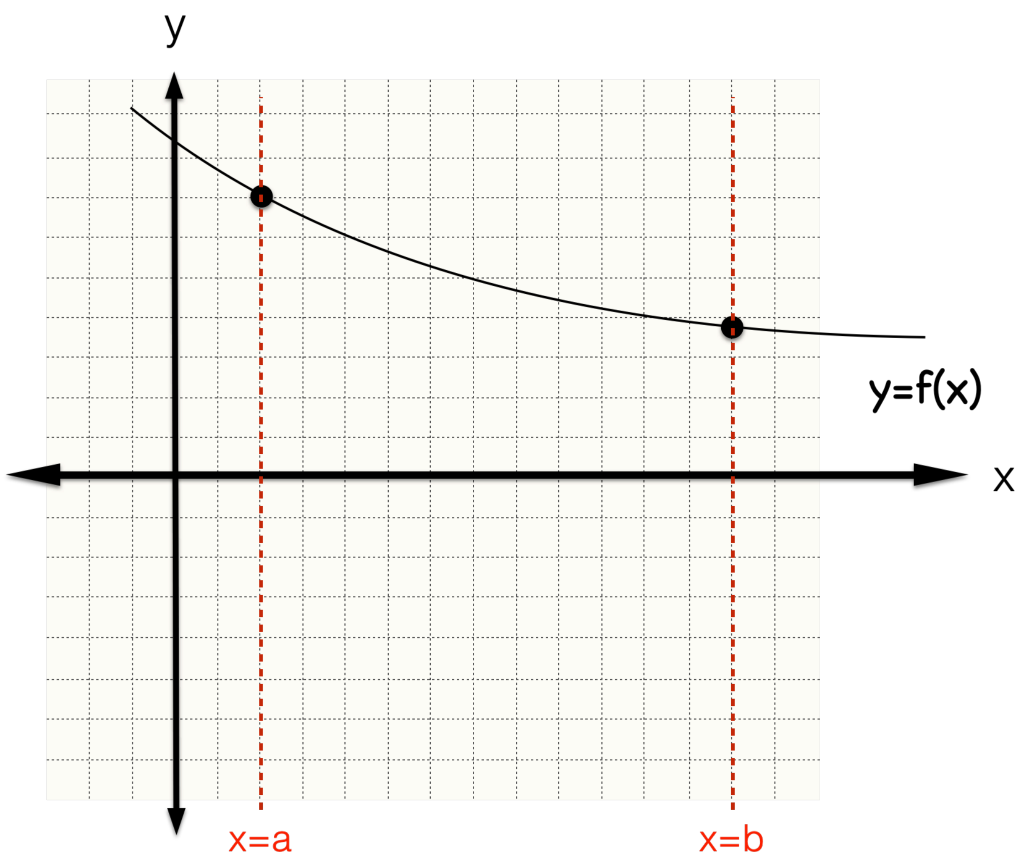
