① $\int x^{1/2}\,dx$; $\int \sqrt{x}\,dx$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$; $n+1 \neq 0$

$n \neq -1$

$F(x) + C$

$$\frac{x^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}} + C$$

$$\frac{x^{3/2}}{3/2} + C$$

$1 + \frac{1}{2}$ mixed number

$$\boxed{\frac{2}{3}x^{3/2} + C}$$

$$\frac{2}{3}x^{1+\frac{1}{2}} + C$$

$x \cdot x^{1/2}$

$x^{3/2}$

$= \sqrt{x^3}$

$= \sqrt{x^2 \cdot x}$

$= \sqrt{x^2}\sqrt{x}$

$x\sqrt{x}$

$$\boxed{\frac{2}{3}x\sqrt{x} + C}$$

(3) $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx$

$x^{1/2}$

$$\frac{x^{-\frac{1}{2}+\frac{2}{2}}}{-\frac{1}{2}+\frac{2}{2}} + C$$

$$\frac{x^{1/2}}{1/2} + C$$

$$\frac{\sqrt{x}}{1/2} + C \qquad \boxed{2\sqrt{x} + C}$$

(5) $\int (x^2+5)\,dx$

$\int (f(x) + g(x))\,dx$

$= \int f(x)\,dx + \int g(x)\,dx$

$\int x^2\,dx + \int 5\,dx$; note $g(x) = 5$

power Rule

$g(x) = 5x^0$

$2+1$

$5x^{0+1} + C$

$$\frac{x^{2+1}}{2+1} + c_1 + 5x + c_2$$

$\frac{x^{0+1}}{0+1}$

$5x + C$

$$\frac{1}{3}x^3 + c_1 + 5x + c_2$$

$c_1 + c_2$

C

$$\boxed{\frac{1}{3}x^3 + 5x + C}$$

⑦ $\int (x^4 - 7)\, dx$

$$\int \left(f(x) - g(x)\right) dx$$

$$= \int f(x)\,dx - \int g(x)\,dx$$

$\frac{x^{4+1}}{4+1} + c_1$ $\quad 7x + c_2$

$$\int x^4\,dx - \int 7\,dx$$

$$\int c\,dx = cx + C$$

$$\frac{1}{5}x^5 + c_1 - (7x + c_2)$$

$$\frac{1}{5}x^5 + c_1 - 7x - c_2$$

$C_1 - C_2$

C

$$\boxed{\frac{1}{5}x^5 - 7x + C}$$

(9) $\int \pi \cos(x)\,dx$

$$\int c\, f(x)\,dx = c \int f(x)\,dx$$

$$\pi \int \cos(x)\,dx \qquad \sin(x) + C$$

$$\pi\left[\sin(x) + C\right]$$

$$\int \underbrace{\cos(x)}_{f(x)}\,dx = \underbrace{\sin(x)}_{F(x)} + C$$

$$\frac{d}{dx}\big(\underbrace{\sin(x)}_{F}\big) = \underbrace{\cos(x)}_{f(x)}$$

$$\int \overbrace{\sin(x)}^{f}\,dx = -\overbrace{\cos(x)}^{F} + C$$

$$\frac{d}{dx}\big(\underbrace{-\cos(x)}_{F}\big) = -\frac{d}{dx}\big(\cos(x)\big)$$

$$= -(-\sin(x))$$

$$= \underset{f}{\sin(x)}$$

$$\int \underset{f}{\sec^2(x)}\,dx = \underset{F}{\tan(x)} + C$$

$$\frac{d}{dx}\Big(\underset{F}{\tan(x)}\Big) = \underset{f}{\sec^2(x)}$$

$$\int \underset{f}{\sec(x)\tan(x)}\,dx = \underset{F}{\sec(x)} + C$$

$$\frac{d}{dx}\Big(\underset{F}{\sec(x)}\Big) = \underset{f}{\sec(x)\tan(x)}$$

$$\int \underset{f}{\csc^2(x)}\,dx = \underset{F}{-\cot(x)} + C$$

$$\frac{d}{dx}\Big[\overset{F}{-\cot(x)}\Big] = -\frac{d}{dx}\big(\cot(x)\big)$$

$$= -\left(-\csc^2(x)\right) = \underset{f}{\csc^2(x)}$$

$$\int \underset{f}{\csc(x)\cot(x)}\,dx = \underset{F}{-\csc(x)} + C$$

$$\frac{d}{dx}\Big(\overset{F}{-\csc(x)}\Big) = -\frac{d}{dx}\big(\csc(x)\big)$$

$$= -\left(-\csc(x)\cot(x)\right) = \underset{f}{\csc(x)\cot(x)}$$

$\pi \sin(x) + \cancel{\pi C} \; C$

$$\boxed{\pi \sin(x) + C}$$

(11) $\int (3x^2 - 2x + 5)\,dx$

use the properties

$3\int \cancel{x^2\,dx}^{\frac{1}{3}x^3} - 2\int \cancel{x\,dx}^{\frac{1}{2}x^2} + 5\int \cancel{dx}^{x}$

