Integration by Substitution

$$\int f[g(x)]\,dx = \int f(u)du$$

This is used when we have a composition of functions.

$$(fog)(x) = f[g(x)]$$

Procedure

Step 1: let $u = g(x)$ which is the "inside function" and is the domain of f

Step 2: Determine $\frac{du}{dx}$ by differentiating u with respect to x

That is, $\frac{du}{dx} = \frac{d}{dx}(u) = \frac{d}{dx}(g(x)) = g'(x)$

Step 3: Solve for dx

That is, $dx = \frac{du}{g'(x)}$

Step 4: Substitute $u = g(x)$ and $dx = \frac{du}{g'(x)}$ into the original integral to obtain a new equivalent integral in terms of u. $\int f[g(x)]\,dx = \int f(u)du$

Step 5: Integrate.

Integrate the following:

1. $\int \sqrt{x+5}dx$

2. $\int (x-5)^4 dx$

3. $\int (3x + 2)^{11} dx$

4. $\int sin(2x)dx$

5. $\int cos(\pi x)dx$

6. $\int sec^2\left(\frac{1}{4}x\right)dx$

7. $\int x\sqrt[3]{2x^2+5}dx$

8. $\int \frac{1}{(3x-7)^2}\,dx$

9. $\int sec(4\pi x)tan(4\pi x)dx$

10. $\int \frac{4}{\sqrt{x+3}}\,dx$

11. $\int \frac{x^2}{\sqrt[3]{2x^3+9}}\,dx$

12. $\int sin^2(x)\cos(x)\,dx$

13. $\int cos^3(x)\sin(x)\,dx$

14. $\int \sqrt{\sin(x)} \cos(x)\, dx$

15. $\int cos^{3/2}(x)\sin(x)\,dx$

16. $\int x(4 - x^2)^5 dx$

17. $\int 2x^3\sqrt{x^4 - 6}dx$

18. $\int 3x^4(x^5 + 3)^{12}dx$

19. $\int \frac{x+1}{(x^2+2x)^3}\,dx$

20. $\int \sqrt{x}cos\left(x\sqrt{x}\right)dx$

21. $\int x sin(x^2)dx$

22. $\int \frac{x^2}{(x+5)^4}\,dx$

23. $\int \frac{sec^2(\sqrt{x})}{\sqrt{x}}\,dx$

24. $\int x^5\sqrt{x^3+1}dx$

25. $\int \frac{x^2}{(x+5)^3}\,dx$

26. $\int 6x^2(4 - x^3)^4 dx$

27. $\int (x^5 + 4x^2)\,(x^3 + 1)^{12}dx$

28. $\int x^3 (x^2 - 1)^{3/2} dx$

Use a change of variables (substitution) to evaluate the following definite integrals.

29. $\int_1^6 \sqrt{x+3}\,dx$

30. $\int_0^1 \frac{x}{(x^2+1)^3}\,dx$

31. $\int_1^2 \frac{4x+12}{(x^2+6x+1)^2}\,dx$

32. $\int_{-\pi/2}^{\pi/2} \frac{\cos(x)}{\sqrt{\sin(x)+1}} dx$

33. $\int_{\pi/3}^{\pi/2} cot^2\left(\frac{x}{2}\right) csc^2\left(\frac{x}{2}\right) dx$

These are some interesting integrals you may want to try.

34. $\int_0^2 x\sqrt{5-\sqrt{4-x^2}}dx$

35. Determine values a and b such that $\int_a^b (u^2 + 1)du = \int_{-\pi/4}^{\pi/4} sec^4(\theta)d\theta$

36. Show that $\int_0^{\pi/6} f(\sin\theta)d\theta = \int_0^{1/2} f(u)\frac{1}{\sqrt{1-u^2}}du$