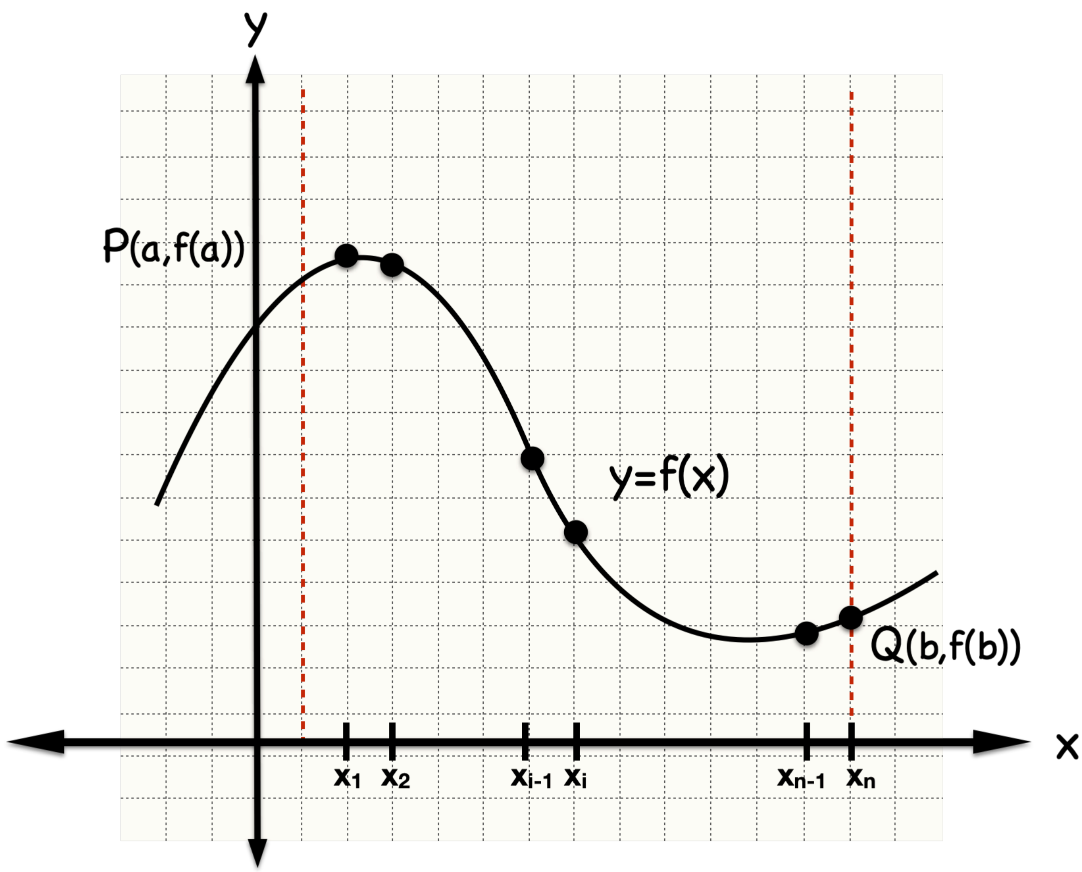
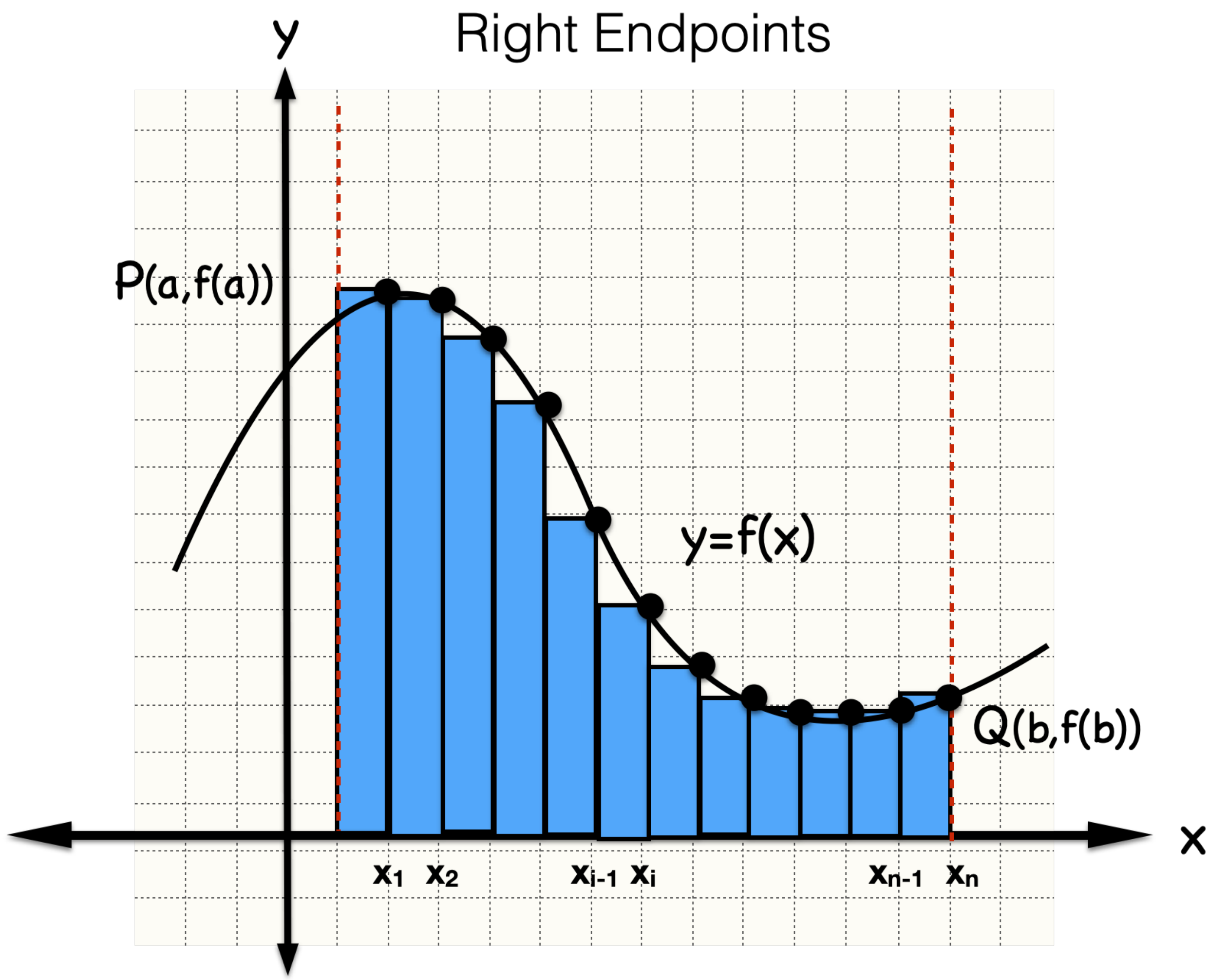
Numerical Approximation to Area Under a Curve