$3\left(\frac{x^3}{3}\right) - 2\left(\frac{x^2}{2}\right) + 5x + C$

$$\boxed{x^3 - x^2 + 5x + C}$$

(13) $\int x(1 - \sqrt{x})\,dx$

$\int f(x)g(x)\,dx \neq \int f(x)\,dx \cdot \int g(x)\,dx$

$\int (x - x\sqrt{x})\,dx \; ; \; \int (x - x^{3/2})\,dx$

$x^1 \cdot x^{1/2}$

$x^{1+\frac{1}{2}}$

$x^{3/2}$

$$\cancel{\int x\,dx}^{\frac{1}{2}x^2} - \cancel{\int x^{3/2}\,dx} \qquad \frac{x^{3/2+1/2}}{3/2+1/2} + C$$

$$\frac{1}{2}x^2 - \left(\frac{x^{5/2}}{5/2} + C\right)$$

$$\frac{1}{2}x^2 - \frac{2}{5}x^{5/2} - C$$

$x^{5/2} \div \frac{5}{2}$ $\qquad + (\cancel{-C}) \to D$

$x^{5/2} \cdot \frac{2}{5}$ $\qquad 2 + \frac{1}{2}$

$$\frac{1}{2}x^2 - \frac{2}{5}x^{5/2} + D$$

$$\boxed{\frac{1}{2}x^2 - \frac{2}{5}x^2\sqrt{x} + C}$$

$x^{2+\frac{1}{2}}$

$x^2 \cdot x^{1/2}$

$x^2\sqrt{x}$

(15) $\int \sqrt{\frac{2}{x}}\,dx = \int \frac{\sqrt{2}}{\sqrt{x}}\,dx$

Recall $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $\qquad \sqrt{2}\int \frac{1}{\sqrt{x}}\,dx$

$x^{1/2}$

$$\sqrt{2}\int x^{-1/2}\,dx$$

$$\sqrt{2}\,\frac{x^{-\frac{1}{2}+\cancel{1}\frac{2}{2}}}{-\frac{1}{2}+\cancel{1}\frac{2}{2}} + C$$

$$\sqrt{2}\,\frac{x^{1/2}}{1/2} + C$$

$$\boxed{2\sqrt{2}\,\sqrt{x} + C}$$

(17) $\int \sqrt[3]{5x^2}\,dx$

Recall $\sqrt[n]{a\cdot b} = \sqrt[n]{a}\cdot\sqrt[n]{b}$

$\sqrt[3]{a\cdot b} = \sqrt[3]{a}\cdot\sqrt[3]{b}$

$\int \sqrt[3]{5}\cdot\sqrt[3]{x^2}\,dx$; $\sqrt[m]{x^n} = x^{n/m}$

$= \sqrt[3]{5}\int x^{2/3}\,dx$

$$\sqrt[3]{5}\left[\frac{x^{\frac{2}{3}+\frac{3}{3}}}{\frac{2}{3}+\frac{3}{3}}\right]+C$$

