Integration by Partial Fraction Decomposition

$$\int_a^b R(x)dx$$

$R(x) = \frac{P(x)}{Q(x)}$ where P and Q are polynomial functions.

We use Partial Fraction Decomposition when integrating Rational functions. Essentially, we are writing a Rational function as sums and differences of ‘simpler” rational functions that can be integrated. Again, like Trigonometric Substitution we will need to consider cases.

Case 1- Linear Factor
The denominator $Q(x)$ is a product of distinct (different) linear $(ax + b)$ factors.

Such as

$$x(x+1) \ \ or\ (x-1)(x+2)\ or\ (2x-3)(4x+7)\ or\ x(x-3)(5x+2)$$

Case 2- Repeat Linear Factors
The denominator $Q(x)$ is a product of linear $(ax + b)$ factors, however some may be repeated (raised to a power).

Such as

$$(x-5)^2 \ \ or\ (3x+7)^3 \ \ or\ x(x+2)^2\ or\ (2x+1)^4(x-8)^5\ or\ x^2$$

Case 3- Quadratic Factors
The denominator $Q(x)$ is a product of distinct (different) quadratic $(ax^2 + bx + c)$ factors.

Such as

$$x^2(x^2+5)\ or\ (x^2+1)(2x^2-3)\ or\ (3x^2-5x+4)(x^2-9)\ or$$
$$(x^2-5)(x^2+x+8)$$

Case 4-Repeat Quadratic Factors
The denominator $Q(x)$ is a product of distinct (different) quadratic $(ax^2 + bx + c)$ factors, however some may be repeated (raised to a power).

Such as

$$(x+4)^2 \ \ or \ \ (x-2)^3 \ \ or \ (2x^2-3x+8)^4 \ \ or \ \ (x^2+5)(x^2-3)^2 \ \ or$$
$$x^2(3x^2-x+5)^3$$

Note- If $\deg(P) > \deg(Q)$, then divide.

Integration by Rationalizing Substitutions

$$\int_a^b R(x)dx$$

$R(x) = \frac{P(x)}{Q(x)}$ where P and Q are not necessarily polynomial functions.

We can convert non-Rational functions into Rational functions by an appropriate substitution. If we have an expression of the form $\sqrt[n]{g(x)}$ we can let $u = \sqrt[n]{g(x)}$ to accomplish this task.