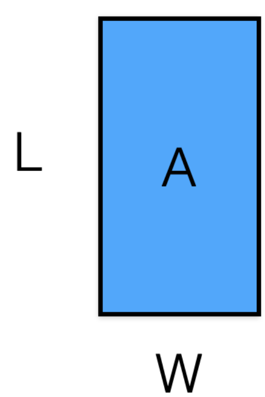
Partition the interval $[a, b]$ into n subintervals of equal length Δx where $\Delta x = \frac{b-a}{n}$



The purpose is to approximate the area under then curve with approximating rectangles.



Where $A = LW$ for each approximating Rectangle.



Specifically, $A \approx A_1 + A_2 + \cdots + A_n$

The Right Endpoint Formula

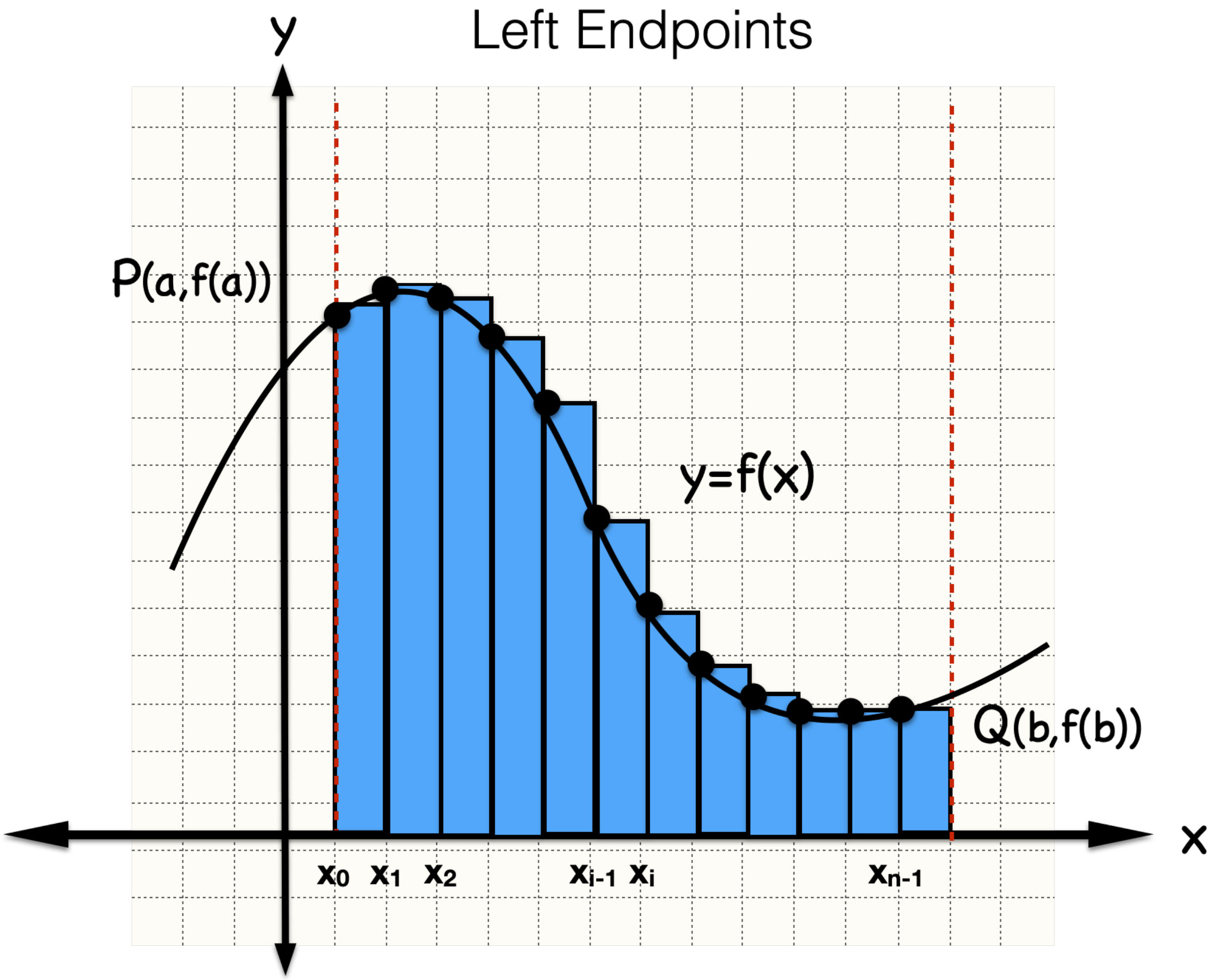
$$A \approx f(x_1)\Delta x + f(x_2)\Delta x + \cdots f(x_n)\Delta x$$

Or

$$A \approx \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$R_n \approx \sum_{i=1}^{n} f(x_i)\,\Delta x$$