$$\sqrt[3]{5}\,\frac{x^{5/3}}{5/3}+C$$

$1\frac{2}{3}$ $\quad 1+\frac{2}{3}$

$$\sqrt[3]{5}\cdot\frac{3}{5}\,x^{5/3}+C$$

$$\frac{3}{5}\sqrt[3]{5}+x\cdot x^{2/3}+C$$

$$\boxed{\frac{3\sqrt[3]{5}}{5}+x\sqrt[3]{x^2}+C}$$

(19) $\int (3x+2)(x-5)\,dx$; $\int f(x)\,g(x)\,dx$

foil

$\neq \int f(x)\,dx \int g(x)\,dx$

$3x^2-15x+2x-10$

$3x^2-13x-10$

$$\int(3x^2-13x-10)\,dx$$

$$\int 3x^2\,dx-\int 13x\,dx-\int 10\,dx$$

$$3\int x^2\,dx - 13\int x\,dx - 10\int dx$$

$$\frac{1}{3}x^3 \qquad \frac{1}{2}x^2 \qquad x$$

$$3\cdot\frac{1}{3}x^3 - 13\cdot\frac{1}{2}x^2 - 10x + C$$

$$\boxed{x^3 - \frac{13}{2}x^2 - 10x + C}$$

(21) $\int\left(2x^3 - \frac{1}{x^2}\right)dx$

$$2x^3 - x^{-2}$$

$$\int\left(2x^3 - x^{-2}\right)dx$$

$$\int 2x^3\,dx - \int x^{-2}\,dx$$

$$2\int x^3\,dx - \int x^{-2}\,dx$$

$$\frac{x^{3+1}}{3+1} \qquad \frac{x^{-2+1}}{-2+1} \qquad + C$$

$$2\cdot\frac{x^4}{} - \frac{x^{-1}}{} + C$$

$4 \qquad -1$

$$\boxed{\frac{1}{2}x^4 + \frac{1}{x} + C}$$

(23) $\displaystyle\int \frac{3x+7}{\sqrt{x}}\,dx$

algebra

$$\frac{3x}{\sqrt{x}} + \frac{7}{\sqrt{x}}$$

$$\frac{3x^1}{x^{1/2}} + \frac{7}{x^{1/2}}$$

$$3x^{\frac{2}{2}-\frac{1}{2}} + 7x^{-1/2}$$

$$3x^{1/2} + 7x^{-1/2}$$

$$\int \left(3x^{1/2} + 7x^{-1/2}\right)dx$$

$$\int 3x^{1/2}\,dx + \int 7x^{-1/2}\,dx$$

$$3\int x^{1/2}\,dx + 7\int x^{-1/2}\,dx$$

$1 \quad 2 \qquad\qquad -1 \quad 2$

$$\frac{x^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+1} \qquad \frac{x^{-\frac{1}{2}+\frac{2}{2}}}{-\frac{1}{2}+1}$$

$$\frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2}$$

$$\frac{2}{3}x^{3/2} + \frac{2}{1}x^{1/2}$$

$$\cancel{3}\cdot\frac{2}{\cancel{3}}x^{3/2} + 7\cdot 2\cdot x^{1/2} + C$$

$$1\tfrac{1}{2}$$

$$2x^{3/2} + 14x^{1/2} + C$$

$$2x^{1}\cdot x^{1/2} + 14x^{1/2} + C$$

$$\boxed{2x\sqrt{x} + 14\sqrt{x} + C}$$

(25) $\int (1+x^2)^3\,dx$

base

algebra

$$(1+x^2)^3$$

$$(1+x^2)(1+x^2)(1+x^2)$$

foil

$$(1+x^2+x^2+x^4)(1+x^2)$$

$$(1+2x^2+x^4)(1+x^2)$$

$$1 + 2x^2 + x^4$$
$$x^2 + 2x^4 + x^6$$

$$1 + 3x^2 + 3x^4 + x^6$$

$$\int (1 + 3x^2 + 3x^4 + x^6)\,dx$$

$$\int dx + \int 3x^2\,dx + \int 3x^4\,dx + \int x^6\,dx$$

$$\int dx + 3\int x^2\,dx + 3\int x^4\,dx + \int x^6\,dx$$

$$x \qquad \frac{1}{3}x^3 \qquad \frac{1}{5}x^5 \qquad \frac{x^7}{7} \qquad +C$$

$$x + \cancel{3}\cdot\frac{1}{\cancel{3}}x^3 + \frac{3}{5}x^5 + \frac{1}{7}x^7 + C$$

$$\boxed{x + x^3 + \frac{3}{5}x^5 + \frac{1}{7}x^7 + C}$$

(27) $\int (3 + \sqrt[3]{x})^2\,dx$

algebra

$$(3 + \sqrt[3]{x})(3 + \sqrt[3]{x})$$

foil

$9 + 3\sqrt[3]{x} + 3\sqrt[3]{x} + (\sqrt[3]{x})^2$

$(x^{1/3})^2$

$x^{2/3}$

$9 + 6\sqrt[3]{x} + x^{2/3}$

$x^{1/3}$

$9 + 6x^{1/3} + x^{2/3}$

$\int (9 + 6x^{1/3} + x^{2/3})\,dx$

$\int 9\,dx + \int 6x^{1/3}\,dx + \int x^{2/3}\,dx$

$9\int dx + 6\int x^{1/3}\,dx + \int x^{2/3}\,dx$

x

$\dfrac{x^{\frac{1}{3}+\frac{3}{3}}}{\frac{1}{3}+\frac{3}{3}}$ $\qquad$ $\dfrac{x^{\frac{2}{3}+\frac{3}{3}}}{\frac{2}{3}+\frac{3}{3}}$