The Left Endpoint Formula



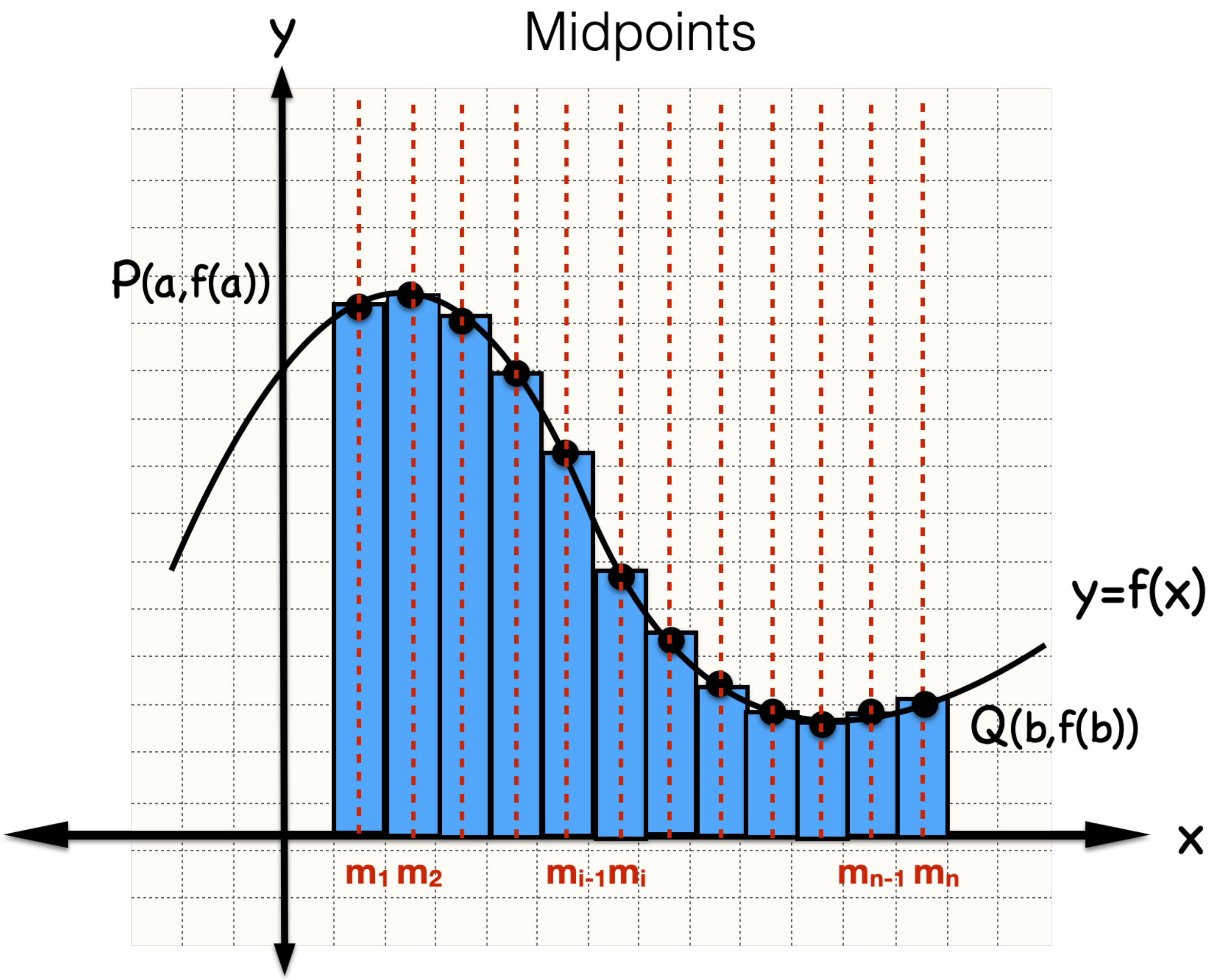
$$A \approx f(x_0)\Delta x + f(x_1)\Delta x + \cdots f(x_{n-1})\Delta x$$

Or

$$A \approx \sum_{i=0}^{n-1} f(x_i)\,\Delta x$$

$$L_n \approx \sum_{i=0}^{n-1} f(x_i)\,\Delta x$$

The Mid-Point Formula



$$A \approx f(m_1)\Delta x + f(m_2)\Delta x + \cdots f(m_n)\Delta x$$

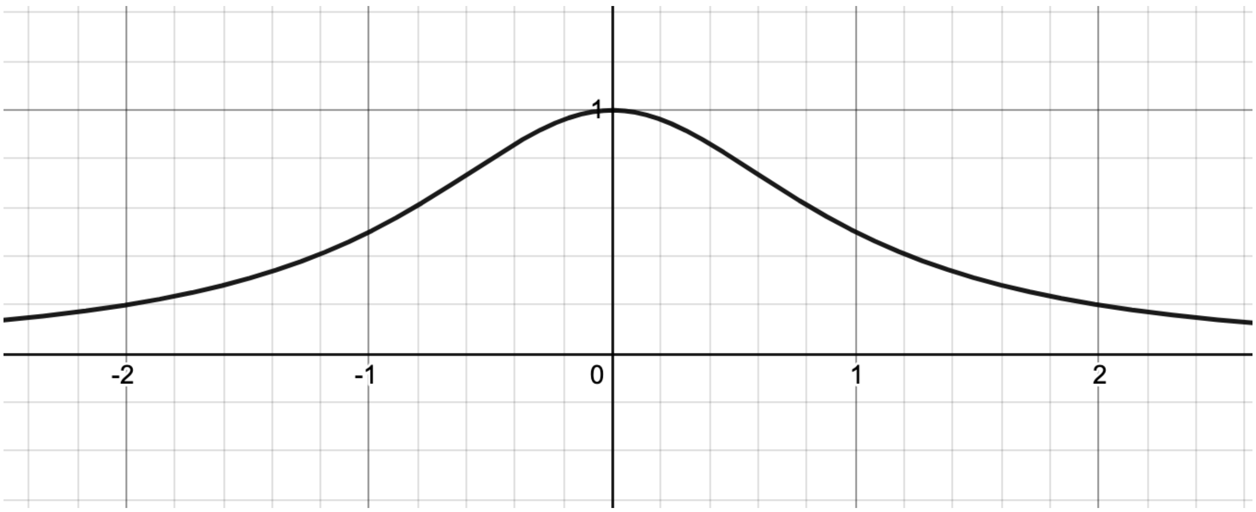
Or

$$A \approx \sum_{i=1}^{n} f(m_i)\,\Delta x$$

$$M_n \approx \sum_{i=1}^{n} f(m_i)\,\Delta x$$

Examples

$$f(x) = \frac{1}{1+x^2} \text{ over } -1 \leq x \leq 1$$



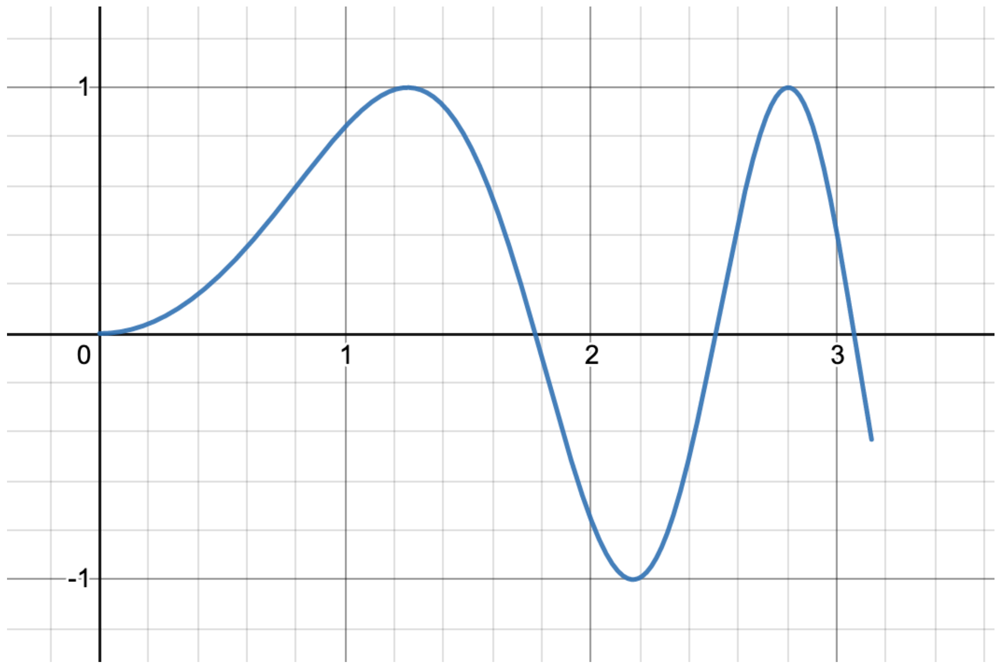
Determine the following approximations to the area over the interval $-1 \leq x \leq 1$

1. L_5

2. R_5

3. M_5

$f(x) = sin(x^2)$ over $0 \leq x \leq \pi$



Determine the following approximations to the area over the interval $0 \leq x \leq \pi$

4. L_5

5. R_5

6. M_5