$\dfrac{x^{4/3}}{4/3}$ $\qquad$ $\dfrac{x^{5/3}}{5/3}$

$\frac{3}{4}x^{4/3}$ $\qquad$ $\frac{3}{5}x^{5/3}$

$$9x + \cancel{6}^{\,3} \cdot \frac{3}{\cancel{4}_{\,2}} x^{4/3} + \frac{3}{5} x^{5/3} + C$$

$$\quad 1\tfrac{1}{3} \qquad\qquad 1\tfrac{2}{3}$$

$$\boxed{9x + \frac{9}{2} x^{4/3} + \frac{3}{5} x^{5/3} + C}$$

$$\boxed{9x + \frac{9}{2} x\sqrt[3]{x} + \frac{3}{5} x\sqrt[3]{x^2} + C}$$

(29) $\int \left(x^5 + 2 - \frac{1}{x^8}\right) dx$

$$\int x^5\,dx + \int 2\,dx - \int \frac{1}{x^8}\,dx$$

$$\int x^5\,dx + 2\int x^1\,dx - \int x^{-8}\,dx$$

$$\frac{x^{5+1}}{5+1} \qquad \frac{x^{1+1}}{1+1} \qquad \frac{x^{-8+1}}{-8+1}$$

$$\frac{x^6}{6} \qquad \frac{x^2}{2} \qquad \frac{x^{-7}}{-7}$$

$$\frac{1}{6}x^6 + \cancel{2}\cdot\frac{x^2}{\cancel{2}} - \frac{x^{-7}}{-7} + C$$

$$\frac{1}{6}x^6 + x^2 + \frac{1}{\ldots} + C$$

$$\boxed{6 \quad \cdots \quad x^7 \quad \cdots}$$

(31) $\int (6\cos(x) + 2\sin(x))\,dx$

$$\int 6\cos(x)\,dx + \int 2\sin(x)\,dx$$

$$6\int \cos(x)\,dx + 2\int \sin(x)\,dx$$

$\sin(x)$ $\quad\quad$ $-\cos(x)$

$$\boxed{6\sin(x) - 2\cos(x) + C}$$

(33) $\int \frac{1+\cos^2(\theta)}{\cos^2(\theta)}\,d\theta$

algebra

$$\int \frac{f(x)}{g(x)}\,dx \neq \frac{\int f(x)\,dx}{\int g(x)\,dx}$$

✓ $\frac{1}{\cos^2(\theta)} + \frac{\cancel{\cos^2(\theta)}}{\cancel{\cos^2(\theta)}}$

✓ $\sec^2(\theta) + 1$

$$\int \left(\sec^2(\theta) + 1\right) d\theta$$

$$\int \sec^2(\theta)\, d\theta + \int d\theta$$

$$\tan(\theta) \qquad \theta \quad + C$$

$$\boxed{\tan(\theta) + \theta + C}$$

(35) $$\int 2\pi \sec(\theta)\tan(\theta)\, d\theta$$

$$2\pi \int \sec(\theta)\tan(\theta)\, d\theta$$

$$\sec(\theta) + C$$

$$2\pi\left(\sec(\theta) + C\right)$$

$$2\pi \sec(\theta) + 2\pi C$$

$$C \quad D$$

$$\boxed{2\pi \sec(\theta) + C}$$

(37) $\displaystyle\int \frac{\sin(\theta) + \sin(\theta)\tan^2(\theta)}{\sec^2(\theta)}\, d\theta$

algebra

$\sec^2(\theta)$

$$\frac{\sin(\theta)\left(\cancel{1+}\ \tan^2(\theta)\right)}{\sec^2(\theta)}$$

$$\frac{\sin(\theta)\cdot \cancel{\sec^2}(\theta)}{\cancel{\sec^2}(\theta)}$$

$\sin(\theta)$

$$\int \sin(\theta)\, d\theta$$

$$\boxed{-\cos(\theta) + C}$$